Preliminary Results on Quasi-Geoid for Western Part of Latvia Using Digital-Zenith Camera and DFHRS V.4.3 Software

K. Morozova[1,2,3], R. Jäger[3], J. Balodis[1], G. Silabriedis[1], J. Kaminskis[1,2], M. Kalinka[1,2], K. Balodis[1] and I. Mitrofanovs[1]

[1] Institute of Geodesy and Geoinformatics, University of Latvia, Riga, Latvia
[2] Department of Geomatics, Faculty of Building and Civil Engineering, Riga Technical Univ., Riga, Latvia
[3] Institute of Applied Research (IAF), Hochschule Karlsruhe – Univ. of Applied Sciences, Karlsruhe, Germany

Abstract

This research presents a quasi-geoid model for the Western part of Latvia based on parametric modelling of continuous polynomial surface by DFHRS (Digital Finite-element Height Reference Surface) software v.4.3 developed by Hochschule Karlsruhe – University of Applied Sciences. Apart from standard observations for quasi-geoid determination – GNSS/levelling points and Global Geopotential Models (GGMs), new kind of measurements – astrogeodetic vertical deflection (VD) observations provided by Digital Zenith Camera (DZC) are used. This instrument has been developed by the Institute of Geodesy and Geoinformatics, University of Latvia (GGI) and provides the accuracy of about 0.10 arcsec, what equivalent to 0.5 mm error in elevation for 1 km length and it gives two times better accuracy than 1st order levelling in Latvia respectively. 44 1st order GNSS/levelling points were observed by GGI staff and 27 1st order levelling points were provided by Latvian Geospatial Information Agency (LGIA), though 12 points were excluded from the processing, because of gross errors; 10 2nd order GNSS/levelling points and 98 points of terrestrial VD observations were also observed by GGI. The research presents the results on different solutions of quasi-geoid in Kurzeme region, using different types of data and the common principle of DFHRS software is also included.

Keywords: DFHRS, geoid, GNSS, levelling, quasi-geoid, Vertical Deflections

1   Introduction

In terms of the project, the main task is to compute the gravity field and develop a precise quasi-geoid model of up to 1 cm in accuracy using all available data.

In the year 2016 the quasi-geoid model for the Eastern part of Latvia *(Balodis et al.,* 2016) has been computed using the Karlsruhe University of Applied Sciences developed DFHRS (Digital finite-Element Height Reference Surface) software version 4.0 *(Jäger,* 2018), which allowed for the combination of GNSS/levelling data together with Global Geopotential Models (GGMs).

At the moment, the Institute of Geodesy and Geoinformatics (GGI) is dealing with a new kind of system development and related measurements – Digital Zenith Camera (DZC) and astrogeodetic vertical deflection (VD) observations – which are possible to use in a new version DFHRS 4.3 in respect to the use of GNSS/levelling data together with GGMs (http://icgem.gfz-potsdam.de/home) and terrestrial VD measure-

ments or/and VD derivatives from GGMs. VD measurements ξ(B, L, h), η(B, L, h) allow for independent checking of places with inconsistencies and improvement of the quasi-geoid model. VD test measurements were done in the Riga region and proved the improvement of quasi-geoid twice (*Morozova et al.,* 2017). DZC and processing software was developed by GGI (*Zariņš et al.,* 2016) and currently being carried out in the Western part of Latvia – Kurzeme region.

2   Test area and data

EGM2008 (*Pavlis,* 2008*)* was used as GGM; 24225 geoid undulations and VD derivatives were computed from it. The current number of terrestrial VD points observed with the DZC is 98, and the precision of these observations is ~ 0,10-0,15 arcsec. In terms of this project, all 1st order and 2nd order levelling points in the Western part of Latvia were provided by Latvian Geospatial Information Agency (LGIA), but GNSS observations on the 71 1st order levelling points were carried out by GGI and partially provided by LGIA, and GNSS measurements on the 10 2nd order levelling points were also carried out by GGI. GNSS measurements were performed by 4 hour sessions and 1 sec sampling interval. GNSS data were post-processed using Bernese GNSS Software Version 5.2 (*Dach et al.,* 2015), and one sigma of post-processed data do not exceed 0.004 m. 9 European Permanent Network (EPN) (http://www.epncb.oma.be/) stations (nearest around Latvia) were used as reference stations and transformed to one epoch: 2015.0 using Helmert transformation coordinates. 12 points were excluded because of gross errors in DFHRS software. Fig. 1 introduces the location of GNSS/levelling points and VD observations.
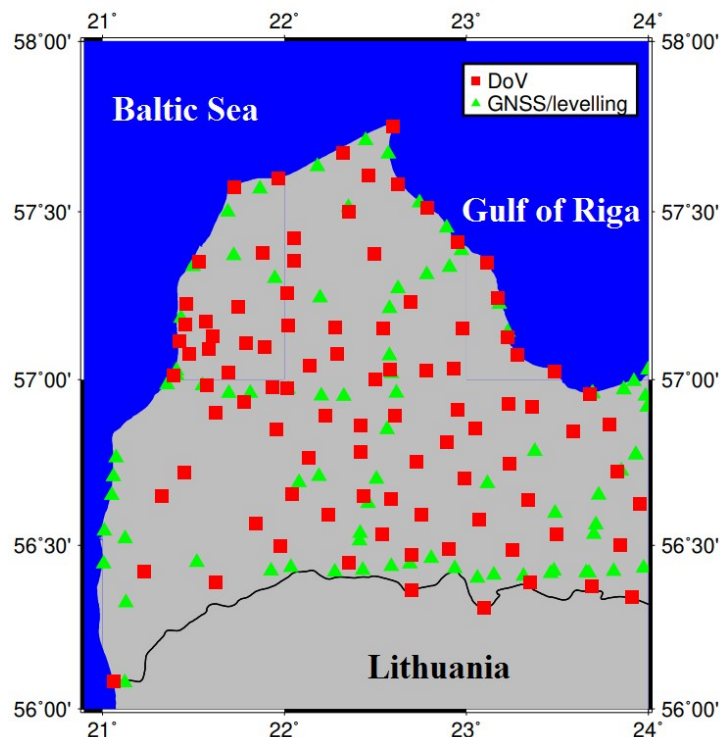

Fig. 1. The location of GNSS/levelling points and VD observations.

The distance between GNSS/levelling points is about 10–15 km in order to provide sufficient data coverage and to develop 1 cm precision quasi-geoid model for Latvia. VD measurements will be continued to homogeneously cover the entire territory and improve the current version of quasi-geoid (especially at the borders).

3   DFHRS modelling method

In the DFHRS concept, a continuous polynomial surface over of a grid of finite element meshes (FEM) (Fig. 2, and thin blue lines in Fig. 3) with polynomial parameters p is used as a carrier function for the HRS. It reads:

$$N = f^T p \tag{1a}$$

$$with\ f(B, L) = [1|B, L|B^2, B \cdot L, L^2| \dots]^T\ and$$
$$p = [p_{00}|p_{10}, p_{01}|p_{20}, p_{11}, p_{02}| \dots]^T \tag{1b,c}$$

The computation of the polynomial coefficients p (1a, c) of the Gauß-Markov-Model (GMM) by the DFHRS software is done by introducing additional continuity conditions along the mesh borders as in a finite element model (FEM), e.g. in mechanics. So, the geoid or quasi-geoid surface N is always continuous overall, and therefore N in the DFHRS approach is also called *N=NFEM (p| B, L, h),* where *B* – latitude, *L* – longitude, *h* – ellipsoidal (geodetic) height.
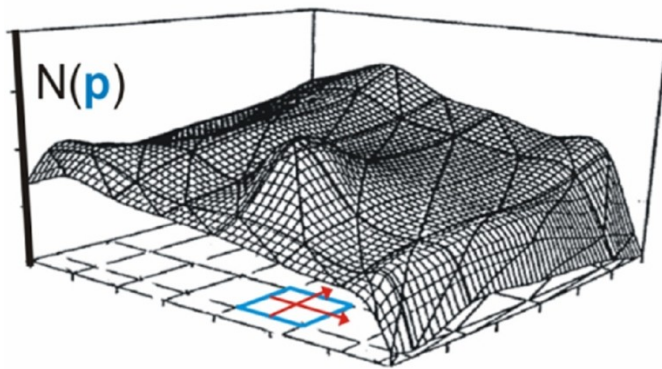

Fig. 2. NFEM (p| B, L, h) model of the quasi-geoid N_{OG}, continuous along the mesh-borders by equations (5) in the GGM of DFHRS (*Jäger,* 2012).

Fig. 3. NFEM meshes (thin blue lines) and "patches" (thick blue lines) for the computation of West European quasi-geoid by DFHRS (*Jäger,* 2018).

For some old normal height systems *H*, a scale-difference factor *Δm* must be considered so that the DFHRS-model of N (1a, b, c) consists of two parts. Thus, the principle of a GNSS-based height determination H requires recording the ellipsoidal height h in the DFHRS software, where the DFHRS (B, L, h)- correction factor N is applied (*Jäger*, 2012*; Jäger and Schneid*, 2004).

$$H = h - N = h - DFHRS(p, \Delta m|B, L, h) = h - (NFEM(p|B, L) + \Delta m \cdot h) \tag{1d}$$

The GGM is used to estimate the parameters of DFHRS as being consistent with the quasi-geoid approach of Molodensky (*Molodensky,* 2001). The observation equations (respective residuals introduced with *v*) for GNSS/levelling points (2a, b), quasi-geoid and VD (ξ, η) observations taken from GGMs like EGM2008 or grids like the EGG97 (*Denker and Torge,* 1998*)* (3a, b, c), and DZC observed VD (4a, b), together with the above continuity *(C)* equations (5) read:

$$h + v = \hat{H} + NFEM(\hat{p}|B,L) + \Delta\hat{m} \cdot h = \hat{H} + f(B,L)^T \cdot \hat{p} + \Delta\hat{m} \cdot h \tag{2a}$$

$$H + v = \hat{H} \tag{2b}$$

$$N_{QG}(B,L)^j + v = NFEM(\hat{p}|B,L) + \Delta\hat{m} \cdot h + \partial N_{QG}(\hat{d}_N^j) \tag{3a}$$

$$\xi^j + v = -\frac{f_B}{M(B)+h} \cdot \hat{p} + \partial B\left(\hat{d}_{\xi,\eta}^j\right) \tag{3b}$$

$$\eta^j + v = -\frac{f_L}{(N(B)+h)\cos(B)} \cdot \hat{p} + \partial L\left(\hat{d}_{\xi,\eta}^j\right) \tag{3c}$$

$$\xi^j + v = \frac{-f_B}{M(B)+h} \cdot \hat{p} \tag{4a}$$

$$\eta^j + v = \frac{-f_L}{(N(B)+h)\cdot\cos(B)} \cdot \hat{p} \tag{4b}$$

$$C + v = C(\hat{p}) \tag{5}$$

The datum parts $\partial N_{QG}(\hat{d}_N^j)$ (3a), $\partial B\left(\hat{d}_{\xi,\eta}^j\right)$ (3b) and $\partial L\left(\hat{d}_{\xi,\eta}^j\right)$ (3c) are related to the Molodensky transformation (*Jäger,* 2012*)*. They permit the removal of long-waved systematic errors "weak-shapes", in the respective observation components via mesh compounds, so-called "patches" (Fig. 3, thick lines). For the same geopotential or grid data basis, the datum parameters in the jth "patch" are identical. The estimation of the NFEM-based quasi-geoid surface parameters $\hat{p}$, the scale $\Delta\hat{m}$ and the patches datum parameters ($\hat{d}_N^j$, $\hat{d}_{\xi,\eta}^j$, $\hat{d}_{\xi,\eta}^j$) in the DFHRS 4.3 software is based on a Least Squares adjustment related to functional model (2a–5). This is accompanied by a statistical quality analysis of all observation data (2a–4b) (variance component estimation, data snooping). The example of the adjustment protocol of the DFHRS-software related to the fitting point heights H (2b) can be found in (*Morozova et al.,* 2018).

4   *Results*

Table 1 presents the assessment of preliminary results of quasi-geoid determination for the Western part of Latvia. 4 solutions are introduced using different data types: GNSS/levelling points, GNSS/levelling points + observed VD measurements by DZC, GNSS/levelling points + VD derivatives from EGM2008 and GNSS/levelling points + VD derivatives from EGM2008 + observed VD measurements by DZC. 69 GNSS/levelling points were used in estimating statistics.

The results show a significant improvement in the standard deviations (SDs) which use both terrestrial VD and VD derivatives from EGM2008. For example, if we compare the 1st and 2nd solutions presented in Table 1, we can see that the use of terrestrial VDs results in an improvement in accuracy of more than double the original determined accuracy. Comparing the 3rd and 4th solutions (there are no significant differences between these two solutions, because the number of VD derivatives was 24225 compared with 98 terrestrial VDs) with the 2nd solution, we can see an improvement of 0.004 m due to VD derivatives. Fig. 4 presents the difference between the solutions that take into account observed VD measurements and those that do not use these data. The maximum difference and impact of terrestrial VD is ± 1 cm, which indicates better local anomalies in comparison to global models.

Table 1. The evaluation results of 4 solutions [in units of m].

Used data	SD	Min	Max	Mean
EGM2008 (without VD)+GNSS/levelling points	0.0429	-0.1280	0.1030	-0.0098
EGM2008 (without VD)+GNSS/levelling points + VD by DZC	0.0204	-0.0530	0.0420	-0.0017
GNSS/levelling points + VD derivatives from EGM2008	0.0161	-0.0310	0.0260	-0.0005
GNSS/levelling points + VD derivatives from EGM2008 + observed VD by DZC	0.0160	-0.0310	0.0250	-0.0002

Fig. 5 depicts the comparison of the solution using GNSS/levelling points, observed VD and EGM2008 GGM with the LV'14 local quasi-geoid, which is currently officially adopted in Latvia and developed by LGIA. This model has been computed with a GRAVSOFT software using the remove-restore technique. 4886 relative gravity measurements, 84 GNSS/levelling points, the free air gravity anomaly model DTU13 from satellite altimetry, the DTM model and GOGRA02s GGM were used as input data. (map.lgia.gov.lv, 2008–2018). Fig. 6 presents the comparison of the same solution with the LV98 developed by *Kaminskis* (2010a). It is the previously used quasi-geoid model, transformed to the new Latvian reference height system – LAS-2000.5 (from BHS-77). The estimated precision of LV98 model was evaluated ~6–8 cm, and it was the first quasi-geoid model in Latvia, where satellite altimetry data made by the ERS-1 were implemented. Apart from ~500 terrestrial gravity measurements, ~12000 digitized gravimetric points were included in modelling (*Kaminskis,* 2010b). By comparing the GGI solutions with the LV'14 and LV98 local models, we can clearly see that better coincidence is found with the LV'14 quasi-geoid model: the differences vary from -0.086 up to 0.047 m, while the differences with the LV98 quasi-geoid model vary from -0.082 up to 0.138 m.
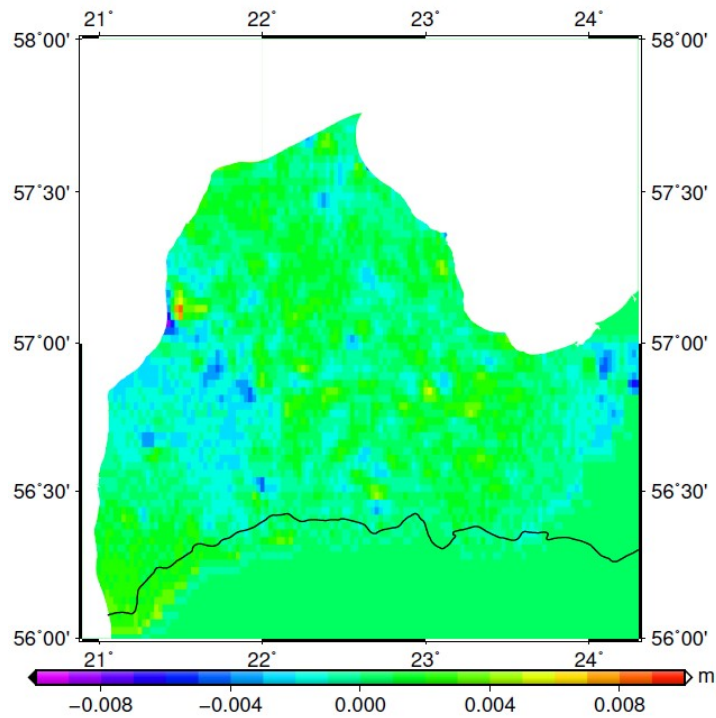
Fig. 4. The difference between the solution with only GNSS/levelling data + derivatives from EGM2008 and GNSS/levelling data + derivatives from EGM2008 + VD.
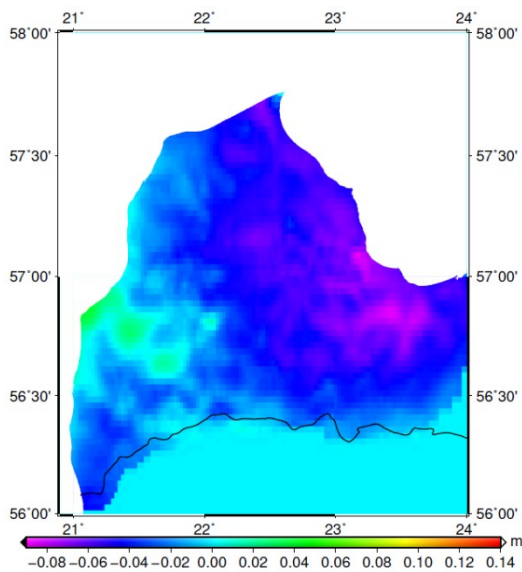

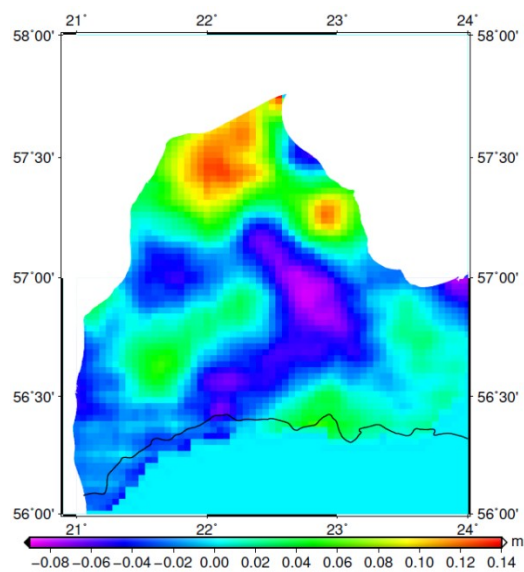Fig. 5. The difference between LV'14 and GNSS/ levelling + VD + EGM2008 solution.

Fig. 6. The difference between LV98 and GNSS/ levelling + VD + EGM2008 solution.

5   Conclusions

The use of both terrestrial VD and VD derivatives gives a significant improvement of the quasi-geoid model, and, in case of systematic errors of the quasi-geoid, VD can "reveal" these in terms of a changed shape of the quasi-geoid. This improvement concerns more the moutain areas, e.g. due to worse levelling fitting points, than in the

flat areas. By analyzing the area of biggest difference of GGI solution and LV'14 (violet color in Fig. 5) we can see that just few GNSS/levelling points were left (Fig. 1) in that area, and most of them were excluded from the quasi-geoid computations, because of VD based detected statistically significant gross errors (12 points). From that, we conclude that these points should be remeasured in combination with further VD observations to enable a profound statistical control of the GNSS/levelling fitting points. Another topic of further research will deal with the 1st order design optimization on the optimal location by using both gravity data and DV respectively in a hybrid network design.

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