*Geophysica* (2022), 57 (1), 3–22

# On barotropic net water exchange applied to the Sound strait in the Baltic Sea

*Bertil Håkansson*

Corresponding mail address: `bertilhak50@gmail.com`

(Submitted: April 12, 2022; Accepted: October 10, 2022)

## *Abstract*

Knudsen *(1900) derived relations for water and salt when baroclinic water exchange dominates in a semi-enclosed basin. Here, are the corresponding relations for barotropic water exchange developed. With these relations at hand, the net volume and net salt exchange can be found as averages over a certain time, say a week, month, or year. These relations are applied to the barotropic water exchange in the Sound strait connecting the Baltic Sea with the Kattegat. The study covers the years 1977 to 2018. Volume in- and outflows are characterized by being of the same order of magnitude, whereas the average time for inflow and outflow is close to* 35 % *and* 65 %*, respectively. The average net volume flow is close to* 50 % *of the average runoff input to the Baltic Sea, suggesting the other* 50 % *of the runoff input is leaving through the Belt Sea strait.*

*The seasonal net volume flow through the Belt Sea strait is estimated by closing the Baltic Sea water budget. During summer, the Sound and the Belt Sea straits show different responses to changes in runoff and Baltic Sea volume. The net volume flow through the Belt Sea strait even change sign being directed inwards during June and July on average.*

*Keywords: Baltic Sea; water and salt exchange relations; barotropic flows; net volume flows*

## *1 Introduction*

The Baltic Sea is an elongated semi-enclosed sea with internal sills, subbasins and seasonally changing input of freshwater. The two entrance areas, Belt Sea strait and Sound strait, connecting Baltic Sea to North Sea, are both narrow and shallow. The Belt Sea and the Sound have sill depths of approximately 15 m and 5 m, respectively, while the length of the Belt Sea strait is about three times longer than the Sound strait.

The Baltic is on average fifty-five meters deep. A halocline between 60 to 80 meters depth, separates the surface from the bottom layer. Compared to the Baltic Sea volume, water exchange and freshwater input from the drainage area are small. Model investigations suggests that the Baltic turnover time is approximately 30 years (*Döös et al.*, 2004), indicating strong inertia to fast changes.

On the other hand, the water exchange has strong impacts on the deep-water density stratification in the sub-basins of the Baltic Proper. The inflowing saline water is spreading into the subbasins as bottom boundary currents or intrude into different layers of the halocline. These inflows of saline water are reach in oxygen and impact the chemistry and biology of the sub-basins deep-water.

Published by the Geophysical Society of Finland, Helsinki

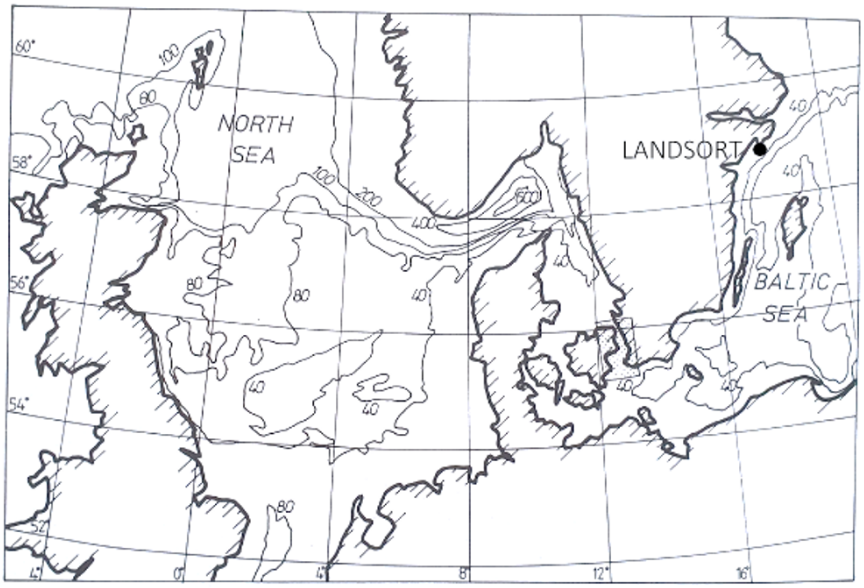


Fig. 1.1: Schematic overview of North Sea and Baltic Proper.

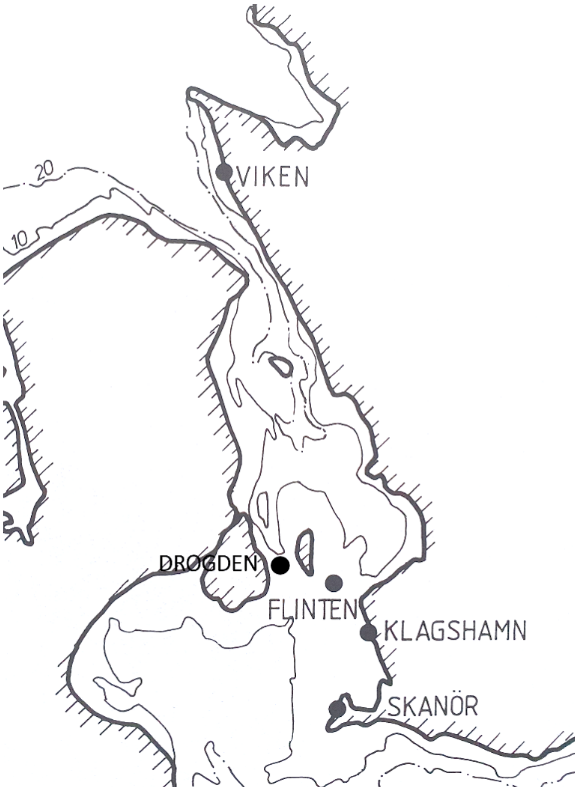


Fig. 1.2: Schematic overview of the Sound strait with sea level stations.

*Knudsen* (1900) established relations for exchange of water and salt under baroclinic flow conditions. J. *Jacobsen* (1925) introduced a formula to estimate barotropic transports through the Sound strait, using current meter recordings. The same barotropic formula was used by T. *Jacobsen* (1980) to find the exchange properties in the Sound and Belt Sea straits. The barotropic approach was followed by *Stigebrandt* (1983), *Omstedt* (1987) and *Gustavsson* (2004), estimating the barotropic transports in both straits using the sea level difference – the forcing function – between central Baltic Proper and Kattegat. Earlier attempts to describe the currents and salinities in the Belt and Sound straits is given by *Matthäus* (2006).

Several investigations have been published since then. For example, *Mohrholz* (2018) revised the time series of Major Baltic Inflows considering both baroclinic and barotropic flows, and concluded that baroclinic inflows mainly occur during summer while barotropic inflows occur during rest of the year. Baroclinic inflows have been studied by *Feistel et al.* (2006), investigating impacts of warm water inflows in the southern Baltic.

*Windsor et al.* (2001) pointed out the main forcing factors for the Baltic Sea hydrography, being water exchange, mixing in the entrance straits and freshwater input. *Gustavsson and* H. *Andersson* (2001) related the north-south air pressure over the North Sea as the driving force of barotropic exchange through the Belt Sea and Sound straits.

However, another key forcing factor is the seasonal variation of the sea level in the Baltic Sea, as mentioned by *Lass et al.* (1996). This variation causes an extra forcing factor for the water volume and salt mass exchange in the Sound and Belt Sea straits.

Freshwater input to the Baltic Sea is annually given by HELCOM Pollution Load Compilation. Earlier compilations of runoff are presented by *Mikulski* (1970) and *Bergström and Carlsson* (1994), who also gives the average seasonal variation of the runoff. Mixing processes in the entrance areas are however less well known from observations and measurements. *Gustavsson* (2004) suggests instead a salinity mixing model for the entrance areas.

This investigation is studying the Sound strait water exchange and in particular the net volume flow. A semi-empirical barotropic model is applied, driven by the local sea level slope between the northern and southern end of the Sound strait (cf. T. *Jacobsen*, 1980; *Håkansson et al.*, 1993; *Mattsson*, 1996; *Jakobsen et al.*, 1997; *Jakobsen et al.*, 2010).

The novelty of this study is that the net volume flow is calculated through the Sound strait. This is possible due to the volume and salt relations for barotropic water exchange defined in Section 2. The relations are of the same type as *Knudsen* (1900) relations derived by him for a two-layer baroclinic channel flow. However, for a barotropic flow, time enters as a specific parameter since flows move in both directions. Furthermore, the Belt Sea strait net volume flow is estimated, by closing the Baltic Sea annual water budget.

In Section 3 the barotropic model for the Sound strait is developed and in Section 4 observations on sea levels, being the data for the forcing function are presented. In addition, the Landsort sea level station data are examined, from which volume changes of the Baltic Sea is calculated to close the water budget. Runoff data from different sources is presented and shortly discussed. In Section 5 the computational results are presented and in Section 6 especially the seasonal variations of average net volume flows are examined. Section 7 summarizes and discusses the results.

## 2 *On water and salt relations for barotropic flows*

*Knudsen* (1900) developed the concept on volume and salt relations for a baroclinic two-layer strait flow, connecting a semi-closed basin with the open ocean. *Welander* (1974) added the diffusive salt fluxes in the conservation of salt. Here, the corresponding relations for barotropic flows are developed, including salt diffusion. Relations of volume and salt for a barotropic water exchange in a strait flow follow below.

Volume exchange during time $T$ reads:

$$V_{\text{in}} = \int_0^T Q_{\text{in}}(t)\ \mathrm{d}t = \overline{Q_{\text{in}}} n\Delta t$$

$$V_{\text{ut}} = \int_0^T Q_{\text{ut}}(t)\ \mathrm{d}t = \overline{Q_{\text{ut}}} m\Delta t$$

Here, $\overline{Q_{\text{in}}}$ and $\overline{Q_{\text{ut}}}$ are average values of inflows and outflows, respectively, during time $T$. Inflows and outflows take place during the fraction of time $n\Delta t$ and $m\Delta t$, respectively, where $\Delta t = T/(n+m)$.

The net volume flow becomes:

$$\text{Net volume flow}(\overline{Q_{\text{NVF}}}) = \overline{Q_{\text{in}}} \cdot \frac{n}{n+m} - \overline{Q_{\text{ut}}} \cdot \frac{m}{n+m} \tag{2.1}$$

For a single strait connecting a semi-enclosed basin with the open ocean the net volume flow is equal to the freshwater input. However, for a multi-strait semi-enclosed basin the net volume flow must be found for each strait.

Salt volume exchange over time $T$ reads:

$$V_{\text{in}} S_{\text{in}} = \int_0^T Q_{\text{in}}(t) S_{\text{in}}(t)\ \mathrm{d}t = \overline{Q_{\text{in}} S_{\text{in}}} n\Delta t$$

$$V_{\text{ut}} S_{\text{ut}} = \int_0^T Q_{\text{ut}}(t) S_{\text{ut}}(t)\ \mathrm{d}t = \overline{Q_{\text{ut}} S_{\text{ut}}} m\Delta t$$

The net salt flux becomes:

$$\text{Net salt flux}(\overline{QS_{\text{NSF}}}) = \overline{Q_{\text{in}} S_{\text{in}}} \cdot \frac{n}{n+m} - \overline{Q_{\text{ut}} S_{\text{ut}}} \cdot \frac{m}{n+m} \tag{2.2}$$

Salt and volume conservation over time $T$ yields

$$\overline{Q_{\text{ut}}} \cdot \frac{m}{n+m} = \overline{Q_{\text{in}}} \cdot \frac{n}{n+m} + \overline{Q_r} \tag{2.3}$$

$$\overline{Q_{\text{ut}} S_{\text{ut}}} \cdot \frac{m}{n+m} = \overline{Q_{\text{in}} S_{\text{in}}} \cdot \frac{n}{n+m}\ , \tag{2.4}$$

where $\overline{Q_r}$ is runoff. Including diffusive processes of salt, the above relations are slightly different, since $\overline{Q_{\text{ut}} S_{\text{ut}}} \neq \overline{Q_{\text{ut}}}\,\overline{S_{\text{ut}}}$.

Introduce two new constants $k_{\text{ut}}$, $k_{\text{in}}$ and let $Q = \overline{Q} + q'$ and $S = \overline{S} + s'$:

$$\overline{Q_{\mathrm{ut}}S_{\mathrm{ut}}} = k_{\mathrm{ut}}\overline{Q_{\mathrm{ut}}}\,\overline{S_{\mathrm{ut}}},\ \text{where } k_{\mathrm{ut}} = 1 + \overline{q'_{\mathrm{ut}}s'_{\mathrm{ut}}}/(\overline{Q_{\mathrm{ut}}}\cdot\overline{S_{\mathrm{ut}}})$$

$$\overline{Q_{\mathrm{in}}S_{\mathrm{in}}} = k_{\mathrm{in}}\overline{Q_{\mathrm{in}}}\,\overline{S_{\mathrm{in}}},\ \text{where } k_{\mathrm{in}} = 1 + \overline{q'_{\mathrm{in}}s'_{\mathrm{in}}}/(\overline{Q_{\mathrm{in}}}\cdot\overline{S_{\mathrm{in}}})$$

Insertion in Eq. (2.2) leads to:

$$\text{Net salt flux} = \left(k_{\mathrm{in}}\overline{Q}_{\mathrm{in}}\overline{S}_{\mathrm{in}}\cdot\frac{n}{n+m}\right) - \left(k_{\mathrm{ut}}\overline{Q}_{\mathrm{ut}}\overline{S}_{\mathrm{ut}}\cdot\frac{m}{n+m}\right) \tag{2.5}$$

For cases when salt flux is conserved and applying Knudsen observation that $\overline{S_{\mathrm{ut}}}/\overline{S_{\mathrm{in}}} = \frac{1}{2}$, a few examples are given below for a barotropic water exchange case, resulting in

$$\frac{\overline{Q_{\mathrm{ut}}}}{\overline{Q_{\mathrm{in}}}} = 2\left(\frac{n}{m}\right)\left(\frac{k_{\mathrm{in}}}{k_{\mathrm{ut}}}\right),\ \text{if } \left(\frac{k_{\mathrm{in}}}{k_{\mathrm{ut}}}\right) \approx 1\ .$$

Follows:

$$\overline{Q_{\mathrm{ut}}} = 2\cdot\frac{n+m}{m}\cdot\overline{Q_{\mathrm{r}}}$$

$$\overline{Q_{\mathrm{in}}} = \frac{n+m}{n}\cdot\overline{Q_{\mathrm{r}}}$$

For example, if the time of inflows and outflows are the same ($n = m$), $\overline{Q_{\mathrm{ut}}}$ and $\overline{Q_{\mathrm{in}}}$ equal $4\overline{Q_{\mathrm{r}}}$ and $2\overline{Q_{\mathrm{r}}}$, respectively. This means that the outflow will carry away not only the total freshwater volume but also the other half time of inflow volume, during half of the time available for outflows.

However, if the number of inflows is half of the outflows ($n = m/2$), $\overline{Q_{\mathrm{ut}}}$ equals $\overline{Q_{\mathrm{in}}}$ equals 3 times $\overline{Q_{\mathrm{r}}}$. The volume carried by in- and outflows are $\overline{Q_{\mathrm{in}}}\cdot n/(n+m) = \overline{Q_{\mathrm{r}}}$ and $\overline{Q_{\mathrm{ut}}}\cdot m/(n+m) = 2\overline{Q_{\mathrm{r}}}$, respectively. This case is like Knudsen relations applied to the Baltic Sea water and salt exchange, the difference being in the volume of in- and outflows, which for the barotropic exchange is 3 times the runoff while for the baroclinic case is twice and equal to the runoff for outflows and inflows, respectively.

Solving the semi-empirical barotropic model, developed below for the Sound strait, will reveal the actual numbers in the relations above.

## *3 On barotropic water exchange in the Sound strait*

The hydraulic equation predicts a balance of along strait pressure gradient and the bottom boundary layer (BBL), quantified with the quadratic friction parametrization. For a vertically integrated quasi-steady flow the Navier–Stokes equation is as follows:

$$U\frac{\mathrm{d}U}{\mathrm{d}x} = -g\frac{\mathrm{d}\eta}{\mathrm{d}x} - Bx\ , \tag{3.1}$$

where $Bx = C_d|U|U/H$.

Here $Bx$ is the BBL parametrization, $U$ is the longshore vertically integrated velocity, $g$ the gravitational constant, $C_d$ the drag coefficient of order $1\times10^{-3}$. The sea surface height is $\eta$ and the along strait distance is $x$.

Scaling Eq. (3.1) with $U = \mathcal{O}(U_o)$, $x = \mathcal{O}(L)$, $\eta = \mathcal{O}(H)$ and using typical values for the sill area $L \sim 10\,\mathrm{km}$, $H \sim 5\,\mathrm{m}$ and $U_o \sim 0.3\,\mathrm{m\,s^{-1}}$, the simplified Navier-Stokes equation above is physically valid if the aspect ratio for shallow water and the Froude number are much less than one, i.e., $H/L \ll 1$ and $U_o^2/gH \ll 1$. Hence, the above conditions are met, applying the above scaling numbers $U_o^2/gH \sim 0.002$ and $H/L \sim 0.0005$. From scaling it also follows that the frictional force is of $\mathcal{O}(1)$: $H/(C_d L) \sim 0.5$.

The along strait pressure gradient balances the BBL frictional force. In other words, the shallow water phase velocity is much larger than the current speed. It was shown by *Pratt* (1986) that most straits dominated by frictional bottom boundary layers have values between $0.7 < C_d L/H < 2.0$.

The corresponding value of the Sound sill is at the higher end of these values. Whereas the Pratt number for the Darss sill are at the lower end, using the same scaling procedure but consider the greater depth of the sill (maximum depth 18 meters but here used 15 m to account for the average sill width (*Gustavsson*, 1997).

For the Sound sill overflow, the following hydraulic formula is obtained after integrating the velocity over the width ($W$) of the sill and the depth ($H$):

$$Q = K^{-1/2} \cdot \frac{\Delta\eta}{\sqrt{|\Delta\eta|}} \tag{3.2}$$

$$K^{-1/2} = \left(\frac{gH}{C_d L}\right)^{1/2} \cdot \mathrm{area} \tag{3.3}$$

This balance of forces was first formulated by *Manning* (1891) for pipe and channel flows. However, he formulated it as $Q|Q| = K\Delta\eta$ where $K$ is referred to as the *specific resistance* (*Jakobsen et al.*, 1997). $\Delta\eta$ is the sea level difference across the sill. Scaling for the Sound sill gives:

$$K^{-1/2} \approx 1 \times 10^5\,\mathrm{m^{5/2}\,s^{-1}} \tag{3.4}$$

where the area ($W \times H = 100\,000\,\mathrm{m^2}$) is given by T. *Jacobsen* (1980). *Jakobsen et al.* (1997) studied in detail the values of specific resistance on the Drogden and Flinten sill and found it to vary between $190 \times 10^{-12}\,\mathrm{s^2\,m^{-5}}$ to $260 \times 10^{-12}\,\mathrm{s^2\,m^{-5}}$. In this investigation the average value, based on their minimum and maximum, is applied.

The barotropic forcing function is for practical reasons given by $\Delta\eta/\sqrt{|\Delta\eta|}$ to account for volume flows in both directions, yielding:

$$Q = K^{-1/2}\frac{\Delta\eta}{\sqrt{|\Delta\eta|}} \Rightarrow K^{-1/2} = 67 \times 10^3\,\mathrm{m^{5/2}\,s^{-1}} \tag{3.5}$$

Applying the semi-empirical model to a measured forcing function it will be possible to evaluate the barotropic water exchange relations developed in Section 2.

## 4 *Observation data on sea levels and runoff*

### 4.1 *Sound strait sea level data*

Sea level data are distributed by the Swedish Meteorological and Hydrological Institute (`smhi.se`) in different forms. Data are levelled in the RH 2000 system (SMHI Oceanographic data centre). However, these sea level measurements do include both land and sea level rise. Land rise is due to rise of land after the last glaciation period and sea level rise is due to climate warming, causing thermal expansion of sea water and melt of land-based glaciers. According to *Church and White* (2011), a global approach is necessary to get estimates of sea level rise and its acceleration over time, using tide gauges. On a global scale sea level rise was found to be $(3.15 \pm 0.30)\,\mathrm{mm\,yr^{-1}}$ between 1993 and 2018 (*World Meteorological Organization*, 2018). Due to the ocean circulation variability, it is difficult to distinguish between sea level rise and land rise using single tide gauge station data. In this study detrending of land and sea level rise is only used to normalize different tide gauge station data.

The along strait barotropic pressure gradient is estimated with sea level measurements from three stations located north and south of the Drogden and Flinten sill. North of the sill, at the opening towards the Kattegat area, the tide gauge station Viken is located. South of the sill, at the entrance to the Arkona Sea in the Baltic, two tide gauge stations are located, Klagshamn and Skanör (Figure 1.1). The former used together with Viken to estimate sea level gradients, the forcing function, and the latter for correction purposes. The sea level data is given in (cm) and sampled hourly.

The correlation between the tide gauge stations Viken and Klagshamn, using Pearson correlation coefficient, was found to be 0.18, which is classified as very low. Hence, the forcing function, being the difference between these two sea levels, is considered a random variable with no causal connection. On the other hand, the Klagshamn and Skanör sea levels, both located south of the Sound sill and within a short distance, show high correlation of 0.95 (Table 4.1).

*Samuelsson and Stigebrandt* (1996) and M. *Andersson and Johansson* (2018) have shown that the sea levels at Klagshamn under-estimate sea levels during inflow conditions

Table 4.1: Statistics of sea level data.

| **Tide Gauge Station** Name | **Period** (Year) | **Linear Trend Slope** (mm/yr) | **Mean Sea Level** (m) | **Coefficient of Variation** | **Pearson Correlation Coefficient** |
|---|---|---|---|---|---|
| Viken | 1977–2018 | 0.91 | 6.26 | 3.23 | |
| Klagshamn | 1977–2018 | 1.70 | 12.21 | 1.51 | |
| Skanör | 1992–2018 | 3.74 | 15.23 | 1.38 | |
| Landsort | 1900–2018 | −2.96 | 5.90 | 3.20 | |
| Klagshamn–Viken | 1977–2018 | | 5.90 | | 0.18 |
| Klagshamn–Skanör | 1992–2018 | | | | 0.95 |

when compared to the Skanör tide gauge data. Since Klagshamn is the longest and Skanör the shortest record of the three it was necessary to compensate for Klagshamn underestimation of sea levels during inflow conditions, to obtain long enough data series.

This correction assumes the sea surface slope is constant along the strait:

$$\left(Z(t)_{\text{viken}} - Z'(t)_{\text{Klagshamn}}\right)/L_{\text{VK}} = \left(Z(t)_{\text{viken}} - Z(t)_{\text{Skanör}}\right)/L_{\text{VS}}$$

where $L_{\text{VS}}$ is the distance between Viken and Skanör and $L_{\text{VK}}$ the distance between Viken and Klagshamn estimated to 83 km and 72 km, respectively. $Z'(t)_{\text{Klagshamn}}$ is the corrected sea level at Klagshamn. This correction is calculated using:

$$Z'(t)_{\text{Klagshamn}} = Z(t)_{\text{Klagshamn}} - \varepsilon \cdot \left(Z(t)_{\text{viken}} - Z(t)_{\text{Klagshamn}}\right), \text{ if } \left(Z(t)_{\text{viken}} - Z(t)_{\text{Klagshamn}}\right) < 0$$

In a nudging process $\varepsilon = 0.09$ was found to maximise the Pearson correlation coefficient of the above penalty function during inflows. In Figure 4.1 the results are presented with correlation coefficient.

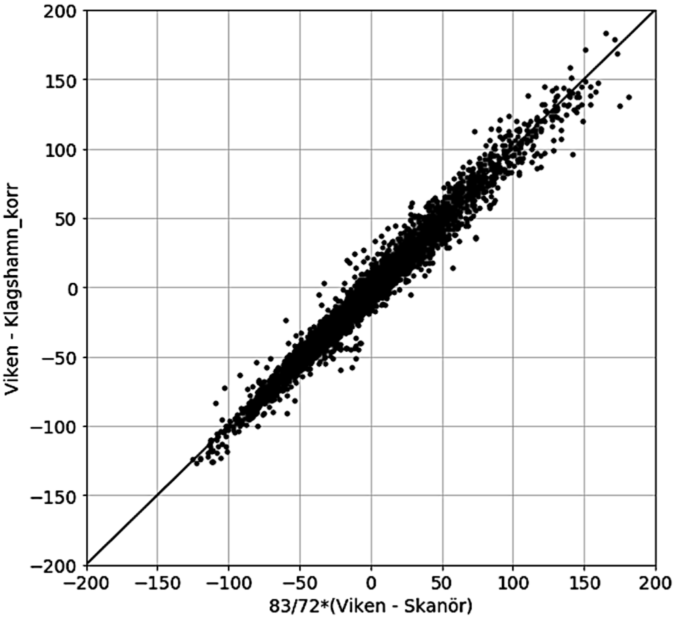


Fig. 4.1: Optimized correspondence between sea level slopes of Viken–Klagshamn and Viken-–Skanör, gives a Pearson correlation coefficient of 0.98.

The freshwater input to the Baltic increases the surface height above the Kattegat surface until a balance is achieved between input and output of water. Since linear detrending also remove the mean sea level, it is added to the detrended time series. The average value between Klagshamn and Viken sea level stations for the period 1977 to 2018 is 5.9 cm (Table 4.1).

## *4.2 Landsort sea level data*

Landsort sea level data sampled hourly for the period 1900 to 2018 was also linearly de-trended. This procedure removes the mean value of Landsort station data. As found for the Sound, the average mean difference between southern Baltic and Kattegat was 5.9 cm, the same mean value was added to the Landsort detrended data.

This coastal station represents Baltic Proper sea-levels rather well. In this study, the average seasonal variability is used to close the Baltic water budget. To achieve the seasonal variability of monthly resolution, hourly Landsort sea levels were monthly averaged for the common period (1977–2018). The results are shown in Figure 4.2, in which also the monthly runoff data are plotted. The sea level magnitude spans about 20 cm during a year. The lowest sea level corresponds with the maximum monthly runoff in May. Whereafter, the sea level rises again to slightly above annual mean and with minor variability.

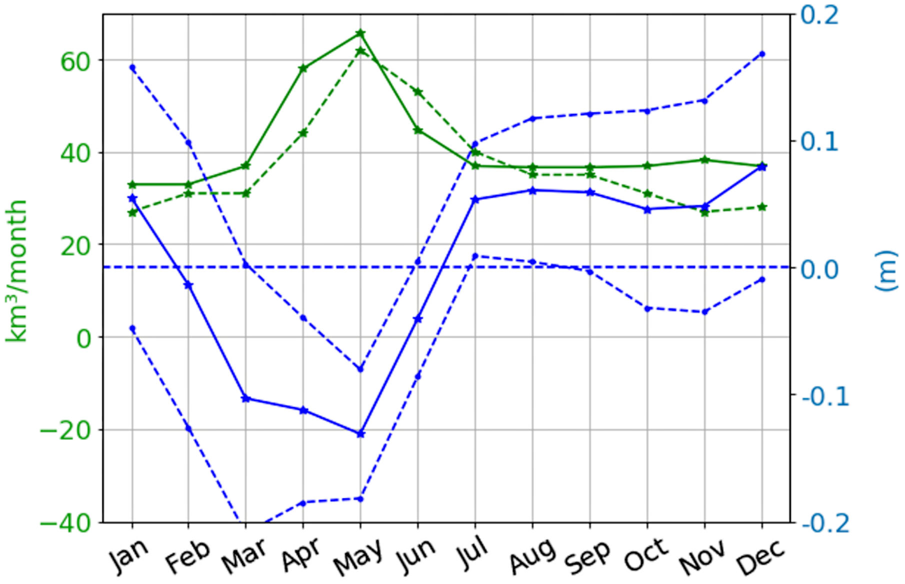


Fig. 4.2: Monthly mean runoff Bergström & Carlsson (solid green), runoff Mikulski (dotted green) and Baltic Sea monthly mean sea level (solid blue) with 96 % confidence intervals (doted blue).

This annual change was described already by *Svansson* (1975), for the single year 1964. *Stramska et al.* (2013) presented similar results using satellite altimetry data and found a magnitude of 18 cm.

### 4.3 *Freshwater input to the Baltic Sea*

Freshwater input to the Baltic Sea include both runoff and net precipitation. The latter is estimated to be around 10 % of the former, according to *Rutgersson et al.* (2002). In the present study, only runoff is considered.

Annual runoff is published in HELCOM Pollution Load Compilation, covering the period 1995 up to 2017. A longer time-period is calculated by the Baltic Nest Institute at Stockholm University, covering the period 1977 up to 2017. For the period in common, these time series show very small differences with average values of 15 713 $m^3 s^{-1}$ and 15 588 $m^3 s^{-1}$. Hence, the longer time series is used further on in this study.

Monthly time series of runoff is estimated by *Mikulski* (1970) and *Bergström and Carlsson* (1994), covering the periods 1951–1960 and 1970–1990. The latter data are used to close the Baltic water budget (see Section 6).

## *5 Results*

### *5.1 Average conditions on volume flows and sensitivity tests*

The histogram of volume flows is presented in Figure 5.1. The tails are covering the extreme inflow and outflow transport events. The extreme inflows occur with higher transports compared to the extreme outflows. Whereas the average volume flows, rounded to integers, show an inward flowing transport ($\overline{Q_{\text{in}}} = 25\,665\,\text{m}^3\,\text{s}^{-1}$) slightly lower than outflowing transport ($\overline{Q_{\text{ut}}} = -26\,345\,\text{m}^3\,\text{s}^{-1}$). On average, outflows occur during 65 % and inflows 35 % of the time. With these data at hand, it is possible to find the average net volume flow through the Sound strait (cf. Eq. (2.1)), being $\overline{Q_{\text{NVF}}} = -8148\,\text{m}^3\,\text{s}^{-1}$.

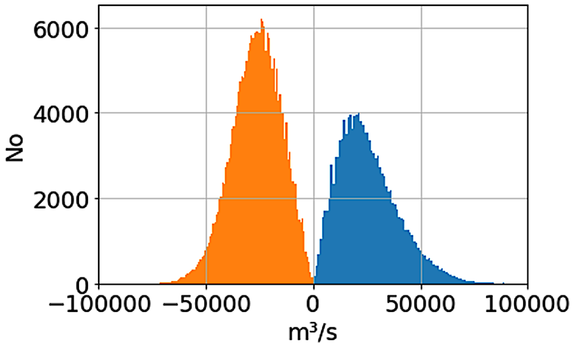


Fig. 5.1: Histogram of barotropic volume outflow (negative) and inflow (positive).

The Sound strait barotropic flow export capacity (cf. Table 5.1), measured as net volume flow, is close to 50 % of the annual average runoff. Hence, the other 50 % must be exported through the Belt Sea strait (Table 5.1).

Table 5.1: Statistics of volume flows for the hourly time series from 1977 to 2018. All numbers rounded to integers, whereas the mean net volume flow is calculated from Eq. (2.1) before rounding to integers.

| | | |
|---|---|---|
| No of hours | 365 598 | |
| Mean Inflow | 25 665 | $\text{m}^3/\text{s}$ |
| Mean Outflow | −26 345 | $\text{m}^3/\text{s}$ |
| Mean Inflow time | 35 | % |
| Mean Outflow time | 65 | % |
| Net Volume Flow | −8148 | $\text{m}^3/\text{s}$ |
| Mean Runoff | 15 900 | $\text{m}^3/\text{s}$ |

What processes determine the volume flows and their respective time fractions? Model runs were made, to investigate the sensitivity of net volume flows, in- and outflow time and in- and outflow volume flows on different mean sea level differences. In Figure 5.2 the results are presented. The variable ($\overline{Q_{\text{ut}}}/\overline{Q_{\text{in}}}$) shows a weak dependence on mean sea level difference, while the time share variable ($n/m$) show a larger dependence. The most sensitive variable is the net volume flow $Q_{\text{NVF}}/\overline{Q_{\text{NVF at } -5.9}}$, varying from zero to above 2.5 for sea level differences from zero the −20 cm.

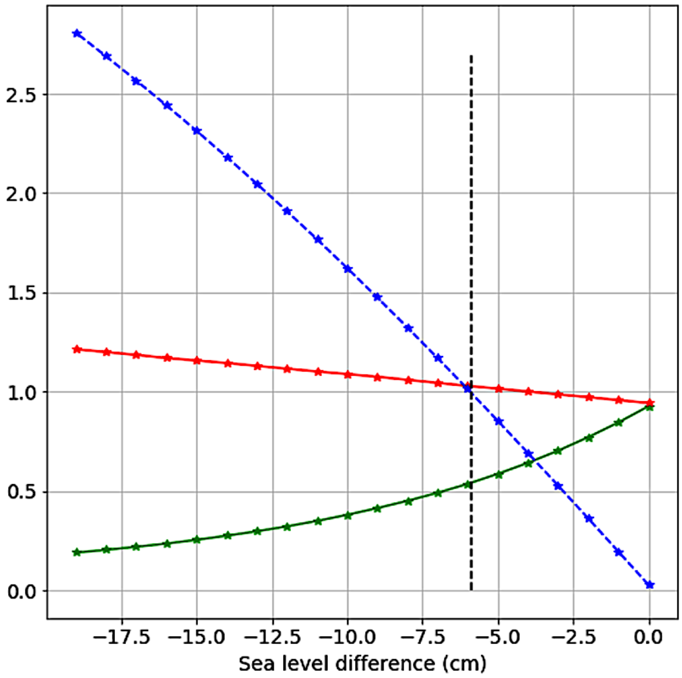


Fig. 5.2: Sensitivity of Inflow/Outflow time ($= (\overline{n}/\overline{m})$ – solid green), Inflow/Outflow volumes ($= (\overline{Q_{\mathrm{ut}}}/\overline{Q_{\mathrm{in}}})$ – in solid red) and net volume flow/mean net volume flow ($= Q_{\mathrm{NVF}}/\overline{Q_{\mathrm{NVF\ at\ -5.9}}}$ – in dotted blue) as function of mean sea level difference between Kattegat and Baltic Sea. The dotted black line shows the present case of $-5.9$ cm average sea level difference.

Hence, it appears that two processes are involved. The weak dependence of $(\overline{Q_{\mathrm{ut}}}/\overline{Q_{\mathrm{in}}})$ is a result of a random process of the forcing function, indicating these, ought to be of equal magnitude. This forces the other process, the mean time share $(\overline{n}/\overline{m})$ to adapt to the mean sea level difference, which in turn is governed by the average total runoff input.

It can also be concluded from the sensitivity evaluation, that if the Sound strait was the only connection between the Kattegat and the Baltic. The total runoff must leave through this strait. In this very special case keeping all other variables unchanged, the mean sea level difference will increase to about $-13$ cm from the present $-5.9$ cm.

## 5.2 *Annual volume flows and runoff*

To match data on annual runoff input to the Baltic, annual net volume flows were calculated from the hourly data series. The results are presented in Figure 5.3, where annual runoff and net volume flows are shown. Correlation of these two variables ($R^2 = -0.21$) indicate they are uncorrelated. This rather unexpected result should have impacts on the volume exchange through the Belt Sea strait, as is estimated and evaluated in Section 6.

## 5.3 *Monthly Volume flows and Major Baltic Volume flows*

The choice of monthly mean statistics for the averaging process is based on time series of volume inflow and outflow. The average monthly time fraction for inflows is 35 % with a standard deviation of ±10 %. Hence, for volume inflows making impact

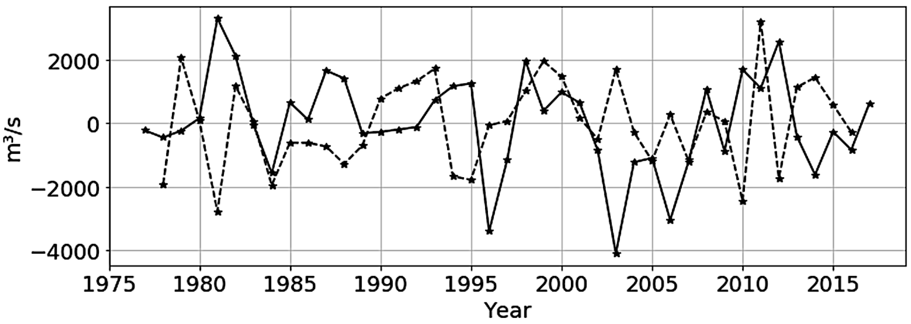
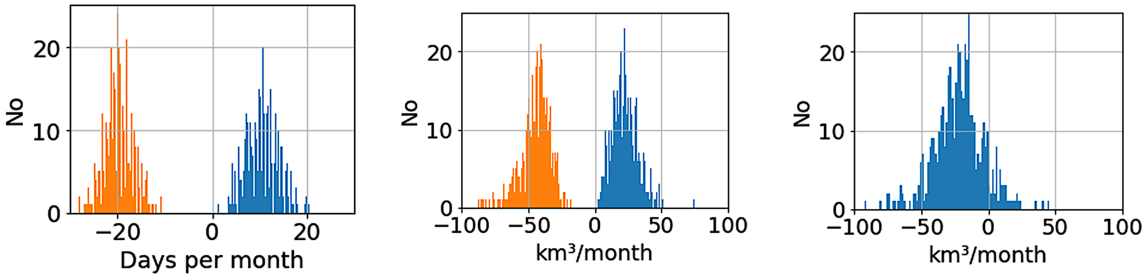


Fig. 5.3: Annual means of net volume flow (solid black) and annual runoff (dotted black) deviation from average.



(a) Monthly mean histogram of relative inflow time (positive) and outflow time (negative).

(b) Monthly mean histogram of volume inflow (positive) and volume outflow (negative).

(c) Monthly mean histogram of net volume flow ($Q_{\mathrm{NVF}}$ through the Sound strait).

Fig. 5.4

on deep-water properties in the Baltic Sea, an averaging period of one month should be enough.

Major Baltic in- and outflows are generally characterized by substantially long in- or outflow times and high transport rates. If these variables coincide there is a major Baltic flow. These volume flows are represented by the histogram tails in Figures 5.4a and 5.4b. Major Baltic Inflows are also characterized by high saline waters, causing impacts on deep-water salinity and oxygen content in the Baltic. For volume flows these can be traced to impacts, but it is better to involve the salt mass flux from these volume flows, which will be delt with in Part 2.

The threshold for major inflows and outflows is here arbitrarily given by monthly volume flows larger than or equal to the monthly mean plus/minus one standard deviation. In Table 5.2 statistics of the major volume in- and outflows through the Sound strait are presented. Major inflows are more frequent during the 1990th than during other decades, while Major outflows are evenly distributed between decades. The monthly distribution of major flows is concentrated to late autumn, winter, and early spring, while absent during summer (Figure 5.5).

### 5.4 *Monthly mean net volume flow*

The net volume flow, given by Eq. (2.1), is applied to the Sound strait as monthly averages. Corresponding histogram and time series is presented in Figure 5.4c and in

Table 5.2: Statistics of major volume in- and outflows through the Sound during 1978 to 2018.

| Major Volume Inflows | | | |
|---|---|---|---|
| Years | No | Days | $m^3/s$ |
| 1980–1989 | 12 | 14.34 | 15 108 |
| 1990–1999 | 23 | 15.25 | 16511 |
| 2000–2009 | 16 | 14.95 | 14574 |
| 2010–2018 | 16 | 15.25 | 15 462 |

| Major Volume Outflows | | | |
|---|---|---|---|
| Years | No | Days | $m^3/s$ |
| 1980–1989 | 25 | 23.2 | −24 014 |
| 1990–1999 | 24 | 23.2 | −23 582 |
| 2000–2009 | 18 | 22.9 | −24 076 |
| 2010–2018 | 25 | 23.2 | −23 380 |

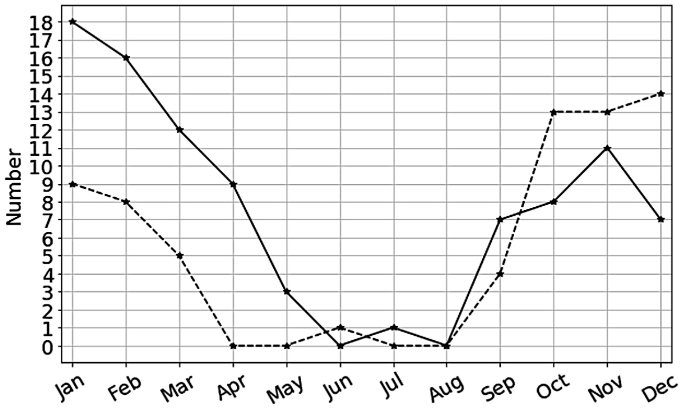


Fig. 5.5: Monthly distribution of Sound strait Major in- and outflows during the period 1978–2018, inflows (dotted), outflows (solid). Thresholds are set by flows larger than mean ±1 std.

Figure 5.6. Since the monthly mean transport rates $\overline{Q_{\mathrm{in}}}$ and $\overline{Q_{\mathrm{ut}}}$ is of similar magnitude it is the time share (average inflow: 35 % and outflow: 65 %) that differentiate the net volume transport capacities. This is close to the last of two theoretical options presented in Section 2.

The monthly mean net volume flow during a standard year is shown in Figure 5.7. The largest net volume outflow occurs during November to March, whereafter it is slightly weaker. However, the variability for each standard month is large but is at minimum in June, July and August.

Since the forcing function was classified as a random variable it might be assumed the net volume flow is stochastic variable. Therefore, the variable was tested whether the mean, variance and autocorrelation function are independent of time. In Table 5.3, these parameters are presented. All parameters show similar results for all four 10 years period. The autocorrelation function (Figure 5.8), the mean and variance independency of time, indicate the net volume flow is a stochastic variable.

Table 5.3: Mean and variance of net volume flow during four periods of the total time series.

| **Period** | **Mean ($m^3/s$)** | **Variance ($(m^3/s)^2$)** |
|---|---|---|
| 1978–1988 | −8586 | 45 706 936 |
| 1988–1998 | −8133 | 47 575 896 |
| 1998–2008 | −7754 | 45 018 821 |
| 2008–2018 | −8177 | 55 078 045 |

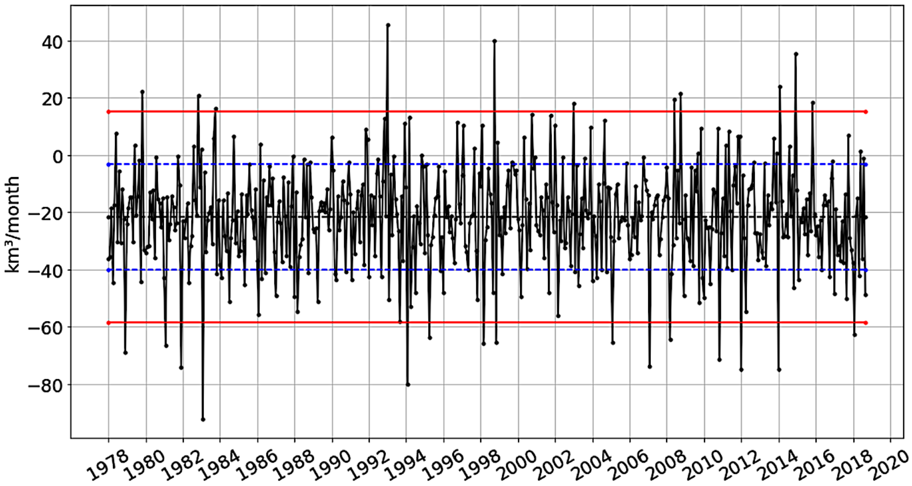


Fig. 5.6: Time series of monthly mean net volume flows, covering 489 months. Dotted and solid line represent one and two standard deviations, respectively.

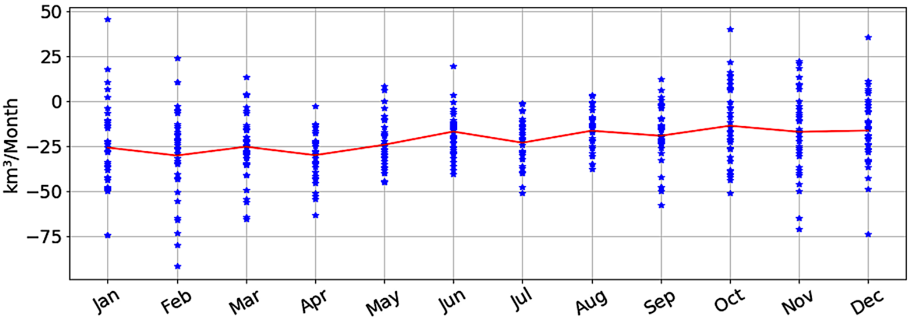


Fig. 5.7: Monthly scatter plot with monthly mean distribution of net volume flows.

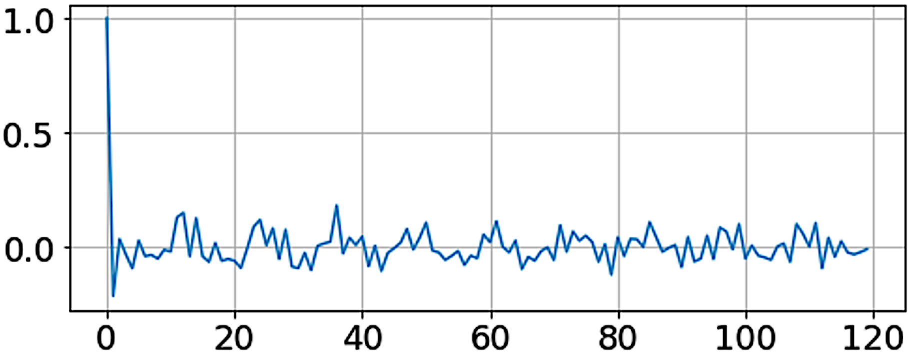


Fig. 5.8: Autocorrelation function of monthly net volume flows.

## 6 *Mean seasonal net volume flows in the Sound and the Belt Sea*

Monthly mean net volume flows during a statistical mean year are calculated from the monthly mean barotropic volume flow through the Sound strait. Monthly mean runoff inflow to the Baltic is given by *Bergström and Carlsson* (1994). While monthly mean Baltic volume changes is estimated using Landsort station sea level data. The water volume budget for the Baltic Sea reads:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \mathrm{Sound}_{\mathrm{NVF}}(t) + \mathrm{Belt\ Sea}_{\mathrm{NVF}}(t) + \mathrm{RO}(t) \tag{6.1}$$

Where $\mathrm{RO}(t)$ is the monthly mean runoff input to the Baltic, $\mathrm{d}V/\mathrm{d}t$ the monthly mean volume changes in the Baltic and the $\mathrm{Sound}_{\mathrm{NVF}}(t)$ and $\mathrm{Belt\ Sea}_{\mathrm{NVF}}(t)$ are the average of the monthly mean net volume transports through respective strait. Here $(t)$ is the month from January to December, covering an average year of mean monthly data. The basic data used for the estimation of the Baltic Sea water budget is presented in Table 6.1. An estimate of the $\mathrm{Belt\ Sea}_{\mathrm{NVF}}(t)$ is obtained as a rest product of Eq. (6.1). The time derivative of the Baltic volume changes is solved by applying the Euler method with a two-step technique. The average annual monthly mean net volume flow through the Sound and the Belt Sea straits is presented in Table 6.2 and Figure 6.1.

Table 6.1: Input data for scaling purposes and estimates of Baltic Sea volume changes.

| | | |
|---|---:|---|
| Baltic Sea area | 377 000 | km$^2$ |
| Baltic Sea volume | 21 700 | km$^3$ |
| Baltic Sea mean depth | 57 | m |
| Sound sill mean depth | 5 | m |
| Belt Sea sill mean depth | 15 | m |
| Sound strait length | 83 | km |
| Belt Sea strait length | 249 | km |

The seasonal net volume outflow is largest during January to May and almost of equal magnitude in the two straits. This occurs due to decreasing sea level in the Baltic Sea and increasing runoff. Sea level rises again in June, July and August (cf. Figure 4.2), whereafter it is constant rest of the year. The net outflows through the two straits are of equal magnitude and less than the annual mean. The most striking result is that net volume flow in the Belt Sea strait is reversing to net volume inflow during June and July, when the sea level rises in the Baltic Sea and runoff is decreasing.

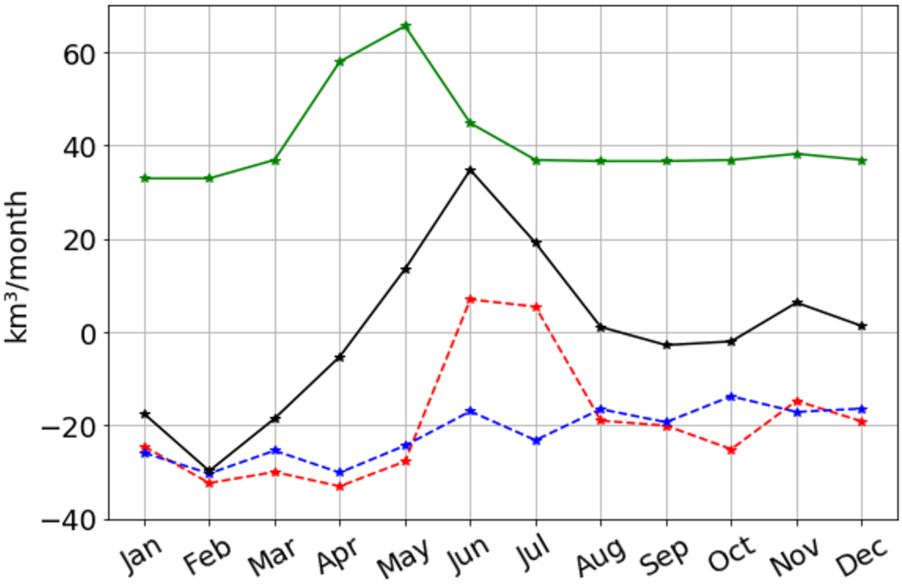


Fig. 6.1: Seasonal variation of monthly mean net volume inflow through the Sound (dotted blue), runoff Bergström & Karlsson (solid green), Baltic volume changes (solid black) and estimated monthly net volume flow through the Belt Sea strait (dotted red).

Table 6.2: Seasonal distribution of net volume flow through the Sound and Belt Sea straits during an average year. The monthly mean covers the period 1978 to 2018.

| **Month** | **NVF Sound Strait** km$^3$/month | **NVF Belt Sea strait** km$^3$/month | **Runoff** km$^3$/month | **Baltic** km$^3$/month |
|---|---|---|---|---|
| Jan | −25.9 | −24.5 | 32.9 | −17.5 |
| Feb | −30.4 | −32.4 | 32.9 | −29.8 |
| Mar | −25.4 | −30 | 36.9 | −18.6 |
| Apr | −30.2 | −33.1 | 58 | −5.3 |
| May | −24.4 | −27.8 | 65.6 | 13.5 |
| Jun | −17 | 7 | 44.8 | 34.8 |
| Jul | −23.2 | 5.4 | 36.9 | 19.1 |
| Aug | −16.5 | −19 | 36.6 | 1.1 |
| Sep | −19.3 | −20.1 | 36.6 | −2.8 |
| Oct | −13.8 | −25.1 | 36.9 | −2 |
| Nov | −17.1 | −14.8 | 38.2 | 6.3 |
| Dec | −16.4 | −19.2 | 36.9 | 1.3 |
| Mean | −21.6 | −19.5 | 41.1 | 0.0 |
| Sum | −259.6 | −233.6 | 493.2 | 0.1 |

## 7 *Summary and discussion*

The volume and salt transport for a barotropic water exchange through a strait is calculated to achieve net volume transports. This approach is like *Knudsen* (1900) relations, but for barotropic water exchange. These relations are solved by first applying the hydraulic semi-empirical model that calculates volume inflows and outflows and time share for in- and outflows during a specific averaging time $T$. Here, this time-period covers both the whole time series, the annual and monthly mean time series. For each of these cases the net volume flow was calculated using the derived relations presented in Section 2.

A sensitivity analysis of average transport rates, in- and outflow time and net volume flow as function of the mean sea level difference between the northern and southern end of the Sound strait was performed. The results indicate the mean net volume outflow is highly dependent on the mean sea level difference, since both the mean volume outflow and mean outflow time is both increasing with increasing mean sea level difference. The mean sea level difference is, in turn, related to the freshwater input and the transport capacity of the Belt Sea and the Sound straits. Theoretically assuming the Belt Sea strait is closed, all net volume flow takes place through the Sound strait. Under this assumption the average sea level difference between the Baltic Sea and the Kattegat, raise to approximately 13.5 cm.

The average outflow and inflow relative time was found to be 65 % and 35 % for the years 1978–2018. The average outflow and inflow volume is $-26\,345\,\mathrm{m^3\,s^{-1}}$ and $25\,665\,\mathrm{m^3\,s^{-1}}$, which is slightly three times larger than the average net volume flow of $-8148\,\mathrm{m^3\,s^{-1}}$. Hence, these results are close to the last example given in Section 2, suggesting that volume inflow is equal to volume outflow, being about 3 times larger than the mean net volume flow, while inflow time is half of the outflow time.

Following the assumptions made in Section 2, inflow salinity should be twice as high as outflow salinity and diffusion of salt should be of equal magnitude for outflow and inflow salt mass. In this case, the barotropic relations for salt and volume exchange is close to the baroclinic relations derived by *Knudsen* (1900). This might be the explanation why his results that inflowing salinity is twice the outflowing salinity is justified over time.

The mean net volume flow through the Sound strait was found to be 50 % of the average runoff ($\approx 15\,900\,\mathrm{m^3\,s^{-1}}$) to the Baltic Sea, suggesting the Belt Sea strait is exporting the other 50 %. However, these statements remain to be shown. Nevertheless, by closing the mean annual water volume balance of the Baltic Sea the Belt Sea straits monthly mean net volume flow was estimated. The budget involves both runoff and variations in the Baltic volume as well as the net volume flow of the Sound. The net volume of the Belt Sea strait was estimated as a rest product of the water balance. The seasonal variation of the Baltic volume is an essential part of the total budget during the first eight months of the year. It even postulates that the net volume flow through the Belt Sea strait becomes positive thus, inwards during the June and July months. Indeed, different from the Sound strait.

This inward directed net volume flow in the Belt Sea strait, will certainly, ease inflows of warm and salty water into the Baltic. For example, in August of 2002 and 2003 warm water with high salinity inflows took place according to *Feistel et al.* (2006). These

Belt Sea strait warm salty inflows are of baroclinic nature. They coincide with periods when barotropic net volume flows are low and even positive (inwards) on average in the Belt Sea strait.

Hence, modelling the two straights with the same barotropic forcing function but with different specific resistances, is probably an over-simplification. Especially, during the summer season, when baroclinic water exchange occurs in the Belt Sea strait.

An unexpected result is that the annual average runoff is uncorrelated with the annual average net volume outflow. This is however not surprising since the net volume flows are a result of several other factors than just runoff. The importance of the sea level seasonal change of the Baltic Sea as one factor. The other being the water exchange in the Belt Sea strait, where both baroclinic and barotropic water exchange occurs. Last, but not least it is shown here that the barotropic water exchange is about 3 times larger the net volume flow and is a weak stochastic variable. All processes, have potential to blur the relationship between net volume flows and runoff.

## *Acknowledgement*

Thanks to the Swedish Meteorological and Hydrological Institute and the Baltic Nest Institute at Stockholm University for providing data. Also, the two referees are highly appreciated, providing important comments and suggestions for changes.

## *References*

Andersson, M and L Johansson, 2018. Extremvattenstånd i Malmö. Diarienummer: 2018/955/9.5. SMHI.

Bergström, S. and B. Carlsson, 1994. River runoff to the Baltic Sea: 1950-1990. *Ambio*, **4-5**, 280–287.

Burchard, H., K. Bolding, R. Feistel, U. Gräwe, K. Klingbeil, P. MacCready, V. Mohrholtz, L. Umlauf and E.M. van der Lee, 2018. The Knudsen theorem and the Total Exchange Flow analysis framework applied to the Baltic Sea. *Progress in Oceanography*, **165**, 268–286.

Church, J.A. and N.J. White, 2011. Sea-Level Rise from the Late 19th to the Early 21st Century. *Surv. Geophys.*, **32**, 585–602.

Döös, K., H.E.M. Meier and R. Döscher, 2004. The Baltic Haline Conveyor Belt or The Overturning Circulation and Mixing in the Baltic. *AMBIO A Journal of the Human Environment*, **33** (4-5), 261–266.

Feistel, R., G. Nausch and E. Hagen, 2006. Unusual Baltic inflow activity in 2002-2003 and varying deep-water properties. *Oceanology*, **48** (S), 2006.

Gustavsson, B.G., 1997. Interaction between Baltic Sea and North Sea. *Deutsche Hydrographische Zeitschritt*, **49** (2/3).

Gustavsson, B.G., 2004. Sensitivity of Baltic salinity to large perturbations in the climate. *Climate Research*, **27**, 237–251.

Gustavsson, B.G. and H. Andersson, Sept. 2001. Modeling the exchange of the Baltic Sea from the meridional atmospheric pressure difference across the North Sea. *J. Geophysical Research*, **106** (C9), 19: 731–19, 744.

Håkansson, B, B Broman and H Dahlin, 1993. "The inflow of water and salt in the Sound during the Baltic major inflow event in January 1993". In: *ICES statuary Meeting*. C.M.1993/C:57.

Jacobsen, J.P., 1925. Die Wasserumsetzung durch den Öresund, den grossen und kleinen Belt. *Medd. Komm. Havunders., Ser. Hydr.*, **2** (9), 1–72.

Jacobsen, T.S., Dec. 1980. *The Belt Project, Sea water exchange of the Baltic, Measurements and Methods*. Denmark: The National agency of Environment Protection.

Jakobsen, Fl., I.S. Hansen, N.-E. Ottesen Hansen and F. Ostrup-Rasmussen, 2010. Flow resistance in the Great Belt, the biggest strait between the North Sea and The Baltic Sea. *Estuar. Coast. Shelf Sci.*, **87**, 325–332.

Jakobsen, Fl., M.J. Lintrup and J. Steen Möller, 1997. Observations of the specific resistance in the Öresund. *Nordic Hydrology*, **28**, 217–232.

Knudsen, O., 1900. Ein hydrographischer Lehrsatz. *Ann. Hydrogr. Marit. Meteorol.*, **28**. Translated from German to English by Burchard, Bolding, Feistel, Gräwe, Klingbeil, MacCready, Mohrholtz, Umlauf and Lee (2018), 316–320.

Lass, H., U. and W. Matthäus, 1996. On temporal wind variations forcing saltwater inflows into the Baltic Sea. *Tellus*, **48A**, 663–671.

Manning, Robert, 1891. On the flow of water in open channels and pipes. *Transactions of the Institution of Civil Engineers of Ireland*, **H**.

Matthäus, W, 2006. "The investigation of saltwater inflows and their effects on the Baltic ecosystem – a historical survey". In: *Historisch-Meereskundliches Jahrbuch*. Vol. 12.

Mattsson, J., 1996. Some comments on the barotropic flow through the Danish Straits and the division of the flow between the Belt Sea and the Öresund. *Tellus*, **484A**, 456–464.

Mikulski, L., 1970. Inflow of river water to The Baltic Sea in the period 1951-1960. *Nordic Hydrology*, **4**, 216–227.

Mohrholz, V., 2018. Major Baltic Inflow Statistics – Revised. *Front. Mar. Sci.*, **5**. DOI: `10.3389/fmars.2018.00384`.

Omstedt, A., 1987. Water cooling in the entrance of the Baltic Sea. *Tellus*, **39A**, 254–265.

Pratt, L.J., 1986. Hydraulic control of sill flow with bottom friction. *J. Physical Oceanography*, **16**, 1979–1980.

Rutgersson, A., A. Omstedt and J. Räisänen, 2002. Net precipitation over the Baltic Sea during present and future climate conditions. *Climate Research*, **22**, 27–39.

Samuelsson, M. and A. Stigebrandt, 1996. Main characteristics of the long-term sea level variability in the Baltic Sea. *Tellus*, **48A**, 672–683.

Stigebrandt, A., 1983. A model for the exchange of water and salt between the Baltic and the Skagerrak. *J.Phys. Oceanogr.*, **3**, 411–427.

Stramska, M., H. Kowalevska-Kalkowska and M. Swirgon, 2013. Seasonal variability in the Baltic Sea Level. *Oceanologia*, **55** (4), 787–807.

Svansson, A., 1975. *Physical and chemical oceanography of the Skagerrak and the Kattegatt – I Open Sea Conditions*. 1. Fishery Board of Sweden, Institute of Marine Research.

Welander, P., 1974. Two-layer exchange in an eustary basin, with special reference to the Baltic Sea. *J. Phys. Oceanography*, **4**, 542–556.

Windsor, P., J. Rodhe and A. Omstedt, 2001. Baltic Sea ocean climate: an analysis of 100 yr of hydrographic data with focus on the freshwater budget. *Clim. Res.*, **18**, 5–15.

World Meteorological Organization, 2018. *State of the Climate in 2018*. URL: `https://public.wmo.int/en/our-mandate/state-climate-2018`.