

RG 2000 – the New Gravity Reference Frame of Sweden

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Abstract

The increased need for improved geoid models for Global Navigation Satellite Systems (GNSS) height determination calls for additional gravity observations and quality assurance of existing data. In this perspective, a modern gravity system and the renovation of an already existing high order gravity network is considered as a moderate strategic investment which provides a firm foundation for further activities. Here the new gravity reference frame RG 2000 for Sweden is presented. RG 2000 is realized by absolute gravity observations at 109 stations. The absolute points are connected via old and new relative gravity observations, including another 216 points. Points and observations have been chosen so that good overlap with the older Swedish reference frames, RG 62 and RG 82, is achieved, allowing to evaluate the older frames and transformations between them. RG 2000 is based on a zero permanent tide system with epoch 2000.

Keywords: Gravity, Reference Frame, Absolute gravimetry, Gravity network, FG5, A10, RG 2000

1 Introduction

Tremendous development in surveying engineering over the last decades promoted the use of Network Real Time Kinematic (RTK) GNSS positioning in practically all European countries as nowadays' standard tool for surveyors. While the uncertainties from densified Network RTK networks for construction work are approaching the sub-centimetre level not only in the horizontals but also in the vertical, this high accuracy may easily get lost while converting the GNSS-derived heights to “gravity related heights” in the national height frame using a regional geoid model. This is due to uncertainties in the latter. Therefore, surveyors are constantly asking for “better geoid models”.

Thanks to recent dedicated satellite gravity field missions (CHALLENGING Minisatellite Payload (CHAMP), Gravity Recovery And Climate Experiment (GRACE), GRACE-Follow On and Gravity Field and Steady-State Ocean Circulation Explorer (GOCE)), the improvements in global geopotential models are on the same level as the developments in GNSS, with an uncertainty at the centimetre level for a resolution of about 100 km (Förste *et al.*, 2014). However, for the precise geoid models

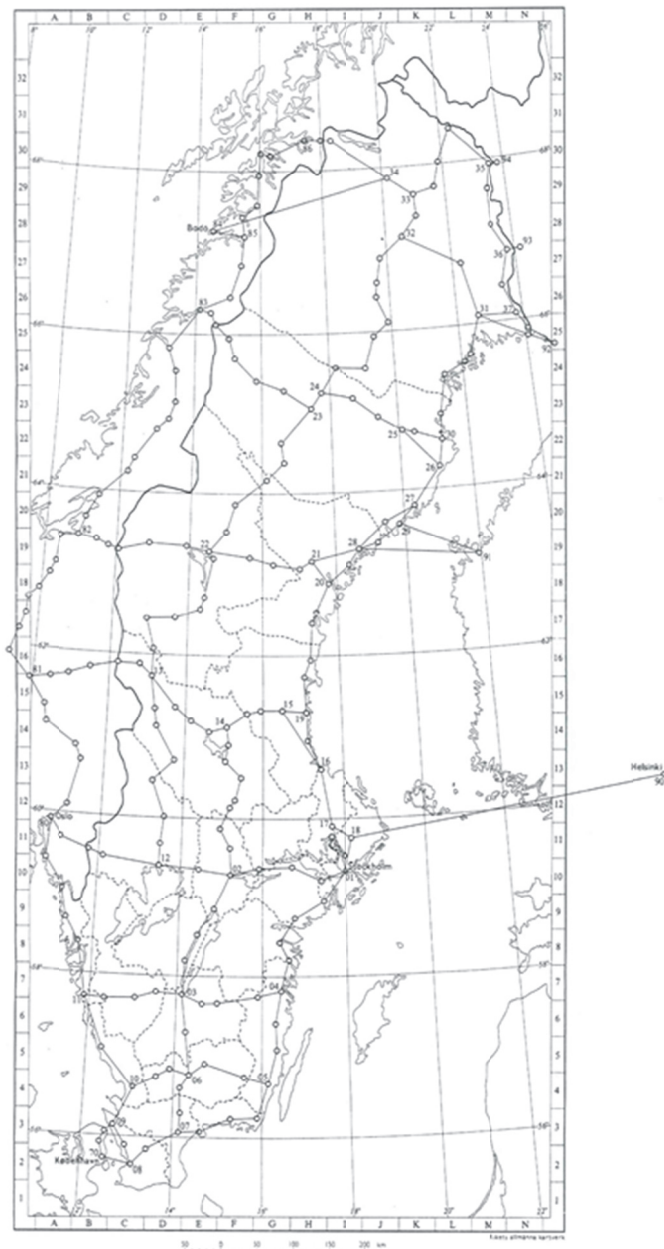


Fig. 1. The RG 62 network with its 185 observation points. Figure taken from *Pettersson* (1967).

that surveyors are asking for, accurate terrestrial gravity observations with much higher spatial resolution (typical 3–5 km spacing) are needed (*Ågren and Sjöberg, 2014*). Unfortunately, the existing gravity frames in Sweden, RG 62 and RG 82, did no longer have sufficient quality for the current needs in improving geoid models. Therefore, existing gravity observations for geoid calculation need to be improved and densified. One important improvement step is to have a modern gravity network which serves as base for new gravity observations for the geoid.

The RG 62 frame (Fig. 1) was established between 1960–1966 with the use of a Worden Master gravimeter (*Pettersson, 1967*) and was connected to Potsdam via the European Calibration System (ECS) 1962 (*Gantar and Morelli, 1962*). It was at this time also known as “The First Order Network”. Despite that the 185 points of RG 62 in general covered the area of Sweden, there were many

gaps in the coverage, where the nearest point was more than 100 km away (see Fig. 1).

An improvement of the situation was achieved with the introduction of RG 82 (*Haller and Ekman, 1988*). It was realized by a so-called Zero and First Order Network. The Zero Order Network of RG 82 (Fig. 2) was mainly observed with the two LaCoste & Romberg gravimeters G54 and G290 (*LaCoste & Romberg, 2004*) in 1981–82 and was based on four absolute gravity observations (two in Sweden (Mårtsbo and Gothenburg), one in Finland (Sodankylä) and one in Denmark (Copenhagen)) by the Italian instrument IMGC (Istituto di Metrologia Gustavo Colonnetti) in 1976 (*Cannizzo and Cerutti, 1978*). It consisted of 25 main points (black dots in Fig. 2) and at least one spare

point per main point. 12 of the points were included in the four Fennoscandian land uplift gravity lines, which were established during the 1960s and 1970s within a Nordic Geodetic Commission (NKG) project in order to investigate the land uplift through observations with relative gravimeters. The points on these lines were chosen in a way, so that the gravity difference between the points was less than 2 mGal and that the scale of the instruments would not affect the result of the observations.

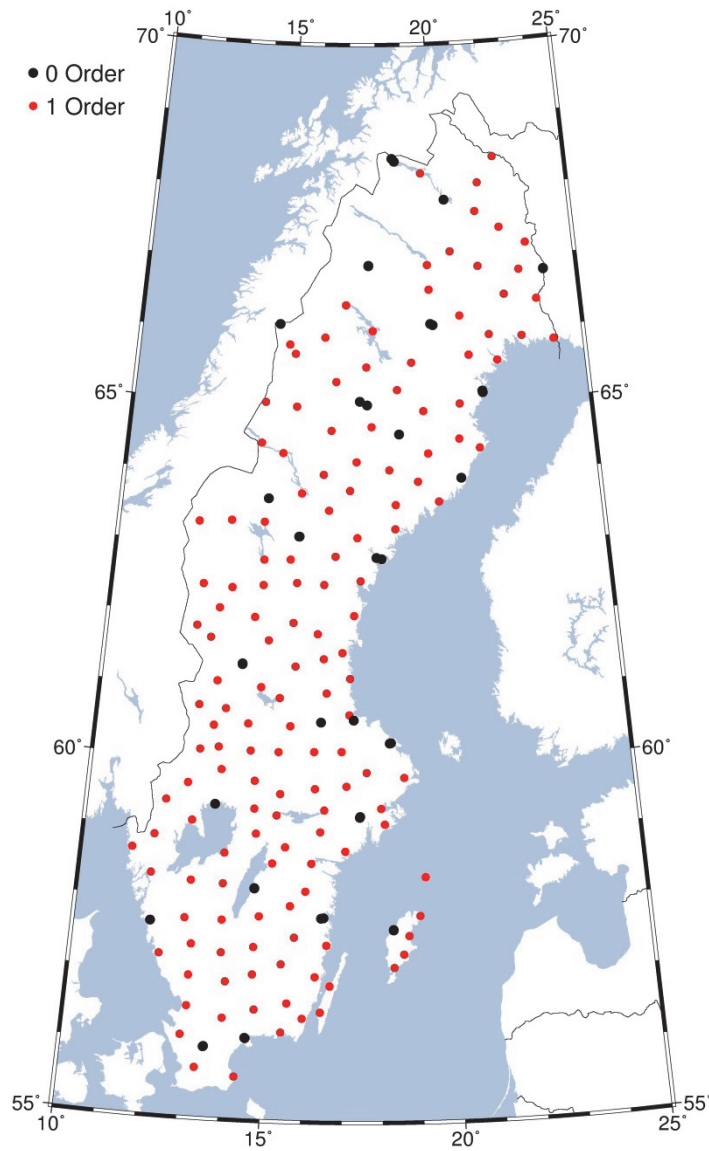


Fig. 2. The RG 82 network.

The First Order Network in RG 82 was a densification of the Zero Order Network and consisted of 149 points (red dots in Fig. 2), where the observations started in 1984 with low priority and thus were not finished until October 2002 (*Engfeldt, 2016a*). Here, the same two LaCoste & Romberg model G gravimeters were used as for the Zero Order Network. With this densification, the whole RG 82 covered the area of Sweden with at least one point every 50th kilometre, except in remote places (Fig. 2).

Between 1991 and 2007, 12 new stations for absolute gravity were established in Sweden (Fig. 3). One station, Mårtsbo, was already established and visited by IMGIC in 1976. Since 2004, all 13 stations are almost annually visited by modern absolute gravimeters of the FG5 type (Niebauer et al., 1995). In this regard, it was inexpedient to have 13 well-observed (instrument- and time-wise) stations while the existing gravity network was based on a few absolute gravity observations at a few stations with an outdated absolute gravimeter. In 2010, Lantmäteriet developed a strategic plan for the Geodetic infrastructure in Sweden (Lantmäteriet, 2010). There it was decided to establish a new gravity reference network and frame, RG 2000, that supports the modern absolute gravity observations and serves as base for new gravity observations for geoid improvements. Due to its location in the Fennoscandian postglacial rebound (PGR) area, Sweden is subject to crustal deformations with a maximum land uplift of about 1 cm/a (Kierulf et al., 2014) affecting also the gravity field. Precise geodetic observations therefore need to be reduced to a common reference epoch. The reference epoch of RG 2000 was chosen to be year 2000 to be compliant with the national height system RH 2000 (also year 2000.0) and the 3D reference frame SWEREF 99 (year 1999.5).

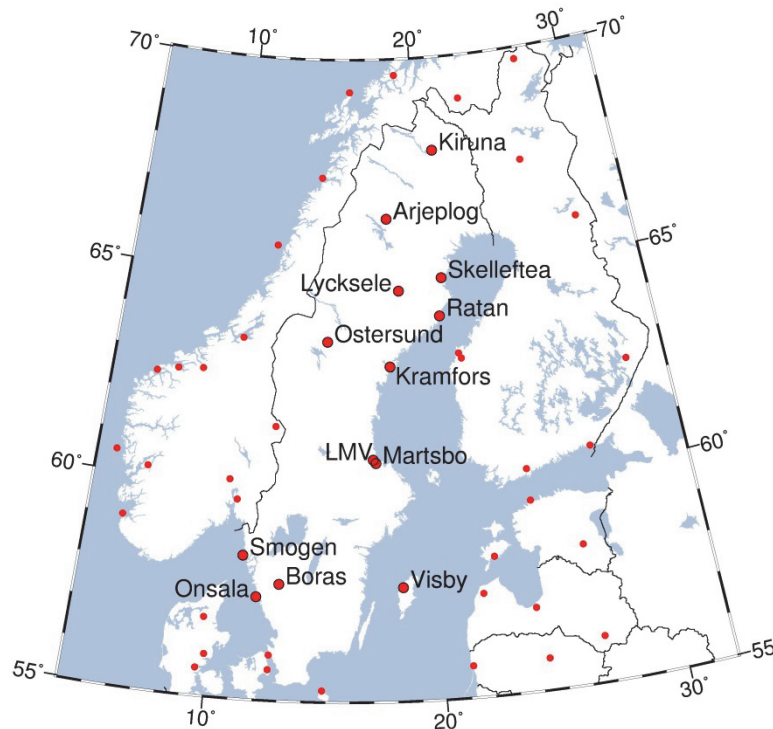


Fig. 3. Location of absolute gravity stations in northern Europe observed with FG5. Swedish stations are named.

In the following section, we explain the strategy, the measurements and methods to realize RG 2000. We then introduce the formal definition and realization of RG 2000 and discuss the differences and improvements to the previous gravity reference frames RG 62 and RG 82.

2 *RG 2000 observations*

2.1 *Strategy*

The foundation for RG 2000 is 13 stations (17 points) with repeated absolute gravity observations using FG5 gravimeters. These points are densified by 96 absolute points observed with another type of absolute gravimeter, A10 (*Micro-g LaCoste*, 2008), which is portable and can be used outdoors, but with less accuracy than the FG5 instrument. The absolute gravity points are connected and controlled by relative gravity observations in a network including another 216 points. The relative observations are a mix of old measurements used for RG 82 and new observations performed to control or improve the network. The points have been chosen so that good overlaps between the different frames are achieved.

The points realizing RG 2000 are divided into different classes (based on type of observation and quality of the points, see Table 1) rather than networks of certain order as are RG 62 or RG 82. Points with FG5 absolute gravity observations form Class A (red dots in Fig. 4). Since FG5 is designed to observe indoors only, all these points are situated indoors. Class B are points (black dots in Fig. 4) observed with A10 gravimeters. These points cover Sweden with one A10 observation every 50–70 km. The requirements for Class B points are a flat surface big enough for the A10 instrument and, if possible, the distance between the back of the car used for transportation and the point should be less than 15 meters to continuously connect the instrument to an external power supply in the car. Class C and D points (blue and green dot in Fig. 4, respectively) were observed with relative gravimeters (LaCoste & Romberg G and Scintrex CG5). The difference between Class C and D is that points of Class C are considered as better than those of Class D. For the latter, one of the three following items makes the point less suitable to use for further observations: (i) something about the surface where the point is situated (e.g. uneven/tilted bedrock or stone or church step); (ii) something about the surroundings of the point (e.g. strong variations in local hydrology); (iii) the quality of the observation is considered poor in comparison to the rest (e.g. the instrument has not worked well or RG 82 points were not rediscovered under soil today).

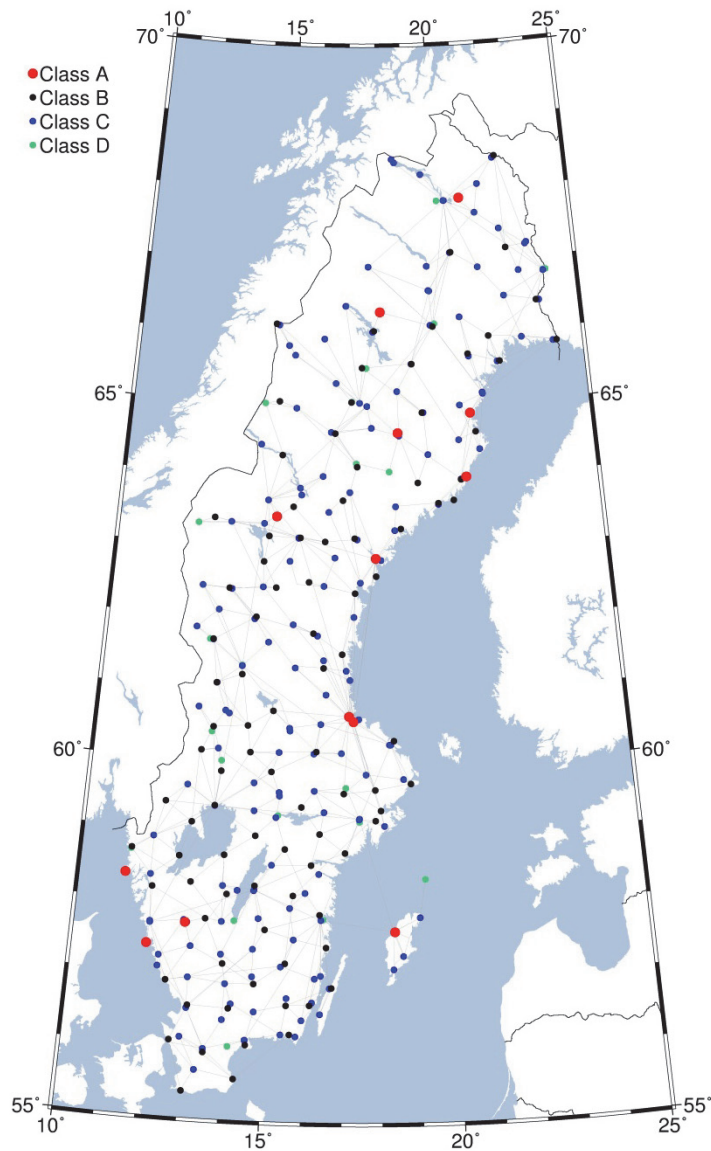


Fig. 4. The RG 2000 gravity network with colour-coded point classes. Grey lines show the relative ties.

Table 1. The classification of 318 RG 2000 points in the main adjustment.

	Number of points	Description
Class A	17	Observed with the FG5
Class B	96	Observed with the A10
Class C	181	Observed with relative gravimeters, considered as very good
Class D	24	Observed with relative gravimeters, considered as less suitable (see text for explanation)

The first step in the RG 2000 work was to investigate which points from the old networks RG 62 and RG 82 still existed and could be potentially used in RG 2000, in addition to the FG5 absolute gravity stations. Of special importance was to identify points which could be used for the portable A10 absolute gravimeter. It was found that

most of the RG 82 points and about half of the RG 62 points were still available and usable for relative gravimeters (Engfeldt, 2016a). However, of the 185 points of RG 62, only 23 were marked with a benchmark, so that they could be identified. Many of the RG 62 points are situated on church steps, where the place on the step is precisely described for only 8 of them (unless “middle of the stone slab” is counted as exact, then more than half of them are exact). Eventually, there were less points than expected which fulfilled the above-mentioned requirements for the A10. Especially the demands about the surface were the reason why only 42 of the old points, 16 from RG 62 and 26 from RG 82, could be used as Class B points. Therefore, 55 new Class B points were established for RG 2000 and more relative observations than initially expected were required (see Sect. 2.4). In total, 318 points were classified according to the four classes (Fig. 4 and Table 1). Note that we have used a total of 329 points in the adjustment (see Sect. 4.2). The 11 unclassified points (1 in Denmark and 10 destroyed) were part of old observation sequences and strengthen the other observations.

2.2 FG5 observations for RG 2000

In October 2006 Lantmäteriet purchased an FG5 absolute gravimeter with the main purpose to study the gravity change due to PGR and to continue time series started by other institutions (see Olsson *et al.*, 2019). Since 2007 it is used for regular absolute gravity observations at the previously mentioned 13 absolute gravity stations in Sweden (Fig. 3) with the highest possible accuracy to date. These observations and stations are thus also excellently suited to form a firm base for the new gravity frame and its realization. For the Class A points, we have used observations from the FG5-220 from IfE (Institut für Erdmessung, Leibniz Universität Hannover, Germany) and FG5-233 (Fig. 5) from Lantmäteriet. The Swedish standard procedure to measure absolute gravity is as follows:

- Two orientations, 24 hours in north orientation and 24 hours in south orientation (these two orientations are used to minimize possible Eötvös effects (Křen *et al.*, 2018)),
- 24 sets in every orientation,
- 50 drops (free fall observations) per set,
- All observations not within the 3-sigma level are regarded as outliers and are removed directly by the g-software (*Micro-g LaCoste*, 2012).



Fig. 5. The Lantmäteriet FG5-233 absolute gravimeter, observing at Smögen AA, Sweden.

The data from FG5-220 were taken from *Gitlein (2009)*. Data from FG5-233 were processed using the *g*-software with final International Earth Rotation and Reference System Service (IERS) polar coordinates, calibrated rubidium frequencies and standard modelling of gravitational effects due to Earth tides (zero permanent tide system), ocean loading (FES2004 model, *Lyard et al., 2006*) and varying atmospheric pressure as implemented in the *g*-software. For further details on the processing, we refer the reader to *Olsson et al. (2019)*.

For RG 2000, an FG5 observation on a specific station is understood to be the mean of all available FG5 observations, reduced to a common epoch (see further Section 3), with the two specified instruments at that station.

2.3 *A10 observations for RG 2000*

Between 2011 and 2015 totally 97 Class A and B points were observed for RG 2000 in five campaigns with the A10-020 (Fig. 6) owned by IGIK (Centre of Geodesy and Geodynamics, Institute of Geodesy and Cartography, Warsaw, Poland) (*Engfeldt, 2016a,b*). At least one point already observed by FG5 was observed during every campaign, where it was used as reference value to check that the A10 results were reliable over time. The check of these points showed a standard deviation of $3.9 \mu\text{Gal}$ ($1 \text{ Gal} = 0.01 \text{ m/s}^2$) from the later adjusted *g*-value at the epoch 2000.0.



Fig. 6. The IGIK absolute gravimeter, A10-020, observing in Tullinge AA, Sweden.

The observations with A10 were conducted such that at each point two setups were performed with the upper part of the instrument oriented in two different directions, 120 degrees in between (there are three possible ways to mount the upper part on the lower part, 120 degrees in between). During A10-020 observations in Finland, the influence of the Eötvös effect was investigated and proven to be insignificant (*Mäkinen et al.*, 2010). Thus, the setup itself meant no degradation of the observations. One single setup consists of 8 sets, each set consists of 120 drops per two minutes. Two setups usually took less than 2 hours per point. In case the results from the two orientations differed less than 10 μGal they were considered satisfactory, otherwise one more orientation was performed to improve the result. The eight points with the largest differences between the orientations from the earlier campaigns were re-measured in 2013/15 to check the occurrence of gross errors.

In 2012, the A10-019 absolute gravimeter owned by DTU Space (Danish Technical University Space, Kongens Lyngby, Denmark) also observed 2 newly established points for RG 2000 in southern Sweden.

2.4 *Relative gravity observations*

The relative gravity observations and instruments used in the calculations and adjustments are the following:

- RG 82 Zero Order campaign in 1981–82, LaCoste & Romberg G54 and G290,

- RG 82 First Order campaign in 1984–96, 2001–02, LaCoste & Romberg G54 and G290,
- RG 2000 campaign in 2015–17, LaCoste & Romberg G54 (Fig. 7A) and Scintrex CG5-1184 (Fig. 7B),
- Additional observations, in 2004–14, between the campaigns mentioned above (very little data, mainly connecting the FG5 points to RG 82), LaCoste & Romberg G54, Scintrex CG5-740 and CG5-198,
- Observations from the NKG land uplift gravity lines (*Mäkinen et al.*, 1986, 2005) between 1975–2003. We chose to use only the observations from LaCoste & Romberg model G gravimeters, in total 17 different gravimeters used in different Nordic institutions,
- Additional observations 1975-91 (very little data, mainly connecting RG 62 points to RG 82), LaCoste & Romberg G54 and G290.

The new relative observations (RG 2000 campaign) were performed to strengthen the network and, in combination with the old relative observations from RG 82 and the A10 observations at points included in the old gravity frames, connect RG 2000 and the old frames. Furthermore, the new A10 points are connected to at least one point in the old RG 82 networks or one of the FG5 points, which enables a rough check for gross errors in the A10 observations (*Engfeldt*, 2016b). This check was performed between 2015 and 2017 at all Class B points.

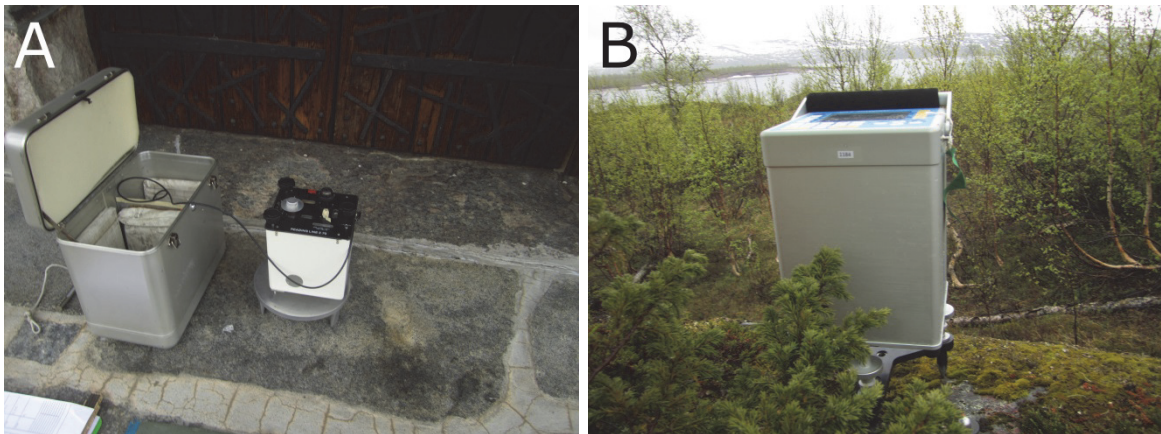


Fig. 7. A) Relative observations with the LaCoste & Romberg G54 in Karesuando AA, Sweden and B) with the Scintrex CG5-1184 in Umbukta A, Sweden.

In the RG 82 network, the observations along the land uplift gravity lines were not used as ordinary relative gravity data, but as precomputed differences. These are mean values for each instrument and year that can be found in the tables in Chapter 7 of *Mäkinen et al.* (1986) and we chose to use them as precomputed differences also for RG 2000. For the observations on the land uplift gravity lines after the year 1983, the relative observations were used as normal. Some of the relative observations between the

main points in the Zero Order Network of RG 82 and the spare points in the same network were unfortunately not written in any protocol book, which means that they are missing. The differences derived in RG 82 were thus also used in the computation of RG 2000, again in form of precomputed differences, see Section 4.

2.5 Vertical gravity gradient determination

When using the observations at our absolute gravity points as a basis for relative gravimetry, the absolute gravity value must first be reduced from the height of observation (~ 1.20 m for FG5) to the benchmark at the ground level. For transferring the gravity value from 1.200 meters to 0.000 meters the vertical gravity gradient must be determined. This gradient is determined from gravity differences. Here, several predefined setups are measured with a relative gravimeter, where it is repeatedly placed at different heights (Fig. 8). These heights are as close to 0.000 meters as possible and anything between as low as it possible to measure on a tripod with a relative gravimeter (Fig. 8) and as high as it is possible to measure with this instrument.



Fig. 8. Measurements for vertical gravity gradient determination at the Class B point Östersund AB, Sweden.

Measurements with such setups were performed at all Class A points in RG 2000. A second-degree polynomial function was then fitted to the observations for each station. For the Class B points a different approach was used (*Engfeldt, 2016a*) as the difference between the sensor height and the ground of an A10 is around 70 cm only. Based on previous experience from the Class A points, the difference between different setups is mostly less than $2 \mu\text{Gal}$, which means negligible in view of the accuracy of an A10. Hence, here the gravity gradient measurements were performed only with one setup repeatedly measuring the difference between two heights.

3 Definition and realization of RG 2000

RG 2000 is established using the postglacial rebound epoch 2000.0. Otherwise, RG 2000 is defined in accordance with international standards and conventions (*Boedecker, 1988*) as customary applied by the absolute gravity community specifically

in the *g* software (*Micro-g LaCoste*, 2012). This means for instance that a zero permanent tide system (*LAG*, 1984; *Boedecker*, 1988) is used.

Each point of RG 2000 is realized by its gravity values and the standard uncertainties (obtained e.g. from the adjustment, see Fig. 13 below). All observations are reduced to the land uplift epoch 2000.0. For RG 2000 the NKG2016LU_gdot model (*Olsson et al.*, 2019, Fig. 9) was used for land uplift corrections. NKG2016LU_gdot is the land uplift model NKG2016LU_abs (*Vestøl et al.*, 2016, in press) converted to gravity change using the factor $-0.163 \mu\text{Gal}/\text{mm}$ (*Olsson et al.*, 2015b).

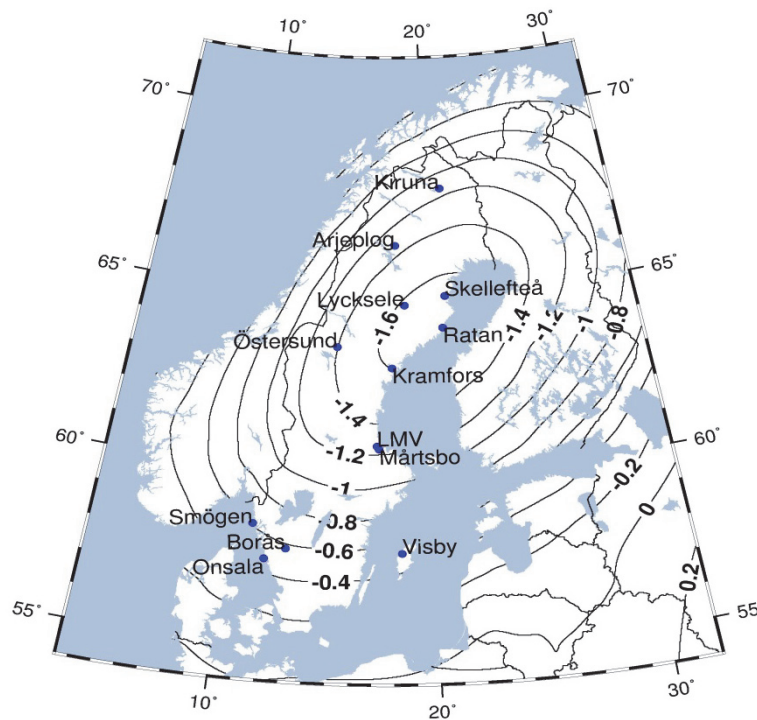


Fig. 9. The NKG2016LU_gdot model, used for RG 2000. Isolines show the postglacial gravity change in $\mu\text{Gal}/\text{yr}$. The uncertainty of NKG2016LU_gdot varies between 0.1 and 0.2 $\mu\text{Gal}/\text{yr}$ (*Olsson et al.*, 2019).

The FG5-233 had until 2017 been five times on service at the manufacturer in the USA (Fig. 10). After the service in 2009/10, a bias/offset was observed, with a shift of about $4 \mu\text{Gal}$ (*Olsson et al.*, 2015a). To deal with suspected offsets, different absolute levels for the instrument between the services have been assumed, which e.g. means that the level of the observations performed between autumn 2006 and summer 2008 should have an absolute level according to the European comparison of absolute gravimeters (ECAG) 2007, the level of the observations performed between autumn 2008 and winter 2009 should have an absolute level according to the international comparisons of absolute gravimeters (ICAG) 2009 etc. (see Fig. 10 and Table 2). This is further discussed in *Engfeldt* (2016a) and *Olsson et al.* (2019). In the calculation and adjustment of RG 2000, all absolute gravity observations from the FG5-233 were corrected according to the results of the ECAG and the ICAG offsets (see Fig. 10, and Table 2, and details in *Olsson et al.*, 2019). Observations from the A10-020 were corrected with the corre-

sponding numbers in Table 2. They were provided by IGiK and are, based on experience, slightly modified results of the ECAG and ICAG offsets (see Fig. 11 and Table 2). FG5-220 and the A10-019 were not corrected for biases. A10-019 has never participated in such a comparison. For FG5-220, *Olsson et al. (2019)* recommended to leave it as is, and we follow this recommendation for consistency.

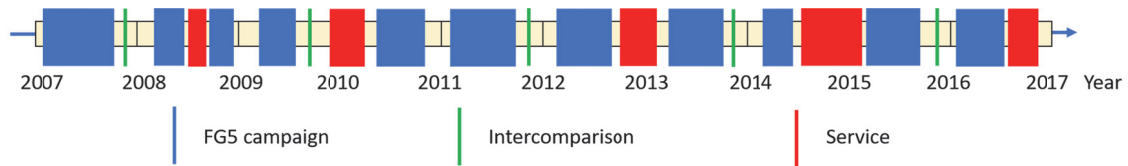


Fig. 10. Overview of the FG5-233 observation periods (blue) for observations included in RG 2000, participation in intercomparisons (green) and scheduled service (red).

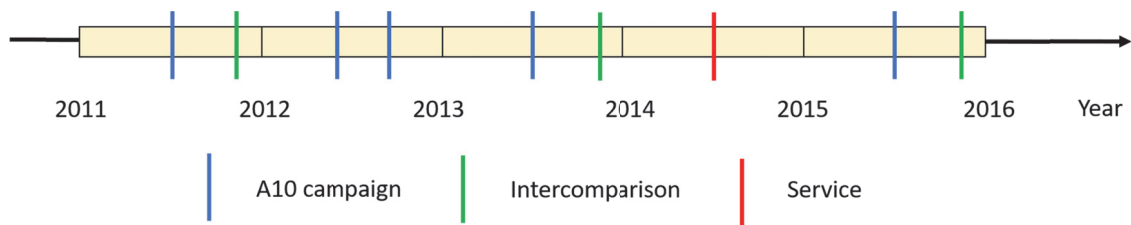


Fig. 11. Overview of the A10-020 observation periods in Sweden (blue), participation in intercomparisons (green) and scheduled service (red).

Table 2. The differences between FG5-233 and CRV’s (Comparison Reference Values) from the ECAG’s and ICAG’s, and between the modified A10-020 values and CRV’s. See *Francis et al. (2010)*; *Jiang et al. (2012)*; *Francis et al. (2013)*; *Francis et al. (2014)*; *Pálinkáš et al. (2017)*.

	ECAG 2007	ICAG 2009	ECAG 2011	ICAG 2013	EURAMET 2015
FG5-233	+1.0	+1.0	+4.7	+2.2	+2.5
A10-020	-	-	+1.0	-4.7	-8.9

4 Calculation and adjustment of RG 2000

4.1 Software

As the software used for RG 82 is not available anymore and a market check did not show a suitable software that fulfilled our requirements, a new in-house software was developed in which the same observation equations and adjustment theory was used as in the RG 82 software. Before using the new software for RG 2000, a test recalculation of the RG 82 network gave identical results. The software contains three parts, Gprep, Gad and Gcross. Gprep prepares the relative gravity data from different instruments and several input files to two input files for Gad in the land uplift epoch 2000.0. In these input files, one for all relative observations and one for precomputed differences, height, gravity gradient and tidal corrections (using ETGTAB, *Timmen and Wen-*

zel, 1995) are done automatically. Gad is the main software applying the least squares adjustment method using the two input files from Gprep and a file with the absolute gravity observations.

In the least squares adjustment with Gad, the three different types of observations were treated using the following observation equations (cf. Koch, 2000):

- 1) Absolute gravity observations:

For an absolute observation i of gravity point j ,

$$l_i - \varepsilon_i = g_j, \quad (1)$$

where l_i is the observed gravity value of observation i , ε_i is the error of observation i , and g_j is the gravity value for gravity point j . We note that there is only one (averaged and reduced) l_i for any g_j .

- 2) Relative gravity observations:

For a relative observation i of gravity point j with instrument n for the instrument level k with drift parameter m ,

$$l_i - \varepsilon_i = \frac{(g_j^0 - IL_k^0)}{SC_n^0} + (t_i - t_k) \cdot DR_m^0 + \frac{1}{SC_n^0} \cdot \Delta g_j - \frac{1}{SC_n^0} \cdot \Delta IL_k - \frac{(g_j^0 - IL_k^0)}{(SC_n^0)^2} \cdot \Delta SC_n + (t_i - t_k) \cdot \Delta DR_m, \quad (2)$$

where l_i is the observed gravity value of observation i , ε_i is the residual of observation i , g_j^0 is the approximate gravity value for point j , IL_k^0 is the approximate instrument level for sequence k , t_i is the time of observation i , t_k is the time for the first observation in sequence k , SC_n^0 is the approximate scale correction for the instrument n , DR_m^0 is the approximate drift parameter of the drift sequence m , Δg_j is the correction to g_j^0 (i.e. the difference between gravity value of point j and the corresponding approximate value), ΔIL_k is the correction to the instrument level, ΔSC_n is the correction to the scale correction and ΔDR_m is the correction to the drift sequence.

- 3) Precomputed differences:

For observation i of difference between gravity point j and gravity point p estimating a scale correction for instrument n ,

$$l_i - \varepsilon_i = \frac{(g_j^0 - g_p^0)}{SC_n^0} + \frac{1}{SC_n^0} \cdot \Delta g_j - \frac{1}{SC_n^0} \cdot \Delta g_p - \frac{(g_j^0 - g_p^0)}{(SC_n^0)^2} \cdot \Delta SC_n, \quad (3)$$

where l_i is the observed gravity value of observation i , ε_i is the error of observation i , g_j^0 is the approximate gravity value for point j , g_p^0 is the approximate gravity value for point p , SC_n^0 is the approximate scale correction for the instrument n , Δg_j and Δg_p are the corrections to the gravity point j and p , respectively, and ΔSC_n is the correction to the scale correction.

The adjustment was then performed iteratively until the solution converged.

Gcross is used to make cross validations, specifically leave-one-out cross-validation, for the absolute gravity observations. Leave-one-out cross validation means that the absolute observation of one certain point was excluded from the input file to form a new input file to Gad, and the difference between the computed g value in the new Gad solution and the observed g value of this certain point is calculated.

4.2 RG 2000 adjustment

The observed agreement between two FG5 observations is about 2 μGal (*Niebauer, 1995 and Micro-g LaCoste, 2006*). However, the repeatability for the instrument is better than that and with long time series using only the mean value, the uncertainty would be even lower. With these things in mind, the *á priori* standard uncertainties of 2.0 μGal and 1.0 μGal for FG5 observations were tested in the adjustment. The latter gave the best result and was our choice in the final adjustment.

The absolute accuracy of an A10 observation is 10 μGal according to the manufacturer (*Micro-g LaCoste, 2008*). However, real performance tests by *Mäkinen et al. (2010)* gave 6 μGal . Thus, all possible integers between 3.0 μGal and 10.0 μGal were tested in our adjustments and 5.0 μGal gave the best result. This is at the same level as in the above-mentioned real performance tests, which additionally supported a use in the final adjustment.

In RG 82 the *á priori* standard uncertainty (or weight, as it was called by *Haller and Ekman, 1988*) of 12 μGal was given to both LaCoste & Romberg G54 and G290. In a later Master thesis (*Jansson and Norin, 1990*), weights of 10.5 μGal and 13.8 μGal were given to G54 and G290, respectively, on advice from professor Lars E Sjöberg (KTH, Stockholm). The observations from G54 were ocular readings, while the observations from G290 were galvanometer readings. The observations from G54, no matter if G290 was observed via ocular or galvanometer readings, are in general more consistent and the instrument was better itself (*pers. comm. with Lars Åke Haller and Jaakko Mäkinen and by own experiences*). Thus, the choice here was to give G290 a higher *á priori* standard uncertainty than the rest of the relative instruments. We first tested 10.5 μGal and 13.8 μGal as in *Jansson and Norin (1990)* for the used instruments. In the end the normal value became 9.0 μGal for all the instruments except G290 (11.0 μGal) and our most consistent instrument, Scintrex CG5-1184 (7.0 μGal). There were cases when the normal *á priori* standard uncertainties were not used. Instead, those were set after certain criteria, depending on how close in g -range and how close in time the observations were performed. For example, the Scintrex CG5-1184 observations of spare points right outside of Class A points (all indoor points) got the very low *á priori* standard uncertainty of 3.0 μGal , while all observations on a loop with big gravity differences, i.e. where it was known in advance that the instrument did not work as well as normal, got the double *á priori* standard uncertainty (for example 18.0 μGal with LaCoste & Romberg G54). The differences of the observations along the land uplift gravity lines are in general spread quite much, so despite they are a mean of observations we used the

standard á priori uncertainty 9 μGal for them. The other precomputed differences got the standard á priori uncertainties 6 or 7 μGal .

The numbers above were found as the best suitable. 329 points (see Fig. 13) were included in the RG 2000 adjustment, of which 328 are in Sweden and 1 in Denmark. During the adjustment, a big gross error in one of the A10 observations was found, so here the relative observations gave the point its g -value.

After the adjustment, the largest difference in gravity value between the absolute observation and the adjusted value was 15.6 μGal (Fig. 12A) in a point observed by A10 in central western Sweden. For a point observed by FG5, the largest difference was 1.3 μGal . The cross-validation result shows two points observed by A10, where the g -value would have been about 28 μGal different if the A10 observation was removed (Fig. 12B). We suspect that for points with a difference larger than 20 μGal (5 in total), less accurate relative observations contribute the most to this difference.

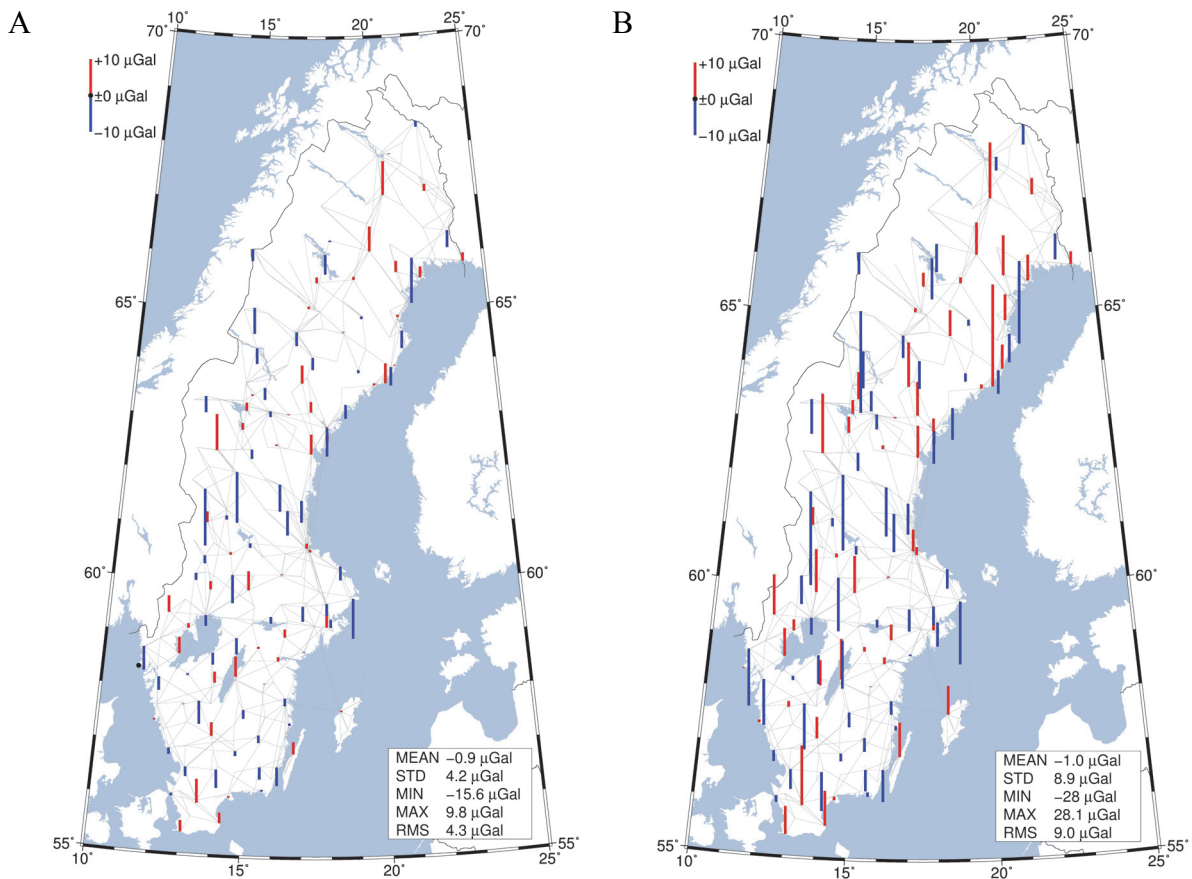


Fig. 12. Adjustment results: A) The residual between observed AG value (FG5 and A10) and adjusted value. B) The cross-validation difference at the points in A.

In the final RG2000 adjustment solution the following statistics resulted:

- Total number of unknowns: 1405
- Total number of equations: 4008
- Total number of absolute instruments: 4
- Total number of relative instruments: 14

- Number of gravity points:	329
- Number of absolute observations:	113
- Number of relative observations:	3721
- Number of precomputed differences:	174
- Number of unknown scale corrections:	9
- Number of unknown drift parameters:	213
- Number of unknown instrument levels:	854

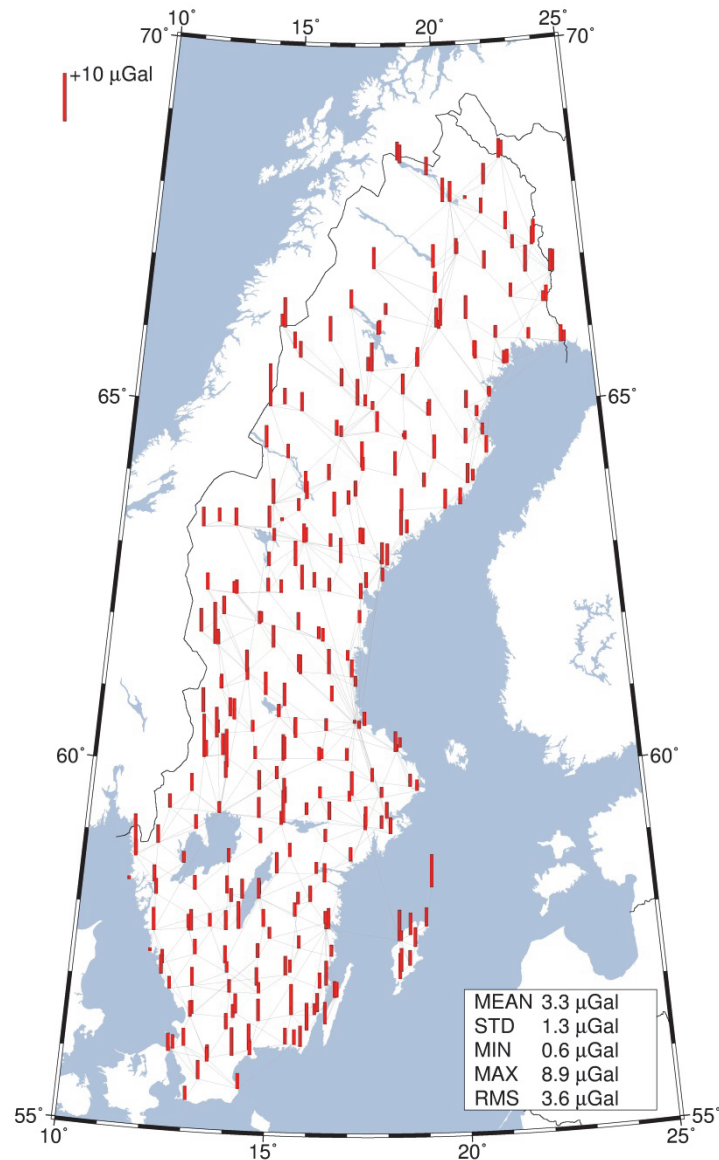


Fig. 13. Estimated uncertainty of all 329 points used in the adjustment. The maximum value in western central Sweden belongs to a point which is destroyed.

All the 15 LaCoste & Romberg instruments which only had been used for observations along the land uplift gravity lines were in the adjustment noted as the same instrument with the known scale factor 1. This is because it is impossible to determine a good scale factor for an instrument when the gravity range of the observations is less

than 2 mGal. The Scintrex CG5-198 was also used too little to get a good scale factor through this adjustment, thus it was also set to 1. The LaCoste & Romberg instruments G54 and G290 were used during many years. Here each instrument first had the same scale factor during the whole time, which later was changed. In the end the scale factors of G54 and G290 were divided into 4 and 3 different time period-dependent scale factors, respectively. The first time period is 1975–1982 for both instruments, where almost all observations were from 1981–82. The second time period is 1983–1996, also for both instruments, when the first third of the RG 82 First Order network was established. The third time period is 2001–2003 for G290 and 2001–04 and 2013 for G54. In 2014, G54 was cleaned on the inside and it consequently got a new scale factor for the time period 2015–17.

The standard uncertainty of unit weight for

- all observations is 0.76,
- the FG5 observations is 1.25,
- the A10 observations is 1.32,
- the relative observations is 0.74, and
- the precomputed differences is 0.68.

The á priori standard uncertainties were slightly overestimated for the absolute gravity observations and slightly underestimated for the relative observations, hence the ideal number for the standard uncertainty of unit weight is 1 for all types of observations. We note that due to the choices made in the adjustment the uncertainties are based on the relative uncertainties of all absolute meters (FG5 and A10) and do not include the absolute uncertainty.

The highest gravity value in RG 2000 is 982428177.6 (Karesuando AA, the northernmost point in the network) and the lowest gravity value in RG 2000 is 981521756.7 (Maglarp AA, the southernmost point in the network) resulting in a gravity range of 906420.9 μ Gal in the RG 2000 network.

4.3 Transformation between RG 2000 and earlier networks

The first transformation between RG 2000 and the previous reference frame RG 82 was performed by deriving a 1-parameter fit, after correction for land uplift, based on 24 of the points included in the Zero Order Network of RG 82. The resolved transformation parameter is 28.2 μ Gal, and standard uncertainty in one common point is 6.1 μ Gal. After that, an inclined plane transformation between the same points and frames was derived. The standard uncertainty in one common point is 4.3 μ Gal for these 24 points and 10.8 μ Gal, if adding the other 176 points included in the RG 82 networks.

Of the 55 new Class B points and the 16 Class B points included in RG 62, 66 were assigned with a RG 82 value in 2016, using the same software and methods as for the RG 82 First Order Network, with the intention to use them in a transformation between the frames. It is noticeable that for these 66 Class B points with RG 82 values, which were actually not included in RG 82, the standard uncertainty in one common point is 8.4 μ Gal, which means less than for the average point included in RG 82. Re-

siduals for the 24 points are shown in Fig. 14. This transformation is hereby the official transformation from RG 82 to RG 2000, thus its inverse is the official transformation from RG 2000 to RG 82.

Due to the well-known bias in the determined absolute level of Potsdam in the ECS 62, and due to poor relative instruments and observations, RG 62 is separated from RG 2000 with between 14.5 and 14.8 mGal. Through the work with RG 2000, totally 95 connections are derived between RG 62 and RG 2000, of which 56 are from observations after 1980 that can be considered good. A second-degree polynomial function was in 2000 derived to transform points between RG 62 and RG 82. When using this function together with a land uplift model and the inclined plane transformation between RG 82 and RG 2000, the standard uncertainty for the difference between RG 62 and RG 2000 in one common point is 50 μGal for the 56 good points. When developing either a new second-degree polynomial function to RG 82 and using the inclined plane transformation to RG 2000 or developing a new second degree polynomial function directly between RG 62 and RG 2000 together with a land uplift model, the standard uncertainty in one point only gets slightly better for the absolute 56 points, about 40 μGal . The reason why the new transformations do not improve the values so much is due to the bad symmetry and geometry in the RG 62 frame (Fig. 15). This can be shown with a 1-parameter fit between the absolute 56 points in RG 62 and RG 2000, corrected for the land uplift in advance. There, the resolved transformation parameter is 14636 μGal and the standard uncertainty in one point is 81 μGal . The insufficient quality of RG 62 is also obvious because it is impossible to get a good transformation from this frame to any other frame. All the relative observations for the RG 62 network have also been adjusted in Gad, but we tested two approaches: (i) using the absolute values of the absolute points in RG 62 and (ii) using the A10 observations from 16 points, corrected to the epoch 1962.0 as absolute values. Both approaches showed large contradictions in the relative observations, as suspected, and different scales for the instruments between Oslo and Copenhagen (the absolute points of RG 62) in comparison to the rest of the network. Since the old transformation has already been used for international geoid computations, we consider it wise to keep the old connection and use the land uplift model and the new transformation from RG 82 to get to RG 2000.

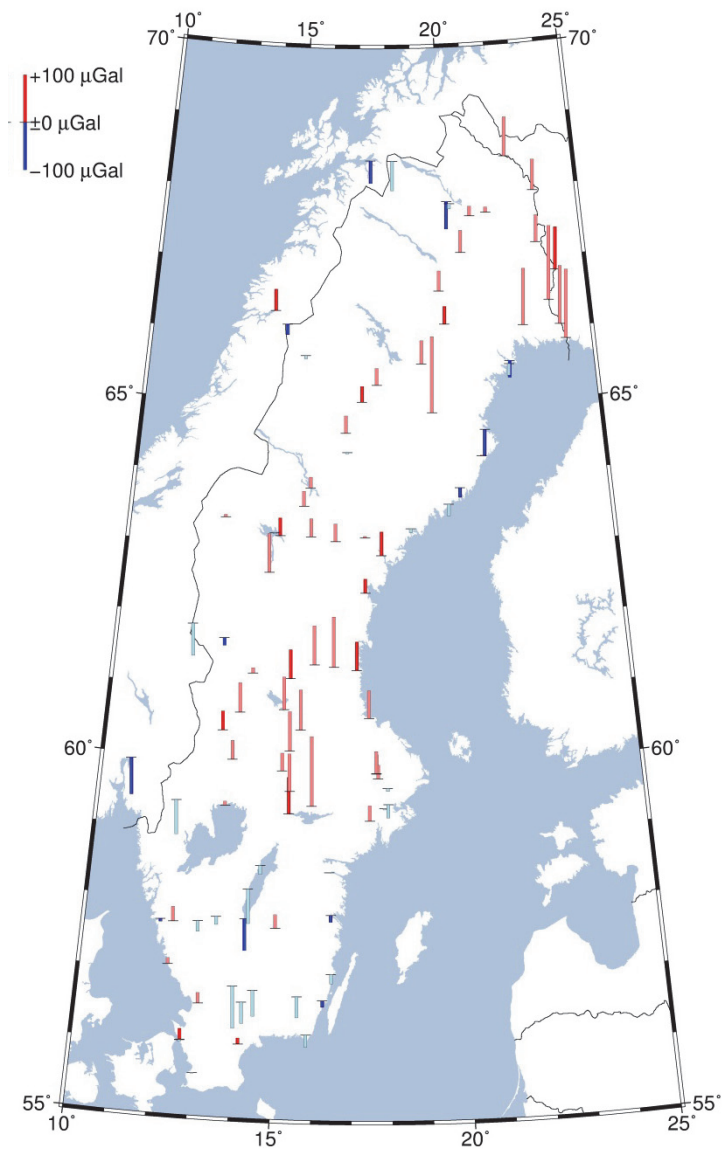


Fig. 14. Residuals in inclined plane transformation between RG 82 and RG 2000 after correction for land uplift.

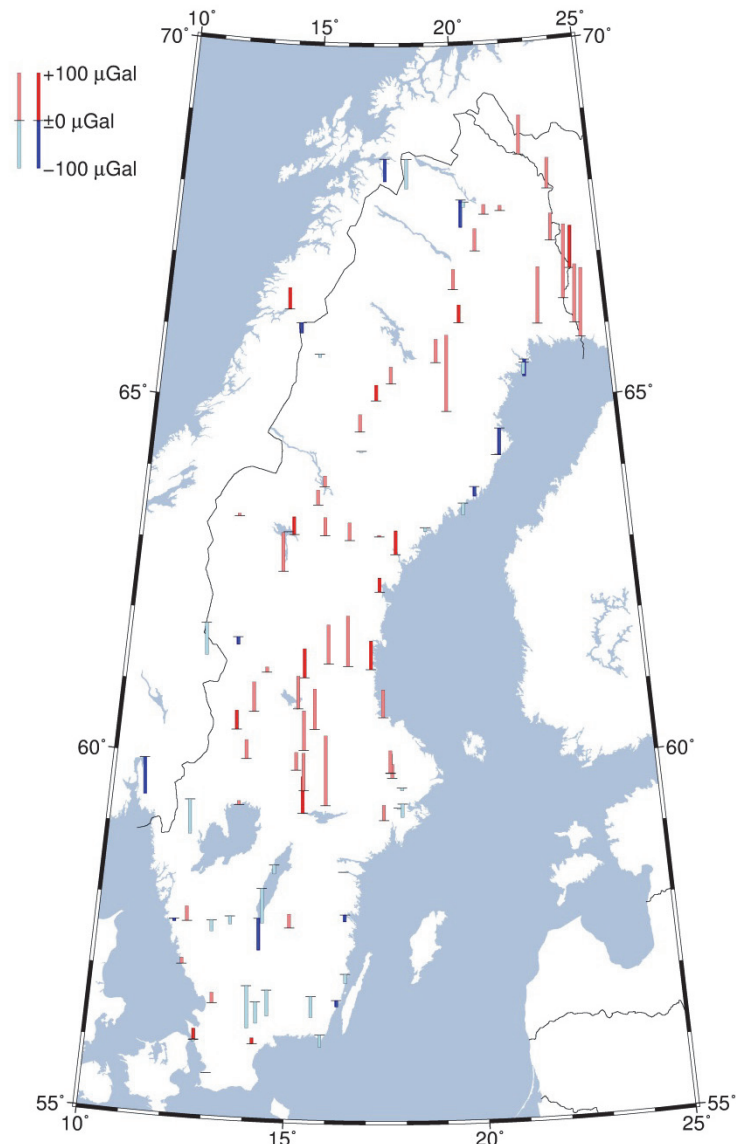


Fig. 15. Residuals in the two step transformation between RG 62 and RG 2000 after correction for land uplift. The darker bars are the 56 points with a quality check and the lighter bars are the other 39 points.

5 *Summary/Conclusions*

RG 2000 is the new gravity reference frame of Sweden. It is based on 13 FG5 absolute gravimeter stations and densified with 96 A10 absolute gravity points. The absolute stations are connected and complemented with relative gravimeter observations so that a spatial resolution of less than 50 kilometres is given everywhere in the country except on remote places where no roads exist. All gravity observations have been reduced to the epoch 2000 by means of the postglacial gravity change model NKG2016LU_gdot. RG 2000 is in the zero permanent tide system. All gravity observations were adjusted in a common adjustment using an in-house software resulting in an RMS for the estimated standard uncertainty for the 329 points of about 3.6 μGal .

When constructing the network much care was taken to include points and observations from the older Swedish gravity reference frames, RG 62 and RG 82, forming a good foundation for evaluating and developing transformations between the frames.

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[1] IAG = International Association of Geodesy

[2] GGHS = Gravity, Geoid and Height Systems