Geophysica (2013), 49(1–2), 33–51

Lithologically Constrained Gridding of Petrophysical Data

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(Received June 15, 2012; Accepted October 15, 2012)

Abstract

We present a numerical method where the information of digital geological maps is used to constrain the interpolation of irregularly sampled petrophysical data onto an evenly sampled grid. The gridding is based on inverse distance weighting and moving window strategy that limits the petrophysical data and map polygons to those overlapping the investigation area. The proportional surface area of the lithological units inside the investigation area are used as basic areal weights that aim to reduce the effect of outcropping geological formations with small surface area but large number of samples. Additionally, the mean, mode or median of the samples within the lithological unit below the grid point is used as an artificial data value that provides an estimate for the base or background value for the petrophysical data. Limiting condition based on standard deviations around the base value and rejection rules based on six level rock classification of the samples are used to confine the range of parameter variation within each lithological unit. Compared to traditional interpolation methods lithological weighting produces maps that are more meaningful geologically, particularly in those areas where only few rock samples are available. The method has been used to prepare petrophysical maps of the bulk density and magnetic susceptibility in Finland.

Keywords: gridding; geological maps; gravity method; magnetic method

1 Introduction

For more than sixty years Geological Survey of Finland (GTK) has collected petrophysical data from Finnish bedrock samples. Primarily, the data contains information about the density and magnetic susceptibility but some samples have been analyzed for remanent magnetization (*e.g*. Q-ratio), electrical resistivity and polarization properties (frequency IP) and porosity. Naturally, the data also contains map coordinates and the name and the hierarchical classification code of the rock. Currently, the petrophysical database contains more than 130.000 samples.

The petrophysics of Finland has been discussed *e.g*. in *Puranen et al*. (1978), *Lähde* (1985), *Puranen* (1989), *Elo* (1997), *Korhonen et al*. (1997), and *Korhonen et al.* (2002a, 2002b). *Puranen et al*. (1978) presented a summary and an analysis of densities

of more 30.000 specimens of the Finnish bedrock. They presented a density map of Finland based on areal means of about 4800 km^2 units. The mean densities of areal units were determined as weighted averages of mean densities of rock types. Areal proportions of rock types, determined from geological maps for each unit area, were used as weights. They also observed distinct relations between areal means of densities and gravity anomalies and presented interpretation of gravity anomalies based on such relations and on conventional gravity modeling. *Lähde* (1985) refined the analysis presented in the paper by *Puranen et al*. (1978) and compared eight geological provinces based on more detailed data. *Puranen* (1989) presented a summary and analysis of magnetic susceptibilities of more than 40.000 rock specimens of the Finnish bedrock. With the aid of para- and ferromagnetic susceptibilities of rocks their iron and magnetite content were estimated. Relations between remanent magnetization and susceptibility were studied to determine the total magnetization of rocks for magnetic interpretations. The methods and relations presented were applied to comparison of geologic provinces in Finland. *Elo* (1997) presented, among other things, principal density components of the Finnish bedrock and analyzed density variations of mafic and metamorphic rocks. *Korhonen et al*. (1997) presented the aims and results of the regional petrophysical programme of the Geological Survey of Finland and discussed questions such as "Near surface versus deeper magnetic sources", "Petrophysics of Middle and Lower Crust", "Correlation of petrophysical properties with rock geochemistry", and "The role of geophysics in the crustal model program of Finland". *Korhonen et al*. (2002a ja 2002b) presented several petrophysical maps as supplements to the Gravity anomaly and Magnetic anomaly maps of the Fennoscandian Shield.

The main problem in the mapping of petrophysical data is the irregular sampling. Samples are often concentrated on few outcropping and unweathered rock formations of small size or only on interesting rock types. Surrounding regions consisting of less durable rocks are covered by soil, sediments and sea or lake water, or uninteresting rock types and, therefore, may not contain any or only a few samples. This disparity biases the overall mean of the petrophysical parameter in the region towards the one with large amount of samples. Moreover, data interpolation and extrapolation over large areas without any sample points can give misleading mapping results. The geological maps are interpretation results derived from direct geological observations and indirect geophysical information, gravity, airborne magnetic and electromagnetic measurements, in particular. Therefore, combined use of petrophysical data and geological maps should provide more accurate mapping results. Moreover, lithological weighting can delineate the geological formations better.

In Finland the geological and areal variation of rock densities has previously been studied by *Puranen et al*. (1978). Because in those days most of the work needed to be carried out manually, they made an overall study of Finland using generalized data and detailed studies of few areas of limited size. The method discussed in this paper follows their principles, but uses modern computer algorithms to solve the problem and extends the method even further. Examples of lithologically weighted petrophysical maps are given for a small study area and for the whole Finland.

2 Method

The lithological weighting of petrophysical data is made using the digital geological maps of Finland in the scale of 1:1 million (*Korsman et al*., 1997) and the petrophysical database of rock density and magnetic susceptibility values measured in laboratory conditions (*Korhonen et al*, 1997). The digital map used in this study consists of 92 lithological units that correspond to different geological rock type, stratigraphy, and age. It must be noted that lithological map is not equal to a bedrock map. For example, in different parts of the country granite rocks belong to different lithological units depending on the age. The original digital map consists of 7922 polygons of which 1959 are duplicates resulting from holes inside the actual polygons. As a whole, the polygon data contain about 450000 vertex points. Because of the high level of details of the geological map of Finland in the scale of 1:1 million, a simplified version in the scale of 1:5 million is shown in Fig. 1.

The petrophysical database contains over 130.000 samples. Fig. 2 shows a map of the locations of the sample points used in this study. The map illustrates that the sampling is highly irregular. Particularly in the sparsely inhabited northern Finland the sampling is incomplete and follows the main roads. In many places the sampling density is discontinuous across the borders of 1:100.000 map sheets inside which bedrock mapping has been made. Areas of high mineral prospecting activity show also finer sample density. Please, note that Fig. 2 and the results hereafter are plotted using the uniform map coordinate system (YKJ) of Finland.

Before actual gridding a point-in-polygon algorithm was used to assign each petrophysical data point an index code (0–92) of the corresponding lithological class based on its location on the geological map. Petrophysical data points outside all polygons were given class number 0. In addition, before gridding the density values were limited between 2200 and 3200 kg/m^3 that roughly equals to the mean $\pm 3.0 \times$ the standard deviation of the original data. The main purpose was to remove outliers, erroneous data values, and exceptional data values taken particularly from ore bodies. The resulting density data consists of 129252 samples. The mean density and the standard deviation are 2725.0 and 134.8 kg/m^3, respectively. Similarly, negative values and values less than 10^{-6} SI units were removed from magnetic susceptibility data. The final $\log_{10}$-normalized susceptibility data consists of 125889 data points, the mean and the standard deviation being -3.163 and 0.8551 ($\log_{10}$ SI), respectively.

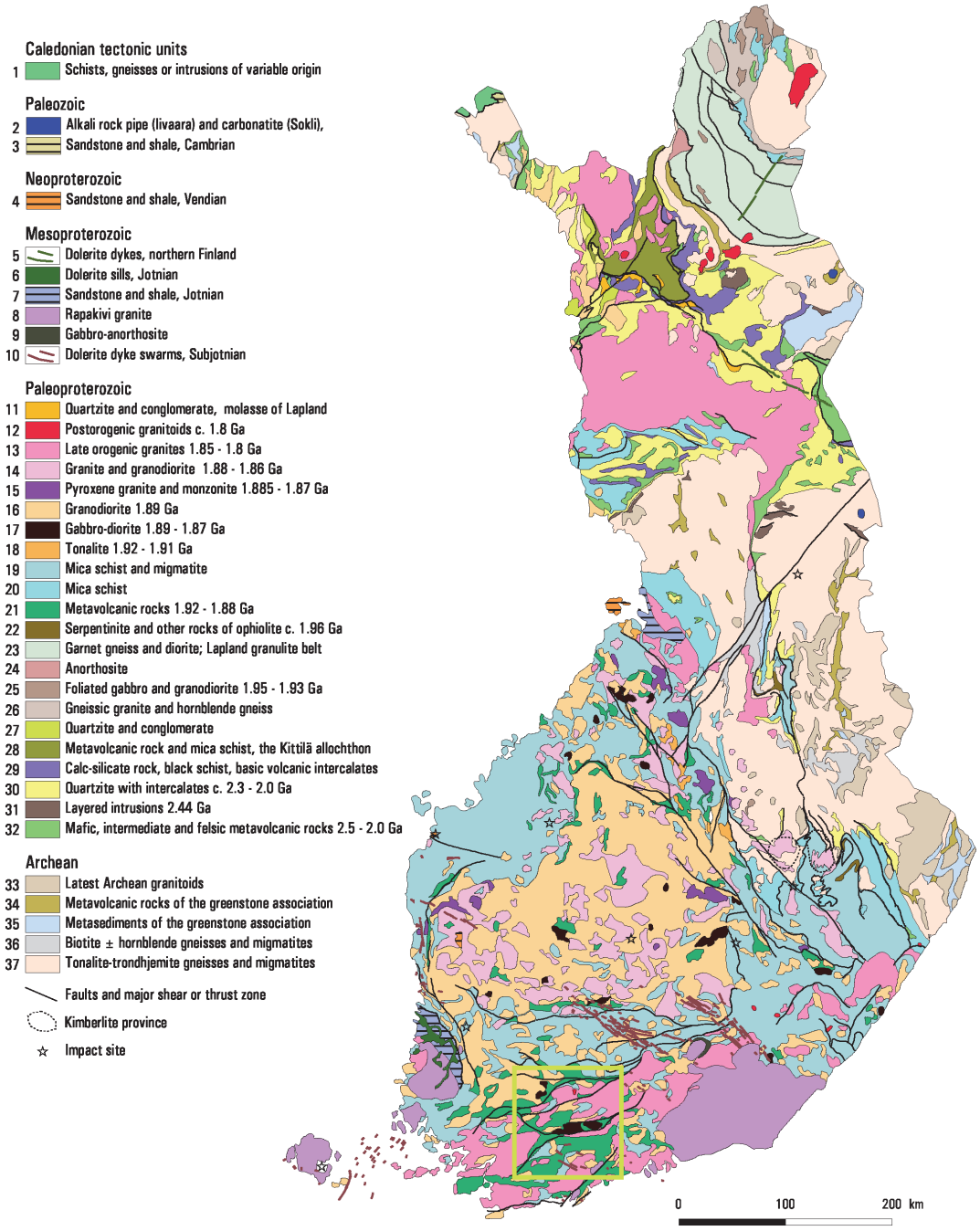

Fig. 1. Geological map (scale 1:5 million) of Finland (*Korsman et al.*, 1997). Yellow rectangle shows the location of the detailed study area discussed in the text.

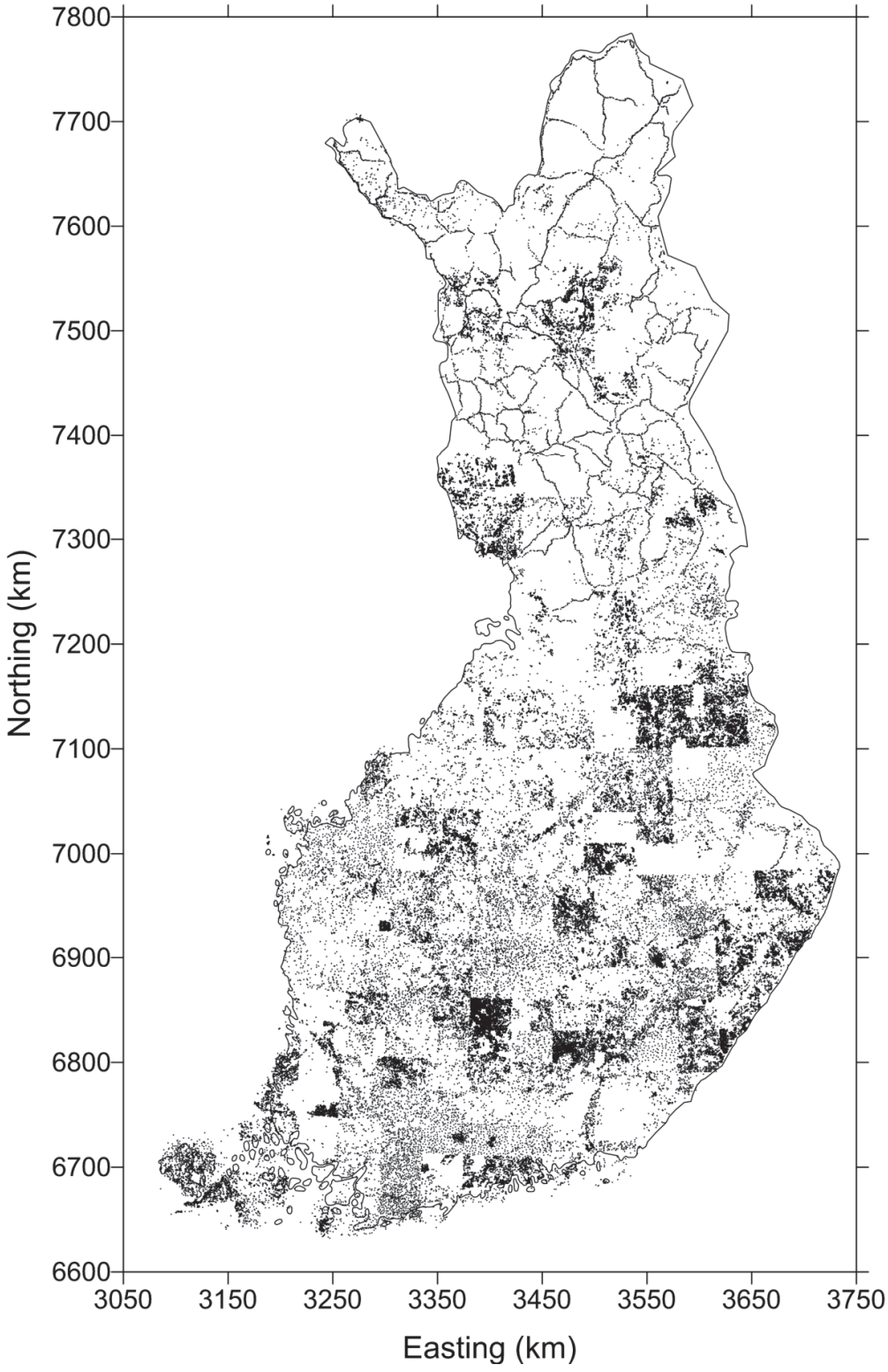

Fig. 2. Location of the petrophysical samples in Finland collected by GTK. Coordinates refer to the uniform map coordinate system of Finland (YKJ).

Because of the vast size of the research area covering the whole Finland (c. 700 km by 1200 km) and the large number of both polygon vertices and petrophysical samples, the gridding is based on moving window strategy. The concept of mapping is illustrated in Fig. 3. The rectangular research area R to be gridded with regular sampling interval is systematically processed by a moving computational window C. For each grid point g inside C the petrophysical data and the lithological polygons are limited to the investigation area I defined by the computational area and user defined margins M around it. This scheme speeds up computations because it allows using only a fraction of the petrophysical data and the lithological polygons for each grid point inside C. Note that the window C does not need to be moved for every grid point. However, to justify the use of statistical discrimination methods a sufficient amount of data points must locate inside I. Therefore, margins M are doubled in size until the number of petrophysical data points N is greater than some predefined minimum value N_{min}. Also note that for smaller areas such as a single map sheet of size 40 km × 30 km, the computational window can be made equal to research area ($C = R$), while margins M should extend beyond the map sheet to enable proper mapping near the borders of the research area.

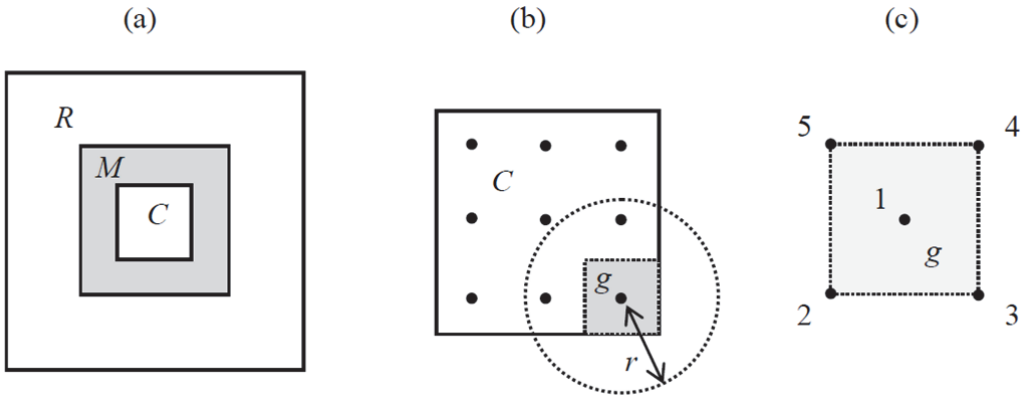

Fig. 3. Moving window strategy: (a) concept of the moving window strategy: total research area R, computational area C, and margins M, (b) the search radius r around grid point g, and (c) the five test points of a grid point g.

Before processing individual grid points, the surface area A_l of the L lithological units inside the current investigation area are computed using the algorithm of *Zerzan* (1989). Lithological weights, w_l, are defined as the surface area of the lithological units (A_l) overlapping with the investigation area divided by the total area (A) of I,

$$w_l = \frac{A_l}{A}, \quad l = 1,2,...,L \tag{1}$$

The lithological weights are passed to the N sample points ($w_l \rightarrow wl_k$, k=1,2,…,N) using the lithological classification codes (0–92) computed beforehand. In other words, sample point k inside polygon l is given the weight $wl_k = w_l$. Note that the lithological weights depend on the current investigation area and cannot be computed in advance.

Each grid point must be located inside some polygon, *i.e*, within national borders and islands. If *C* does not contain any lithological units (*e.g*. the point is located offshore), all grid points inside *C* are given a user defined dummy value, f_0, reserved for missing data. This dummy value is usually the mean or median of the whole petrophysical data. Moreover, for each grid point, *g*, the center and the four corner points around it (Fig. 3c) are tested for underlying polygons. If all five test points are outside the lithological polygons, the grid point is assigned the dummy value. This method prevents assigning values for small islands and points close to the coast line or national borders. Although it would be more accurate to check if the rectangular area of the grid point overlaps with any of the polygons, it is much faster to check the test points only.

If the grid point (the test points) overlap with a lithological polygon, the petrophysical samples within the search radius, *r* (Fig. 3b) around the grid point are sought for. To get some statistical control the search radius is doubled in size until the number of samples *Ng* inside the search area is greater than some user defined minimum number of samples per grid point, Ng_{min}. To emphasize the importance of the nearest sample points inverse distance weighs, *wd*, are computed

$$wd_k = \left(\frac{d_0}{d_k + d_0} \right)^a , \quad k = 1,2,...,Ng , \tag{2}$$

where d_k is the distance of the sample point from the centre of the grid point, *a* is the power of the inverse weighting, and d_0 is a scale length that defines the half weight distance ($wd_k = 1/2$, when $a = 1$ and $d_k = d_0$) that prevents division by zero when $d_k = 0$. Normally, d_0 can be set equal to the grid sample spacing.

After the petrophysical samples within the search radius have been found and their lithological and inverse distance weights have been determined, the final lithologically weighted mean at the grid point can be computed

$$g = \frac{1}{Sw} \sum_{k=1}^{Ng} wl_k \cdot wd_k \cdot F_k , \tag{3}$$

where F_k are the *Ng* sample values inside the search radius, wl_k is the lithological weight of the polygon inside which the sample point *k* is located, and the sum of weights is

$$Sw = \sum_{k=1}^{Ng} wl_k \cdot wd_k . \tag{4}$$

The basic areal lithological weighting, as described above, aims to reduce the importance of outcropping lithological units with small surface area. To provide better grid estimates in sparsely sampled areas an additional lithological weighting method is used where the background values of the petrohysical parameter of the lithological unit or units below the five special points (Fig. 1c) are used as additional, artificial sample values, which are added to weighted mean of Eq. (3). The background value, f_1, can be ei-

ther the mean, median or mode of all the samples inside same lithological unit in current investigation area. The additional lithological background weighting can be done either with or without the areal lithological weights (*wl*). In the latter case the equation for the lithological background weighting becomes

$$g = \frac{1}{Sw}\left[\sum_{k=1}^{Ng} wd_k \cdot F_k + wg \cdot \sum_{k=1}^{5} wd'_k \cdot F'_k\right], \tag{5}$$

where F_k' are the artificial data values below the five special points around grid point, wd'_k are their inverse distance weights, and *wg* is the global weight that is used to emphasize the effect of background weighting with respect to areal weighting. In this case the sum of weights becomes

$$Sw = \sum_{k=1}^{Ng} wd_k + wg \sum_{k=1}^{5} wd'_k \,. \tag{6}$$

The bigger the value of the global weight is, the more the gridding starts to resemble a "painting program" that fills the area of each lithological polygon with its background value and gives less importance to the inverse distance weighting of the actual petrophysical samples.

To improve data quality and to remove local outliers, a limiting condition is imposed on the sample values. In its simplest form the limiting condition is based on the mean and standard deviation of all the samples inside current investigation area. However, the samples represent various geological materials with different petrophysical properties. Therefore, it is much more efficient to define the limiting condition separately on the samples within each lithological unit. The abovementioned background value (f_1) and the standard deviations (*std*) of the samples in each lithological unit are used to remove data values that deviate enough from the background value. The samples rarely follow normal distribution because the digital maps are approximations and some samples always belong to different geological unit. Therefore, the use of median and rather narrow limiting criterion (e.g. $f_1 \pm 1.0 \times std$) can reveal better estimate for the base level of the petrophysical parameter of each lithological unit. Too narrow cutting criterion, on the other hand, flattens the peak-to-peak variation of the data and the resulting maps.

Intrusive mafic dykes inside sedimentary rocks are one of the biggest problems for petrophysical data gridding. This is because most of the samples are taken from more durable mafic rocks and because of the simplifications of the digital geological maps many samples are thus given incorrect classification codes. As a consequence, dense mafic rocks raise the background density of the sedimentary lithological units, for example. To circumvent such problems, a sample rejection method based on the mineralogical rock type is also performed when the mean or the median of the lithological units are computed. The Geological Survey of Finland uses a hierarchical classification system (*Korja*, 1989) where the six main rock classes are: 1= plutonic, 2= intrusive (dykes), 3= volcanic, 4= sedimentary, 5= metamorphic and 6= altered rocks and ore

samples. These are further divided into subclasses such as 1.1= granitoid, 1.5=gabroid, 2.4= andesite, 3.10= tuffite, 3.14= ultramafic vulcanite, 4.1= clastic sediment, 5.2= schist, 5.3= gneiss and 5.5= granulite, for example. The classification has four sublevels (*e.g.* 1.5.1= gabbro, 1.5.1.1= norite), but for our purposes only the first digit is relevant. The rejection is made based on two logical operations. Either a) rock class X is not allowed in lithological unit Y or b) only rock class X is allowed in unit Y. In our case, the simplest form of rejection is accomplished by allowing only sediment rocks (class 4) in sedimentary units (classes 1,2,3,4 and 6). Any number of additional rules can be set. For example, granite units can contain neither sediments nor volcanic rocks.

The new lithologically weighted gridding method has been implemented as a PC microcomputer program PETROCK written in Fortran90 and compiled with Intel Visual Fortran 11 (*Pirttijärvi*, 2005). In addition to the areal lithological weighting defined by Eq. (3) it saves the results with or without the inverse distance weighting and with or without the lithological background weighting. PETROCK also saves the standard deviations that can be used as error estimates of the gridding. Additional information about each investigation area, including the mean, median, maximum, minimum, standard and average deviation, and the number of sample points are also stored. The lithological weighting could be implemented using scripts and build-in algorithms of common GIS software (*e.g.* ESRI ArcGIS or Pitney Bowes MapInfo).

User defined computational parameters that determine the gridding are the grid sample spacing, the size of the computational window, the width of the margins and the minimum number of data points per investigation area (N_{min}). Parameters related to the inverse distance weighting are the minimum number of data points per grid point (Ng_{min}), the minimum search radius (r_{min}), the power of inverse distance weighting, and the scale length (d_0). The reference value for limiting condition (f_1 = mean of whole subarea or the mean, median or mode of each lithological unit), the width (Δ) of the limiting criteria ($f_1 \pm \Delta \times std$), the global weight ($wg$) and the set of rejection rules are important for the additional lithological background weighting. Multiple data values (from a drill-hole) can be replaced with the mean, median or mode (the largest class), or they can all be accepted or rejected totally. Moreover, sample coordinates can be rounded, in which case the mean or median of the samples from a borehole, for example, will be used. The coordinates are rounded only if the lithological code and the rock class of the samples are also the same. In other words, closely spaced samples from sediment rocks and mafic rocks are not mixed together.

The procedure of the lithological weighting can be summarized as follows:

1. Define computational parameters.
2. Read in petrophysical data and polygon data:
   Round sample coordinates and handle duplicate values.
3. For each computational window C inside research area R…
   4. Limit petrophysical data and polygons to those inside investigation area I.
   5. Compute areal lithological weights, wl.
   6. Compute (and save) statistics of the whole subarea.
   7. Compute (and save) background value (f_1) for each lithological unit:

Apply limiting condition ($f_1 \pm \Delta \times std$) and rejection rules
8. For each grid point *g* inside computational area *C*…
9. Check for polygons below grid point (and special points).
If there is a polygon below, then…
10.a Find sample points inside search radius
10.b Compute inverse distance weights, *wd*
10.c Compute weighted values (and errors) for the grid point
10.d Otherwise, give dummy value (f_0) for the grid point
12. Save the results and go for the next grid point

End loop 8 (all *g* inside *C*).

End loop 3 (all *C* inside *R*).

3 Results

Fig. 4 shows some mapping results from an area around the cities of Hyvinkää and Hämeenlinna in the southern Finland. Geologically, the area is characterized by high density gabbros in the middle and the low density granites in the surroundings. The rest are metavolcanic rocks, mica schists, granodiorites, and granitoids (cf. Fig. 1). The grid sampling is 2 km × 2 km, the size of the computational area was 20 km × 20 km, and the margins were 50 km wide. The entire research area with size of 100 km by 100 km consisted of 25 subareas. The minimum number of sample points per subarea and grid point was 1000 and 5, respectively. Duplicate data values were replaced with median. Additional limiting condition was used where the reference value was the median and the limits were ±2.0 × the standard deviation. The inverse distance power and scaling distance were 2 and 2 km, respectively. The global weight of the background weighting was $wg = 2$. The only effective rejection rule was to ignore all but plutonic rocks (class 1) from the lithological unit of the granites (class 14).

Fig. 4 (a) shows the main lithological units (1:5 million scale) the colors of which correspond to those in Fig. 1. Map 4(b) shows the location of the 5607 petrophysical sample points (red dots) and the boundaries of the 1:1 million digital geological map (grey lines). Comparison to the simplified lithological map of 1:5 million scale in Fig. 4 (a) reveals the high amount of details of the digital map that is actually used in the lithological weighting. However, to be able to compare the gridding results with the geology better, the other maps show the less detailed geological map of scale 1:5 million. Fig. 4 (c) shows the results from inverse distance (power of two) interpolation of commercial Golden Software Surfer 9. Despite the use of a smoothing factor of 0.5 (km) Surfer's inverse distance interpolation emphasizes individual data points. In this example, inverse distance interpolation was found better than any other Surfer's interpolation algorithm (*e.g.* kriging, minimum curvature, etc.). Fig. 4 (d) shows the results obtained from Petrock's inverse distance weighting (Eq. 2) without any lithological weighting. Because of the different weighting equation and the requirement for at least 5 data points per grid point the resulting map (d) is much smoother than Surfer's map (c).

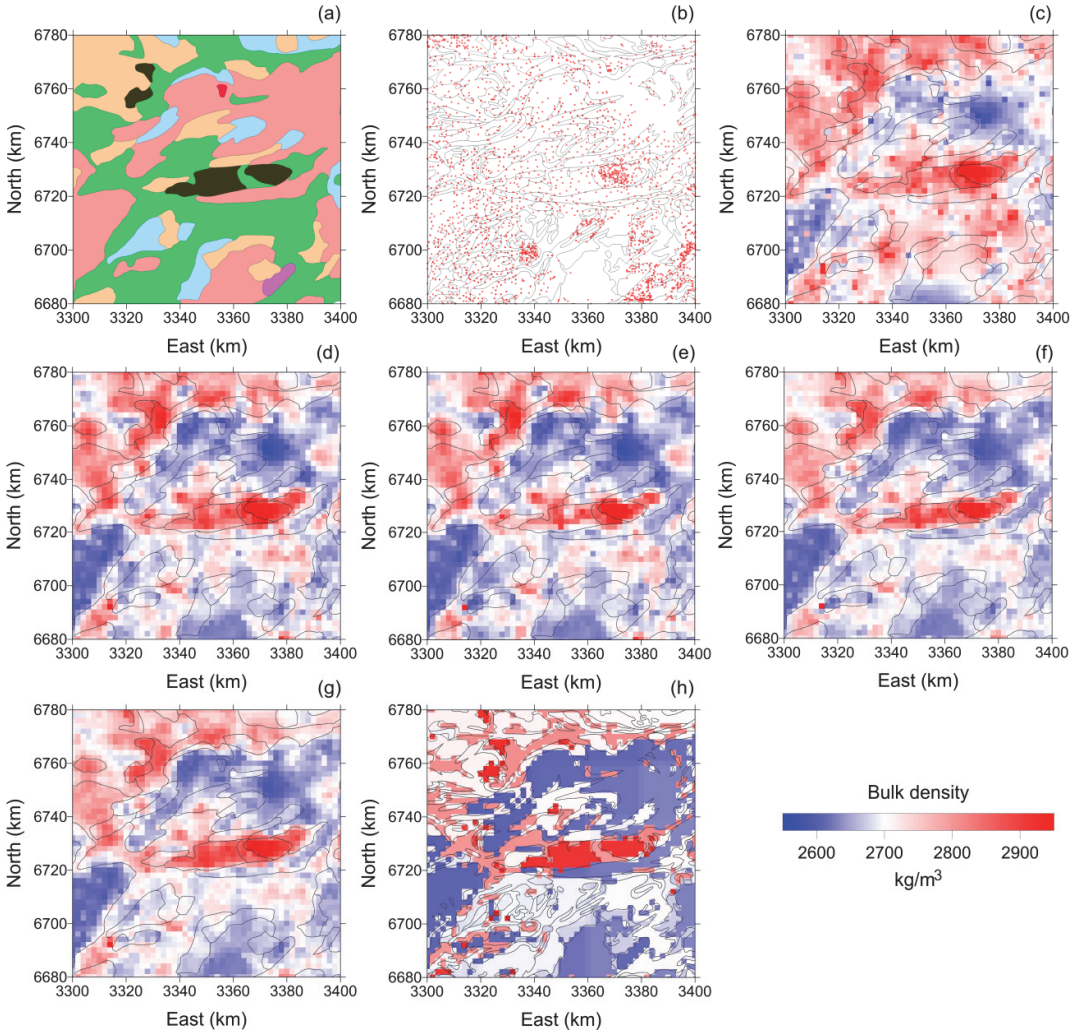

Fig. 4. Examples of gridding density data: (a) geological map of scale 1:5 million (see Fig. 1 for a legend), (b) sample locations (red dots) and boundaries of the geological map of scale 1:1 million (grey lines), (c) Surfer's inverse distance gridding, (d) PETROCK's inverse distance weighting, (e) areal lithological weighting, (f) areal and background weighting, (g) background weighting, and (h) background density map resulting from lithological background weighting.

The next three maps show results obtained from (e) the areal lithological weighting (Eq. 3), (f) areal and background weighting, and (g) lithological background weighting alone (Eq. 5). Comparison of maps 4 (d) and (e) shows that, at least in this case, the areal lithological weighting has quite little effect. The background weighting used in maps 4 (f) and (g) emphasizes the borders of the lithological units and distributes the density values inside each unit more evenly. Fig. 4 (h) shows the median density map of each lithological unit obtained as a byproduct of lithological background weighting. Map (h) represents the background density added to the inverse distance weighting. Because of the moving computational window, there are some block-like features of size 20 km by 20 km present in map (h). However, because of the rather small value of global weight (wg= 2.0) these features are not visible in actual maps (f) and (g). Interestingly the median density of the granites increases from west to east and

the median density of the metavolcanites is lower in the southern part of the map. These results, however, should not be considered unwanted features because the granites in the west and east and the metavolcanites in south and north may be of different origin and geology.

The new lithological weighting method has been used to prepare maps of the rock bulk density and the magnetic susceptibility for the whole Finland shown in Fig. 5. The computation was made on a 2 km × 2 km grid using 50 km × 50 km computational area and 50 km wide margins. Except for the minimum number of samples per investigation area (N_{min} = 2000) all the other computational parameters were the same as in the previous example. To emphasize the background geology, the maps show the results of combined lithological weighting that means both areal and background weighting have been applied. Fig. 6 shows the corresponding maps created using Surfer's inverse distance weighted interpolation (smoothing factor 1.0).

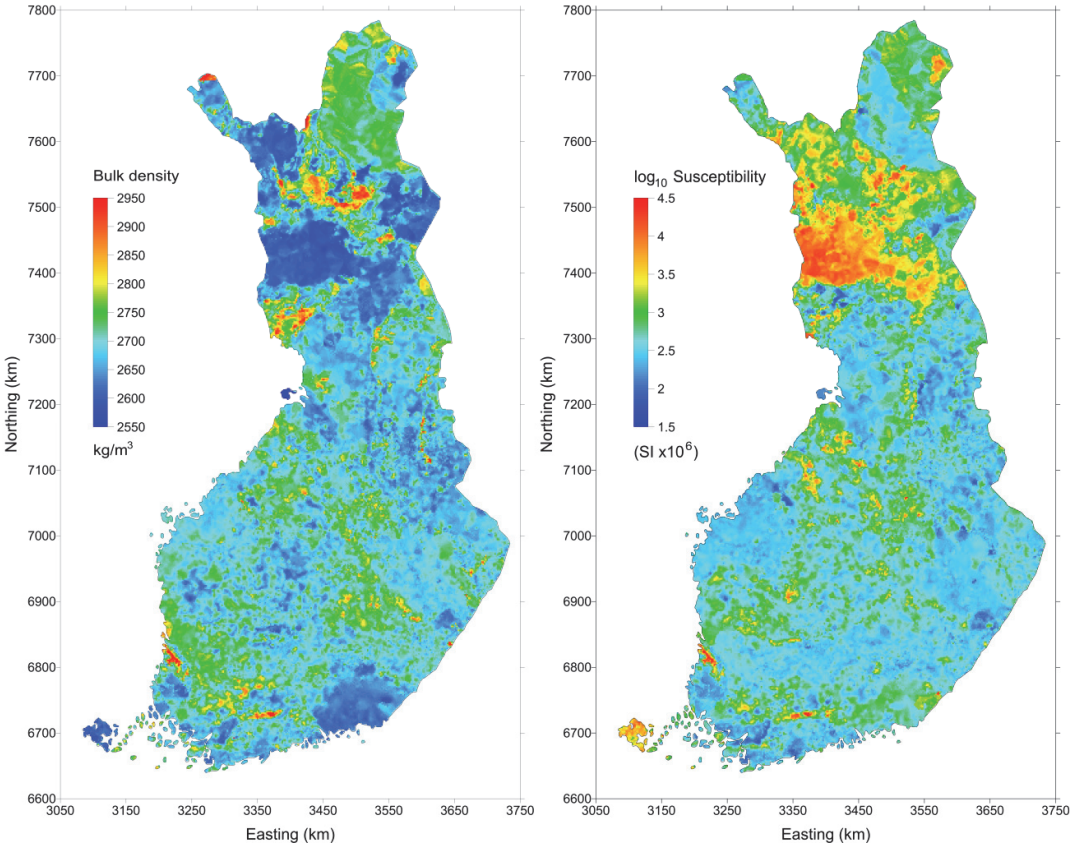

Fig. 5. Maps of bulk density and $\log_{10}$ normalized magnetic susceptibility in Finland computed on a 2 km by 2 km grid using combined lithological weighting.

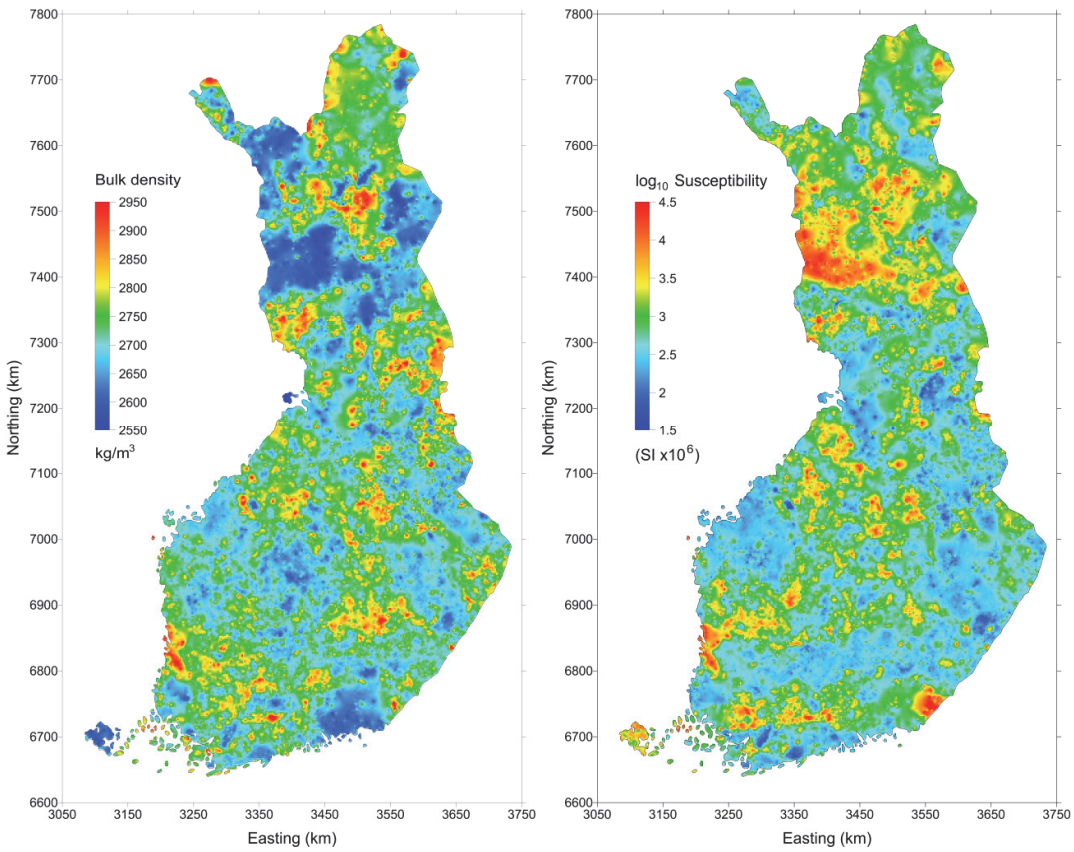

Fig. 6. Maps of bulk density and magnetic susceptibility in Finland interpolated on a 2 km by 2 km grid using conventional inverse distance weighting (Surfer 9).

Comparison between Figs. 5 and 6 shows several differences. Conventional interpolation retains the dynamic variation of the original data better (density: min= 2474, max= 3066 & mean= 2720 kg/m^3 and $\log_{10}$ susceptibility: min= 1.327, max= 5.136 & mean= 2.863 SI×10^6). Because of the limiting condition (median ± 2.0×*std*) the lithological weighting flattens the mapped data (density min= 2492, max= 3001, mean= 2691 kg/m^3, $\log_{10}$-susceptibility min= 1.656, max= 4.616, mean= 2.739 SI×10^6) However, in Fig. 6 areas of high density and susceptibility appear too dominant which makes the maps less accurate geologically. Lithological weighting concentrates high density and susceptibility values inside the corresponding lithological units. As a consequence the overall appearance of the maps is quite different from conventional interpolation and several enhanced features related to the digital geological maps can be seen in Fig. 5. To enhance large-scale gravity and magnetic sources and to show the correlation with the geological map (scale 1:5 million) better, Fig. 7 shows the data in Fig. 5 re-interpolated on a 10 km × 10 km grid. Despite the sparser discretization the lithologically weighted maps show good correlation with the geological maps.

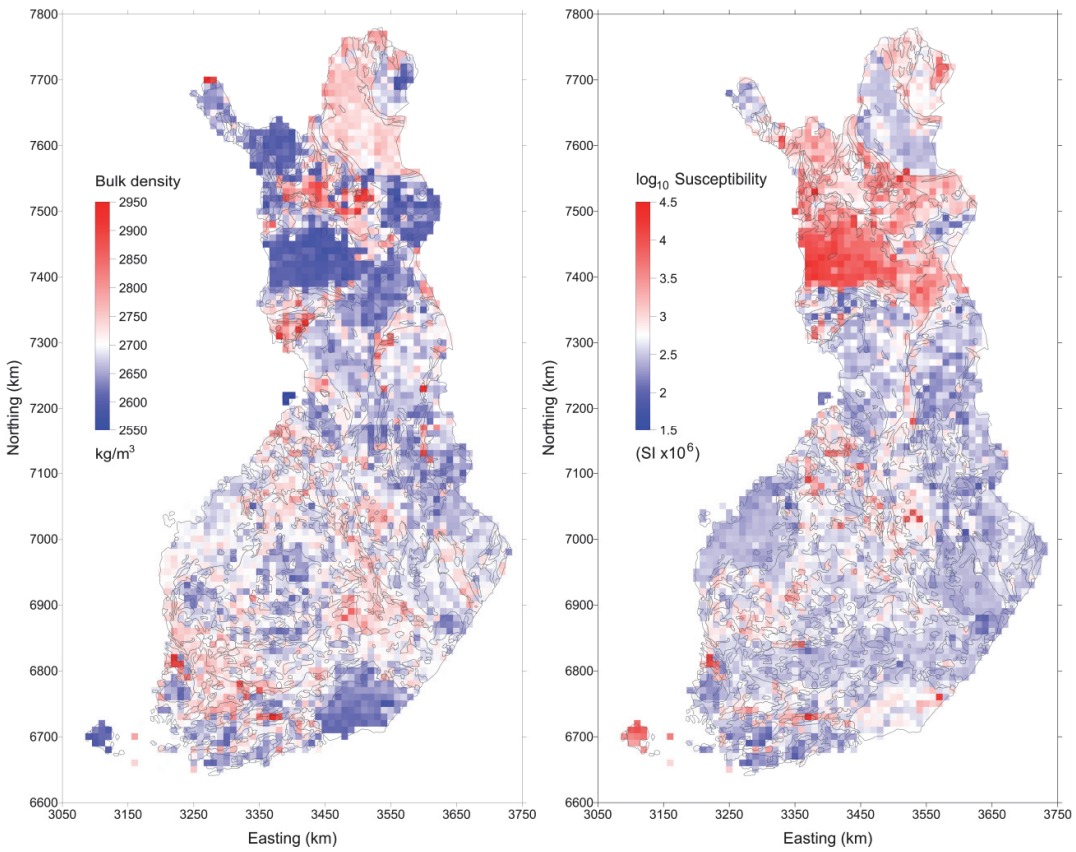

Fig. 7. Maps of lithologically weighted bulk density and magnetic susceptibility in Finland interpolated on a 10 km by 10 km grid and overlain by the geological map boundaries (1:5 million scale).

To improve the gridding rejection rules were applied that ignore all but sediment rocks (class 4) from sedimentary lithological units (classes 1, 2, 3, 4 and 6), and all but plutonic rocks (class 1) from rapakivi granites (classes 8 and 9), and sediment samples from diabases (class 5) and granites (class 39) adjacent to the abovementioned sedimentary units. The effect of the rejection rules can be seen in Fig. 8, which shows the background density and susceptibility maps obtained from background weighting. For example, in Satakunta area (x=3250, y=6800) in South-West Finland the diabases are confined into their corresponding lithological units and the neighboring sediments and granites show up with low density and low susceptibility. It should be noted that the objective is not to create maps such as those in Fig. 8, but to use the lithological information to enhance areal inverse distance weighting. Furthermore, the statistics obtained as a by-product of background weighting gives additional information about the petrophysics of each lithological unit. This information could be used to build large-scale 3D geological models of the crust, for example. However, providing such information is beyond the scope of this paper. Furthermore, more careful analysis of rejection rules is needed before the results regarding the mean density and susceptibility of the 92 lithological units of the digital geological map of Finland could be given.

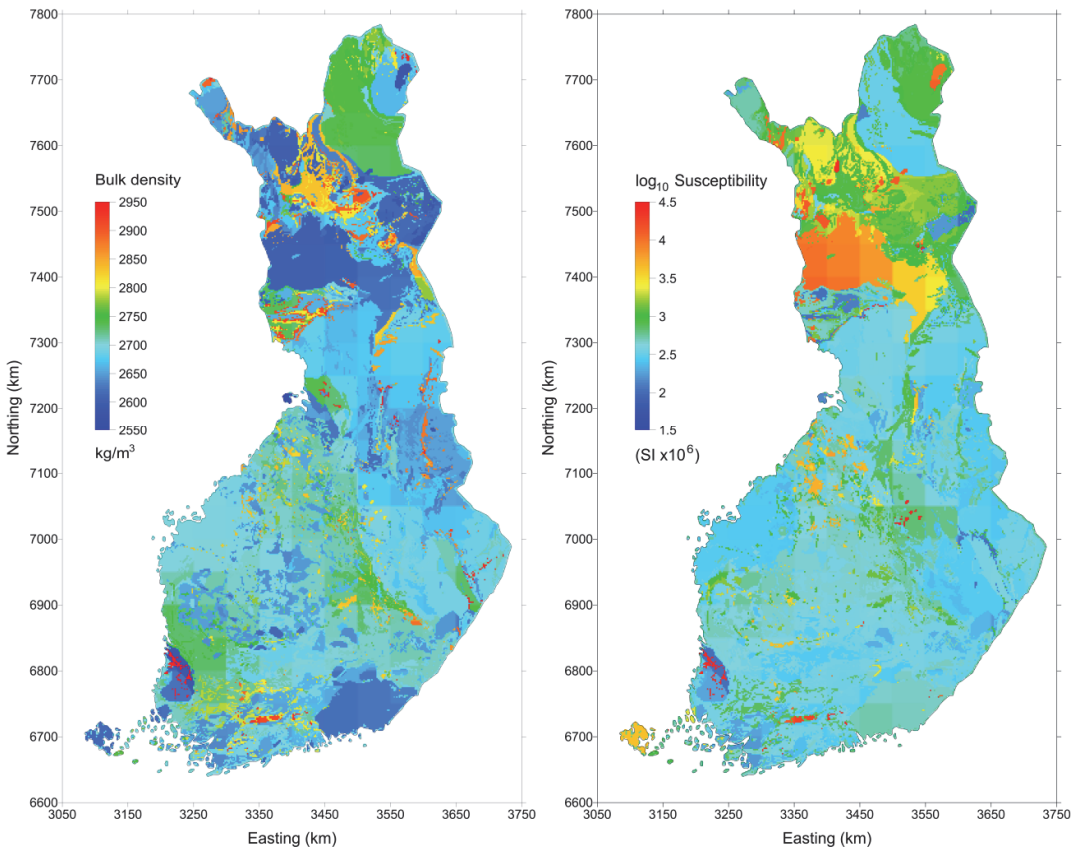

Fig. 8. Maps of the lithological background value of density and magnetic susceptibility in Finland resulting from lithological background weighting.

Fig. 9 shows histograms of the original and the gridded density and magnetic ($\log_{10}$-normalized) susceptibility data. The gridded data correspond to Figs. 5 (a) and (b). The comparison of the histograms shows that the gridded data have much narrower distributions than the original petrophysical data. In practice, gridding has removed most of the high density and high susceptibility values and shifted the mode closer to the mean of the original data. As a consequence, the amount of samples with susceptibility above 0.01 SI ($= 10^{4.0}\times10^{-6}$) is about 12 % in Fig. 9 (b) and only 1.4 % in Fig. 9 (d). This also suggests that the areal proportion of rocks responsible for the magnetic anomalies of Finland is only 1.4 %.

The reader should not compare the histograms too closely with each other because the gridded data are evenly sampled but original data are clustered around active investigation areas (see Fig. 2). As such, Figs. 9 (c) and (d) represent the spatially averaged variation of density and susceptibility in Finland. Comparison of (c) and (d) figures with the geological map in Fig. 1 suggests that the dominant class with a density about 2700 kg/m^3 and magnetic susceptibility about 0.0035 SI ($=10^{2.55}\times10^{-6}$) belongs to gneisses and granites. Because of the large areal proportion of granites in central Finland and Lapland the density histogram 9 (c) is less biased towards high values than the original data in 9 (a). Because of the large areal proportion of the granites with elevated susceptibility in Lapland, the susceptibility histogram (d) still shows a tail towards high

susceptibility values. Some statistical values of the original and gridded data are shown in Table 1. The values show rather small differences, most of which result from the limiting condition (*median* ±2.0×*std*). The sample count of the original data is reduced close to 75000 due to coordinate rounding (0.5 km) and removal of duplicates and that more than 125000 dummy values have been excluded from the gridded data.

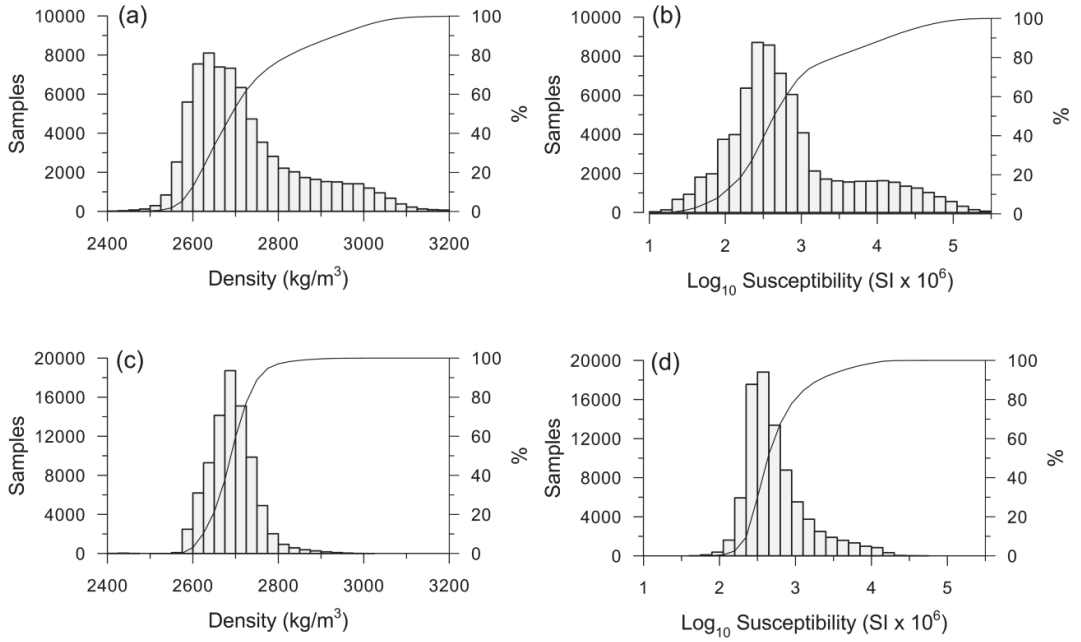

Fig. 9. Histograms of the original petrophysical (a) density and (b) magnetic susceptibility data and the gridded (c) density and (d) magnetic susceptibility data of Figs. 5 (a) and (b). The curves show the cumulative histograms in percents.

Table 1. Statistical measures of the original petrophysical samples and gridded density and (log_{10}-normalized) susceptibility data in Finland.

Statistical parameter	**Original density (kg/m^3)**	**Mapped density (kg/m^3)**	**Original log_{10} susc. ($SI\times10^6$)**	**Mapped log_{10} susc. ($SI\times10^6$)**
Count	74460	85320	73528	85320
Minimum	2204.0	2492.0	1.000	1.656
Maximum	3200.0	3001.1	5.987	4.616
Mean	2725.0	2690.9	2.820	2.739
Median	2691.0	2689.4	2.639	2.634
Standard dev.	130.58	53.23	0.8189	0.4040

The statistics computed as a by-product of background weighting would have given additional and much more detailed statistical information about the petrophysics of the 92 lithological units of the $1:10^6$ digital geological map of Finland. Even more detailed results could be obtained using the DigiKP200 geological maps with scale

1:200000. However, providing such information is beyond the scope of this paper unless more careful analysis of rejection rules per each lithological unit is made.

4 Discussion and conclusions

The three central parts of the new gridding method are 1) the two lithological weighting schemes, 2) the additional limiting condition, and 3) the rejection rules based on rock classification. The areal lithological weighting, which is based on the proportional area of different lithological units inside investigation area, reduces the importance of small but possibly more durable geological formations from which most of the samples tend to be taken from. The background weighting adds statistically computed background value of the lithological unit or units below the grid point to the lithological weighting, and hence, can provide better estimates for the underlying geological units in areas where samples are not available. Therefore, the resulting maps of the spatial distribution of the petrophysical parameters are geologically more realistic than those obtained using normal interpolation methods. The additional limiting condition is used when the background values of each lithological unit are computed. The limiting condition based on the median or the mode can yield the base or background level of the lithological unit better than the mean if the distribution is affected by few incorrectly classified samples.

Incorrect lithological classification of the samples is one of the biggest sources of error in the lithological weighting. The samples are classified based on their location on a digital map, which is only an approximation of the reality. In practice the lithological units contain lots of samples, which may or may not represent that particular unit. For example, unmapped intrusive dykes raise the density of geological host units. If the amount of incorrectly located samples is sufficiently small the additional limiting condition can reduce their effect. However, even the use of median or mode as the reference value cannot reveal the correct base level if the distribution is severely biased towards incorrectly classified samples or the distribution is bi-modal.

In such cases, the rejection rules can effectively remove incorrectly classified samples from lithological units. Rock class rejection is particularly effective in case of sedimentary rocks and granites the background level of which gets biased because of samples from surrounding and outcropping lithological units. Rejection rules based on the main level of the hierarchical rock classification is rather crude approximation of geology. The gridding could be improved if more accurate mineralogical information, *i.e.*, the second or even the third level of the rock classification of the samples were used and more careful analysis of allowed and rejected rocks inside the lithological units were performed. Cross-correlation of the rock types and petrophysical properties could also be used to classify the samples belonging into different lithological units better. However, the large variation of different rocks and the variation of the petrophysical properties within each rock class and lithological unit make it difficult to implement an automated classification method. This problem requires additional work.

Despite of the large amount of petrophysical samples and polygon vertices, gridding the area of whole Finland (700 km by 1200 km) using moving window strategy becomes rather fast operation on modern computers. The results in Fig. 5 (C= 50 km × 50 km, M= 50 km) were computed in less than six minutes using a PC with Intel i3 530 dual core processor. The larger the investigation area is the greater is the amount of polygons and sample points and the search operations start to slow down the computation. On the contrary, decreasing the computational window will also increase computation time because all the subarea operations need to be performed so many times.

Finally, we wish to remind that PETROCK can be used to grid geochemical data as well. We also wish to point out that PETROCK has an important application in three-dimensional gravity and magnetic modelling and inversion where so-called voxel or volume element models are used. When imported into the surface layer of the 3-D models the gridded density or susceptibility data provide constraining *a priori* data.

Acknowledgements

This work has been supported by the Academy of Finland though the three-dimensional crustal model (3DCM) project. PC/Windows version of the software can be downloaded for free downloading at:
https://wiki.oulu.fi/display/~mpi@oulu.fi/Petrohysical+data+ gridding. The package includes auxiliary software to prepare the data and the maps as well as geochemical example data of GTK (*Rasilainen et al*., 2008) and 1:10 million geological map of Finland.

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