Geophysica (2011), 47(1–2), 69–93

The Effects of Basin Dimensions on the Seasonal Depth of Thermocline and Temperature of Hypolimnion in Small Lakes

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(Received: April 2011; Accepted: September 2011)

Abstract

A model is presented for calculation of the seasonal thermocline depth and the temperature of hypolimnion of small lakes. The model input data are the daily means of the surface water temperature (at depth of 0.5 m), incoming global radiation, and wind velocity. The lake parameters are the maximum depth, surface area, a parameter (m) describing the lake bathymetry, and diffuse light attenuation coefficient. The model is formed of two connected parts. 1) A generally used differential equation for the thermocline depth; and 2) A temperature model where the hypolimnion is divided into two parts: a lower non-stratified part with logarithmic velocity profile and an upper part with mixing controlled by buoyancy. The first model has three parameters and the second one has two, determined by calibration with 31 lakes. The surface areas of these lakes varied between 0.004 and 13.6 km^2, and the maximum depths ranged between 5 and 85 m. The combined model was tested with independent data from three lakes with time series of thermocline depth and hypolimnion temperature.

Key words: lakes, hypolimnion, temperature, mixing, dimensions

1. Introduction

1.1 Background of the study

In this study a model is presented for calculation of the seasonal thermocline depth and the temperature in hypolimnion of small lakes. This information is important in many hydrological, meteorological and limnological studies in areas with several small lakes, because seldom there is enough data. According to *Raatikainen and Kuusisto* (1990), the total surface area of lakes is 9% of the surface area of Finland, but in the Lake District in Southern Finland, 20% of the area is covered with lakes. The area of lakes smaller than 10 km^2 covers nearly 35% of the total lake area. This shows the importance of small lakes.

Lake temperature and depth of the thermocline are studied and modelled for various purposes. They are important for the general conditions in lakes and for the interaction with surroundings, which is needed for successful combination of meteorological and hydrological models. Different models also deal with different space and time scales. A typical basic model structure is one-dimensional, vertical, while horizontal variations can often be reduced to two layers, epilimnion and hypolimnion. Epilimnion is strongly coupled to the atmosphere, but heating of the hypolimnion is strongly affected also by the shores, bottom and form of the basin.

Our model differs from several similar models in that we use geometric measures (maximum depth, surface area, and an exponent (m) controlling the basin form) as model parameters, and we calibrate the model using data from several small lakes. However, in applications for single lakes, some individual calibration would be needed, primarily because the sheltering by the shores to wind is very complicated and not fully taken into account in the present simple sheltering function. The model is run on PC and it is very easy to use.

1.2 Models for thermal stratification of lakes

Several empirical approaches exist for explaining the stratification of a lake in a single day. *Gorham and Boyce* (1989) presented a thorough study about seasonal stratification, based on data with large coverage. They gave, as a conclusion, a formula, which can be used to estimate the depth of the thermocline in a fixed day. *Fee at al.* (1996) studied light absorption and the shape of the temperature profile to determine the formation of epilimnion using rather extensive observation data. The method was purely empirical. They found out that transmission of light has a prominent effect on the epilimnion depth in small lakes. They also saw important effects of spring conditions on the later development and believed that the depth of the epilimnion cannot be solved using monthly climatic means. *Virta et al*. (2001) used an experimental formulation by fitting an arctan-log transformation to data of the hypolimnetic temperature and the depth of thermocline measured in several lakes. *Elo* (2007) used this model for morphologically different small lakes and compared the results with the outcome from a 2nd order (k-ε) turbulence model; similar features were found. *Pal'shin et al.* (2008) presented empirical dependences of various thermal characteristics on geographic zonality and morphometric characteristics using lakes in North-western Russia and Finland.

There are many physically based mathematical models concerning the depth of the seasonal thermocline, both explicit models and implicitly in temperature models. The first model for deepening of the thermocline was presented by *Kraus and Turner* (1967). It was further developed by several authors (see *Fischer et al.*, 1979). The model by *Kato* and *Phillips* (1969) was used by *Huttula* (1976) for Lake Pääjärvi, which is one of our calibration and test lakes. The model system was also further developed and applied to a model known as DYRESM (*Imberger and Patterson,* 1981). *Spiegel and Imberger* (1980) analysed dynamics in small and medium lakes.

There are several other models, which can be used for solving the seasonal thermocline depth and the hypolimnetic temperature. *Kirillin* (2002) employed a self-similarity model based on theory developed by *Kitaigorodskii and Miropolski* (1970) for oceanic applications and later by *Zilitinkevich and Rumyantsev* (1990) for lakes.

Wüest and Lorke (2003) presented a model, in which the hypolimnion is divided into two parts: lower non-stratified part with logarithmic velocity profile and upper part with mixing controlled by buoyancy or Väisälä frequency. In the development of the model, measurements with a high-resolution current profiler were utilized (*Lorke et al*., 2002).

More detailed temperature profiles can be solved with 2nd order turbulence models. In k-ε models, turbulence is solved with the equations of turbulent energy and its dissipation rate (*Svensson,* 1978; *Goudsmit et al.,* 2002). The first application of Svensson's (1978) model to Finnish lakes was made by *Malve et al*. (1991) and *Huttula et al.* (1992). This model can also include a separate sub-model for additional mixing in the hypolimnion and formation of temperature profile within the metalimnion. Deep-mixing has been formulated for vertical models by Stigebrandt (1987), used together with Svensson's (1978) model successfully for deeper lakes by *Elo et al.* (1998) and *Elo* (2005, 2007). In the latter study, it was found that the applicability depends also on the data used. *Elo* (2005, 2007) found that in smaller lakes, heat is more blocked to the epilimnion, as an effect of morphology. Solving the temperature profile, k-ε model can be used to determine the depth of the thermocline based on the profile structure but not as an independent variable.

In this study, two interconnected models are used to solve the depth of the thermocline and the hypolimnetic temperature forced by surface temperature, incoming global radiation and wind velocity. Thermocline depth is based on a model presented in *Fischer et al.* (1979), and for the temperature distribution in the hypolimnion the model by *Wüest and Lorke* (2003) is used. The modelling results of *Elo* (2005, 2007) are used to study how well our seasonal results fit with k-ε model results.

2. Lake data

Three data sets were used in the model calibration. The first (FIN1) consisted of 10 lakes in southern Finland. These lakes were monitored for temperature and Secchi depth by personnel of the Lammi Biological Station (LBS) of the University of Helsinki: Nine lakes in August 1997 (*Eeva Huitu and Suvi Mäkelä*, unpublished) and one in August 2003 (*Lauri Arvola*, unpublished). The second data set (FIN2) consisted of 17 lakes also from southern Finland (*Martti Rask, Lauri Arvola, Tarja-Riitta Metsälä and Tiu Similä*, LBS, unpublished). These lakes were monitored for the temperature and colour in July 1985. Other lakes were obtained from the data gathered within the Finnish-Estonian SUVI-project in 1995-2005 (*Arst et al*., 2008). In this project optical properties of several Finnish and Estonian lakes were studied and from them three Estonian lakes and one Finnish lake were selected into this study. These three Estonian lakes form the dataset (EST) and results of the Finnish lake (Lake Pääjärvi) were moved into dataset FIN1. Irradiance was measured in the SUVI lakes at different depths, and diffuse light attenuation coefficients were computed from this data (*Erm et al*., 2002). For the FIN1-data, the attenuation coefficients were computed from Secchi depths using the equation by *Herlevi* (2002), and those for the FIN2-data were obtained from the colour data of *Jones and Arvola* (1984).

Model calibration was based on the mean yearly course of the forcing. Therefore the mean values of the measured hypolimnetic temperature and the depth of the thermocline were used for those lakes with observations from several years. For three lakes there were measurements after August 28; these data were excluded because uncertainty increases when hypolimnetic and epilimnetic temperatures approach each other. The surface area of Lake Oksjärvi was divided by two, because it is composed of two basins.

Data set EST included three cases, the first Finnish data set FIN1 had 11 cases, and the second Finnish data set FIN2 had 17 cases, altogether 31 cases. Table 1 gives the basic characteristics of the lakes. In most cases temperature was measured manually at depth interval of 1 m and the depth of the thermocline was estimated as the inflection point of the depth-temperature curve.

Table 1. The basic characteristics of the study lakes. A is the surface area, H is the maximum depth, h is the depth of the thermocline, K is the diffuse attenuation coefficient, T_{s0} is the surface temperature (depth 0.5 m), and T_m is the temperature of hypolimnion (depth 0.8H).

	Date	A, km^2	H, m	h, m	K, m^{-1}	T_{s0}, °C	T_m , °C	*Mean of years*
FIN1								
Mustakatos	20.8.1997	0.110	6.0	3.2	2.2	20.1	7.0	
Riikosten Valkjärvi	20.8.1997	0.080	8.0	3.6	1.0	17.7	5.8	
Kaukasenjärvi	19.8.1997	0.130	6.0	3.8	1.7	19.9	8.0	
Hervonjärvi	20.8.1997	0.090	11.5	4.3	1.0	20.6	5.1	
Pakkaselanjärvi	20.8.1997	0.130	14.0	3.5	1.2	19.6	4.8	
Lovonjärvi	21.8.1997	0.050	17.5	2.8	2.9	21.3	4.4	
Ormajärvi	21.8.1997	6.530	30.0	7.0	0.6	19.0	7.9	
Säynäjärvi	14.8.2003	0.420	20.0	5.8	0.6	20.2	5.2	
Pannujärvi	20.8.1997	0.370	10.5	5.5	1.1	20.2	9.1	
Oksjärvi	19.8.1997	2.300	16.0	5.9	0.8	19.7	9.0	
Pääjärvi	12.8	13.600	85.0	9.4	3.6	21.0	5.0	2000, 2001
FIN2								
Haarajärven Valkjärvi	17.7.1985	0.035	12.0	2.4	1.4	21.0	5.2	
Haukilampi	16.7.1985	0.023	8.0	1.6	3.0	20.8	5.2	
Pitkäniemenjärvi	18.7.1985	0.144	10.0	2.3	2.4	19.0	5.9	
Rahtijärvi	16.7.1985	0.132	13.0	2.5	2.6	21.0	6.5	
Savijärvi	12.7.1985	0.166	12.0	3.8	2.8	20.5	6.2	
Keskinen-Rautjärvi	18.7.1985	0.149	6.0	2.5	2.1	19.1	7.1	
Haarajärvi	17.7.1985	0.121	14.0	2.9	1.7	20.0	5.2	
Alinen-Rautjärvi	18.7.1985	0.450	12.0	2.7	2.1	19.9	7.7	
Ylinen-Rautjärvi	12.7.1985	0.376	12.0	3.0	2.2	21.6	6.8	

	Date	A, km^2	H, m	h, m	K, m^{-1}	T_{s0}, °C	T_m, °C	*Mean of years*
Alinen-Mustajärvi	18.7.1985	0.007	7.0	2.3	1.4	20.0	5.0	
Huhmari	11.7.1985	0.016	8.0	2.0	1.3	19.5	4.8	
Sorsajärvi	12.7.1985	0.150	13.0	2.5	2.6	21.0	5.0	
Taka-Killo	18.7.1985	0.044	12.0	3.9	0.8	18.8	4.8	
Tavilampi	17.7.1985	0.008	7.0	1.8	1.6	20.8	5.0	
Horkkajärvi	16.7.1985	0.011	12.0	1.5	3.0	20.8	4.0	
Möläkkä	11.7.1985	0.009	15.0	1.6	1.7	20.2	4.8	
Nimetön	17.7.1985	0.004	11.0	1.5	3.0	19.3	4.2	
EST								
Koorkjärv	12.8	0.441	26.8	6.1	0.7	21.6	5.0	1997, 1999–2001
Verevi	11.8	0.126	11.0	3.7	2.2	20.8	4.5	1997–1999
Nohipalu	12.8	0.063	11.7	4.4	1.0	21.8	6.2	1997–2001

3. *Model*

3.1 *Model structure*

The longitudinal section of the model basin is described with a power law formula, an isosceles triangle with sides of equal length forming a special case. The equation of the bottom line is

$$L_y = L_0 \left(\frac{H-y}{H}\right)^m \tag{1}$$

where L_0 is half of the total length L of the model basin, L_y is the corresponding half length at the depth y, and m is the exponent of the power law (its value can be estimated from the depth contour map of the basin, or, better, by calibration of the model to the specific lake). The length L characterizing the length axis of the lake is chosen as the square root of the lake surface area, and wind is assumed to affect along that axis. Eq. (1) was used, *e.g.*, in the work of *Gorham and Boyce* (1989) to correct the formula of the period of internal seiche in a rectangular basin. Figure 1 shows the form of the length profiles with different values of m.

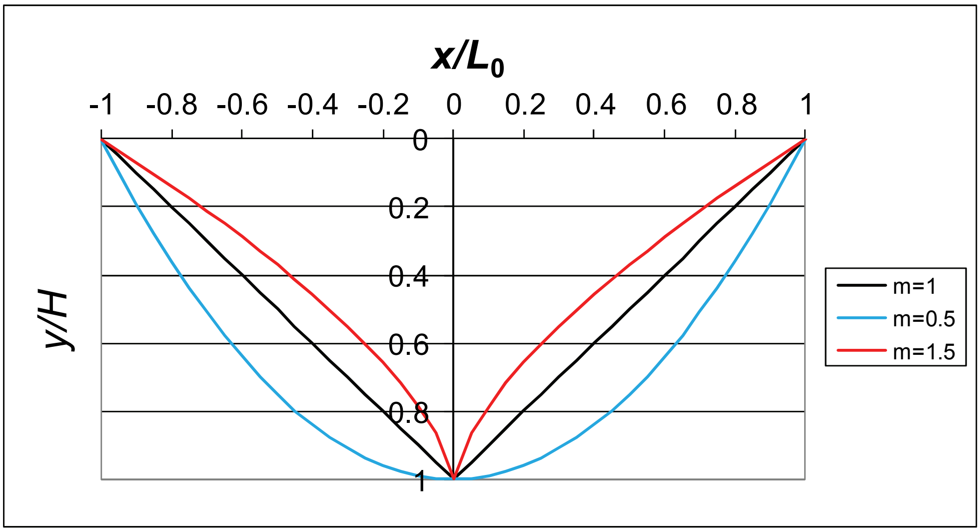

Fig. 1. The length profile of the model basin with different values of the power law parameter *m* in Eq. (1).

The surface temperature T_{s0}, corresponding to the depth of 0.5 m, is given as an input. The mean temperature of the epilimnion, T_s is estimated as T_{s0}. The characteristic temperature T_m of hypolimnion is calculated for the depth 0.8*H*. The model of the seasonal thermocline depth is based on a differential equation, which takes into account the essential influencing factors. In essence, the model of hypolimnetic mixing is one-dimensional (1D). It can be considered partly two-dimensional (2D), because horizontal effects such as uninodal seiche, horizontal distribution of radiational heating, as well as horizontal distribution of turbulent diffusion coefficient are determined and used as horizontal averages in the model.

3.2 Thermocline

The surface layer temperature of lakes is influenced by wind and vertical heat flux from above. This causes diurnal variation to the temperature. Below the surface layer the temperature decreases gradually downwards. A method presented by *Hutchinson* (1957) uses the inflection point of the temperature profile as the depth, where the exchange between the layers above and below is essentially minimized. This interface is very important from the point of view of, e.g., chemistry and biology. The upper layer is called epilimnion and the lower layer is called hypolimnion, and the interface between them is the thermocline. This kind of definition suits well for small lakes as in this study. In large lakes, the exchange between layers may increase considerably and then the thermocline interface is transformed into a thermocline layer.

Thermocline depth can easily be defined numerically, e.g., using spline interpolation, or visually from the depth-temperature graph. Actually there are often two thermoclines, the upper one is the diurnal thermocline and the lower one is the seasonal thermocline. Here we study only the seasonal thermocline. If recording instruments are available, then the seasonal thermocline can be determined from temperature-depth curve averaged over a 24-h period. In this way the diurnal thermocline is filtered out.

In theory, thermocline deepens during summer. Due to internal seiches, the thermocline can rise or deepen dependent on the site. Also a secondary thermocline may exist and give the impression of a risen thermocline. That can be seen especially in spring, but also later during warm, calm periods.

Epilimnion temperature is assumed constant and equal to the measured temperature at the depth of 0.5 m. This depth was selected because there are much more data of the surface temperature than of the temperature of deeper layers. The selection causes model errors in periods with strong heating when the epilimnion is not mixed (see Fig. 2). However, a large vertical temperature change within the epilimnion is not common. E.g., in June and August 1979 in Lake Valkea-Mustajärvi, only in 25% of the measured temperature profiles the temperature difference was more than 1.5°C between 3.5 m and 0.5 m depths (the average was 1.2°C). In late autumn this difference diminishes. In large lakes this difference is smaller due to the more effective wind mixing causing more homogenous epilimnion.

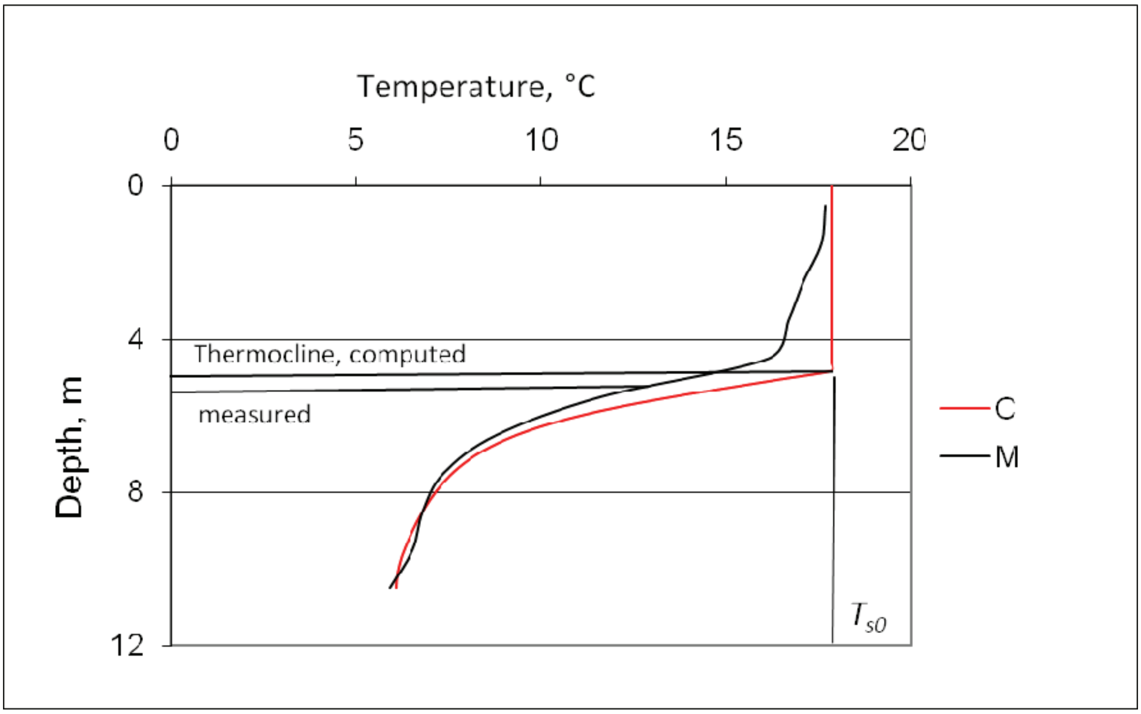

Fig. 2. Measured (M) and computed (C) 24-h averages of the vertical temperature profiles of Lake Valkea-Mustajärvi on the 10th of August 1979 and discrepancy between the measured and computed thermocline depths.

The differential equation for lowering the thermocline can be written in the form (see *Fischer et al.*, 1979) based on the balance of potential and turbulent energy:

$$C_T \left(q^2 + \frac{gh\Delta\rho_{hs}}{\rho_w}\right)\frac{dh}{dt} = C_\kappa^f q^3 + C_s(\Delta u)^2 \frac{dh}{dt} \tag{2}$$

where h is the seasonal thermocline depth, C's are constants (obtained from literature), $\Delta\rho_{hs}$ is the density difference between epilimnion and hypolimnion, Δu is the velocity difference between these layers, and q represents the turbulent energy created by wind and by cooling the epilimnion, described in detail later. On the right side, the second term represents deepening of the thermocline due to the so-called billowing effect in larger lakes. On the left side, the second term in the parentheses describes the energy needed for the deepening the thermocline.

The turbulent energy q is solved with the formula

$$q = \left(u_f^3 + C_4 \eta^3 u_*^3\right)^{1/3} \tag{3}$$

The parameter η is a coefficient measuring the stirring efficiency of wind (its value can be obtained from literature), u_f describes the vertical convection velocity, C_4 is a calibration parameter, and u_* is the friction velocity, given by:

$$u_* = \sqrt{C_w \frac{\rho_a}{\rho_w}} Dw \tag{4}$$

Here w is the wind velocity, C_w is the drag coefficient of wind, ρ_a is density of air, ρ_w is density of water, D is the shading function of the shoreline, dependent on the vertical dimensions of the lake and height of landscape around the lake, and w is the free wind velocity over the lake.

Convection with velocity u_f occurs when the surface water cools and dwells down. *Fischer et al*. (1979) gave the formula

$$u_f^3 = \frac{\alpha_w g h \widetilde{H}}{\rho_w C_p} \tag{5}$$

where α_w is the thermal expansion coefficient of water, C_p is the specific heat of water, and $\widetilde{H}$ is the flux of heat from the surface, positive when directed upwards. In this study, convection is defined with the following method. The heat balance of the epilimnion can be solved with the formulation:

$$\frac{\widetilde{H}}{C_p \rho_w h} = -\frac{dT_s}{dt} - 2\frac{T_s - T_h}{h} \frac{H-h}{2H-h} \frac{dT_s}{dt} < 0 \tag{6}$$

The subscripts s and h refer to epilimnetic water and hypolimnetic water at the depth of $(H+h)/2$, respectively. The second term on the right side describes cooling when water from hypolimnion rises to epilimnion. When $\widetilde{H}$ is eliminated from the above equations, u_f^3 can be solved from a finite difference form

$$u_f^3 = \frac{\alpha_w g h^2}{\Delta t}\left(T_{sp} - T_s - 2\frac{T_s - T_h}{h} \Delta h \frac{H-h}{2H-h}\right) \tag{7}$$

The subscript p refers to the former time step (24 h).

The shading function D is based on measurements of *Venäläinen et al.* (1998) in Sweden in Lake Råksjö, where shading is caused by the forest around, typical in most cases in Finland as well. From their results, shading function can approximated by the form 1–exp(–$\gamma l/d$), where l is the distance from the leeward shore. From this the mean square of the shading function for a rectangular surface can be computed as

$$D^2(d/L_0) = 1 + \frac{d}{2\gamma L_0}\left(4e^{-\gamma L_0/d} - e^{-2\gamma L_0/d} - 3\right) \tag{8}$$

The value of the constant γ is 0.06. For the height d of the surrounding forest, the value of 20 m can be used, and L_0 is the square root of the surface area of the basin. In some cases there can be directional variation, e.g., due to open fields at the shore. This is not taken into account in the formulation of the present model, but in principle it can be taken into account when calibrating for an individual lake.

For solving the change of the thermocline depth, the differential equation (2) can be given in a difference form as

$$\Delta h = \frac{C_1 q^3 \Delta t}{C_2 q^2 + \dfrac{gh\Delta\rho_{hs}}{\rho_w} - C_3 \Delta u^2} \tag{9}$$

3.3 Vertical mixing and heating in the hypolimnion

Wind tilts the water surface and induces a horizontal pressure gradient in the hypolimnion. This gradient causes motion in the hypolimnion as internal seiches. E.g., in the largest test lake, Lake Pääjärvi, measurements indicated that the uninodal internal seiche period is about 16 h, and the related current velocities can exceed 30 cm/s in a specific internal furrow of the basin. These large velocities naturally depend on the lake topography, which ought to be known in details. However, it can be assumed that lakes in this region have usually similar furrows, which may have been formed in similar geological processes, and currents may have shaped them. The main idea is that the present model is used to solve the effect of general flow field, and the special individual lake calibration includes the effects caused by microtopography. This is suitable for lakes with surface area smaller than 20 km^2. In larger lakes, Coriolis acceleration becomes important as it changes the flow patterns and also the structure of internal seiches becomes more complicated.

The main idea in the model of *Wüest and Lorke* (2003) is that the current velocity in the hypolimnion is generally so small that fully developed turbulent flow approximation cannot be used for the whole profile. Direct measurements (*Lorke et al.,* 2002) in some stratified lakes with an acoustic high-resolution profiler have shown that the flow above a thin laminar bottom layer can be divided into two parts: an upper part, where buoyant effects are predominant, and a lower part, thickness z_c, where stratification is neutral and a logarithmic velocity profile can be used. This division has been presented earlier by *Thorpe* (1988). Here it must be emphasized that the real lake bottom can be very irregular and estimation of the existence of fully developed turbulence cannot be based on Richardson number computed from average data.

The turbulent diffusion coefficient is formulated as (*Wüest and Lorke*, 2003)

$$K_t(x,y) = \begin{cases} \dfrac{b_2\left[V^*(x)\right]^3}{k\left[N(y)\right]^2 z}, \text{if } z > z_c = \dfrac{\sqrt{b_2}V^*(x)}{kN(y)} \\ kzV^*(x), \quad \text{if } z \le z_c \end{cases} \tag{10}$$

where V^* is the friction velocity of the hypolimnetic flow, k = 0.4 is von Kárman constant, N is the Väisälä frequency, b_2 is a model calibration parameter, z_c is the thickness of the lowest part of hypolimnion, and z is the distance from the local bottom. Here the mixing efficiency γ_{mix}, the ratio of the buoyancy flux to the dissipation, has been replaced by the calibration parameter b_2. A typical value of γ_{mix} is 0.15; b_2 may deviate from this due to leakages of the model. The distance from the bottom is

$$z(x,y) = H\left[1 - \left(\frac{x}{L_0}\right)^{1/m}\right] - y \tag{11}$$

Next we derive the equation for V^*. First we assume that we can use the logarithmic profile as a first guess and then correct the results by calibration. The reason for this idea is that the velocity profile is logarithmic below the height z_c and above this the velocity has only a small change.

As wind tilts the surface, a horizontal pressure gradient is formed and the thermocline tilts. In balance the slope s of thermocline can be solved as

$$S = \frac{C_w \rho_a D^2 w^3}{g \Delta\rho_{hs} h} \tag{12}$$

This balance hardly can be reached, because the wind varies. However, a characteristic feature of the hypolimnetic flow can be seen. When wind calms, the stress is released making the thermocline to oscillate. The related deflection S at the distance L_h is in a frictionless case

$$S = L_h \frac{C_w \rho_a D^3 w^3}{g \Delta\rho_{hs} h} \cos\left(\frac{2\pi}{T_i} t\right) \tag{13}$$

where T_i is the period of internal seiche for a basin with the mean depth of the hypolimnion equal to $\bar{d}$. It can be solved from

$$T_i = 4L_h \sqrt{\frac{\rho_s\left(\frac{1}{d}+\frac{1}{h}\right)}{g\Delta\rho_{hs}}} \tag{14}$$

$$d = \frac{1}{L_0}\int_0^{L_0} Z\,dz = H - h + \frac{m}{1+m} H \left(\frac{L_h}{L_0}\right)^{1/m}$$

The length L_h is obtained from Eq. (1). The characteristic vertical velocity of thermocline can be obtained from the maximum value of the time derivative of the deflection, which can be formulated as

$$\dot{S}_{\max} = \frac{2\pi C_w \rho_a}{g} D^2 w^2 \frac{L_h}{\Delta\rho_{hs} h T_i} \tag{15}$$

This idea has been used for rectangular basins at least by *Huttula* (1976), *Spigel and Imberger* (1980) and *Gorham and Boyce* (1989). If the thermocline is assumed to be a plane, the maximum cross sectional velocity $V(x)$ in the vertical x can be solved using continuation equation:

$$V(x)Z(x)B = b_1 B\dot{S}_{\max} \frac{L_h + x}{2L_h}(L_h - x) \tag{16}$$

where B is the breadth, b_1 is the scaling factor for the hypolimnetic velocity, the second calibration parameter of the temperature model. The height $Z(x)$ of the thermocline above the bottom can be solved as

$$Z(x) = H\left[1 - \left(\frac{x}{L_0}\right)^{1/m}\right] - h \tag{17}$$

With Eq. (16) the maximum cross sectional mean velocity can be formulated as

$$V(x) = b_1 \dot{S}_{\max} \frac{(L_h - x)(L_h + x)}{2L_h Z} \tag{18}$$

Next we compute the friction velocity for verticals using the neutral stratification approximation and the logarithmic velocity profile. Discharge per breadth is

$$Q(x) = \int_{z_0}^{z} v(x,z)dz = \frac{V^*(x)}{k}\left[Z \ln\left(\frac{z}{z_0}\right) - (Z - z_0)\right] \tag{19}$$

where z_0 is the roughness parameter. The value z_0 = 0.000.6 m was determined for Lake Pääjärvi with an acoustic current profiler by *Virta and Pulkkinen* (1989). Due to larger roughness elements (logs, stones), z_0 can be larger. E.g., *Imberger and Patterson* (1981) have suggested values in the range 0.01–0.2 m. In this study, z_0 = 0.01 m is taken, based on early tests with the model. On the other hand, the discharge with the mean velocity V can be calculated as

$$Q = (Z - z_0)V(x) \tag{20}$$

Using Eqs. (18–20), the friction velocity V^* can be solved for each vertical. In order to use a 1D model for the vertical heat flow, we computed horizontal averages of K_t for depths $i\Delta y$ ($i = n_h, \ldots, 50$) from Eq. (10) using horizontal grid points with spacing of $L_0/100$; n_h is the ordinal of the uppermost grid point in the hypolimnion.

In some situations and some parts of a lake, molecular effects may be dominant in mixing. This is taken into account by adding the molecular heat diffusion coefficient K_m to K_t:

$$K_{tm} = K_t + K_m \tag{21}$$

The temperature in the hypolimnion was calculated with the differential equation

$$\frac{\partial}{\partial y}\left(K_{tm} L_y \frac{\partial T}{\partial y}\right) = -L_y \frac{\partial T}{\partial t} \tag{22}$$

This equation was solved in difference form using the iterative Gauss-Seidel method. The vertical grid spacing was $\Delta y = H/50$. In the upper layer, the epilimnion temperature was used as a boundary condition ($T=T_s$), and the uppermost depth step was $n\Delta y-h$, where n is the ordinal of the uppermost grid point in the hypolimnion. At the bottom, the no-flux condition $\partial T/\partial y = 0$ was used.

Warming in the hypolimnion due to global radiation was separately calculated using an absorption law for solar radiation. In shallow lakes, part of the light entering the bottom is reflected and the rest warms the bottom. The warmed bottom has only a small influence on the water temperature above, and this effect is rather similar with the effect of reflected radiation. Therefore these two small effects can be combined using relatively large value for bottom reflectivity: 0.5.

4. *The use of the model*

In our model, the time step is $\Delta t = 24$ h and daily means are used as the input. The starting time of the calibration is 15 May, somewhat later than the mean date of ice break-up. Calculations were made until the date when the seasonal thermocline depth had been measured (Table 1). In the test runs and in the applications, for practical reasons the calculations were started when the surface temperature had first risen above 6°C and continued until overturn with the thermocline approaching the bottom. For the initial hypolimnetic temperature, 4.5°C was used, except 4°C at the depth H. The initial value of the depth of the thermocline was $H/45$. The values of the parameters C_1, C_2 and C_4 were determined with the calibration. C_1 and C_2 include the influence of wind, and C_4 includes possible systematic errors associated with Eq. (7). C_3 is important only in large lakes, so it can be assumed 0 in our case. The density difference is $\Delta\rho_{hs} = \rho_h - \rho_s$ where the subscript *s* refers to epilimnion and *h* to hypolimnion at the depth of $(h+H)/2$. The density distribution in the hypolimnion is solved with the temperature distribution obtained with the temperature model.

4.1 Calibration

Data sets, which can be used for the model calibration, are rare. We ought to have several lakes with measured vertical temperature, global radiation, and wind velocity. In our case, with measurements of vertical temperature profile in different lakes in different years, it is reasonable to use the average yearly course of meteorological parameters and use the rather coarse time resolution of 24 hours.

The temperature model and the seasonal thermocline depth model were calibrated separately using the data presented in Table 1. In the temperature model, the variable *h* was obtained from the thermocline depth model. Correspondingly, the vertical hypolimnetic temperature needed in the thermocline depth model was obtained from the temperature model.

In the model, one assumption is that the free wind velocity above a lake is the same for all lakes. The yearly course was obtained from the monthly means over the period 1971–2000 in Jokioinen (*FMI*, 2002). Average monthly mean values of global radiation in Jokioinen were computed for years 1974–1981 from data delivered by (FMI, Climate Service Centre). The surface temperature (depth 0.5 m) was obtained using the mean course of the surface temperature of a deep lake (Lake Pääjärvi from 6 years) and that of a shallow lake (Lake Valkea-Mustajärvi, from 3 years). The temperature for other lakes was then interpolated from them based on the maximum depth. The error functions (the sums of the squares of the differences between the measured and calculated values), as a function of the model parameters, were very complicated for both models with several local minima. For this reason the absolute minimum of the error function was obtained manually by changing one parameter at a time. The initial values for the parameters were found from the literature.

In the calibration we used the value of 1.0 for the power parameter *m* in Eq. (1). This is because we do not have depth maps with sufficient accuracy for all lakes. Also the few examples we used showed that the forms of some smaller lakes were so complicated that Eq. (1) cannot describe it. For the test lakes we estimated the value of this parameter both from depth contour maps and by individual optimizations of the model.

The model of the thermocline depth has 4 parameters to be determined by calibration (Eq. 9). The temperature model has two parameters: b_1 (Eq. 16) and b_2 (Eq. 10). However, the actual number of parameters is not as large: C_1 and C_2 are rather well known in the literature, C_3 can be taken as 0 in small lakes, and the value of C_4 ought to be near 1. The parameter values obtained with the calibration are shown in Table 2. Figures 3 and 4 show the comparison of the modelled depth of thermocline and hypolimnetic temperature with corresponding measurements.

Table 2. The parameters of (a) The model of the seasonal thermocline depth h, and (b) The hypolimnion temperature T_m. R is the correlation coefficient and RMS is the root-mean-square error.

a)

C_1	C_2	C_4	R	RMS
0.131	0.40	0.70	0.940	0.67 m

b)

b_1	b_2	R	RMS
10.0	0.345	0.898	0.62 °C

Imberger and Patterson (1981) have collected information concerning the parameter values in Eq. (2) or Eq. (9) based on several studies. The values of the parameters may vary rather much due to differing conditions. They may, e.g., depend on the used wind shading functions and different time step in the calculation.

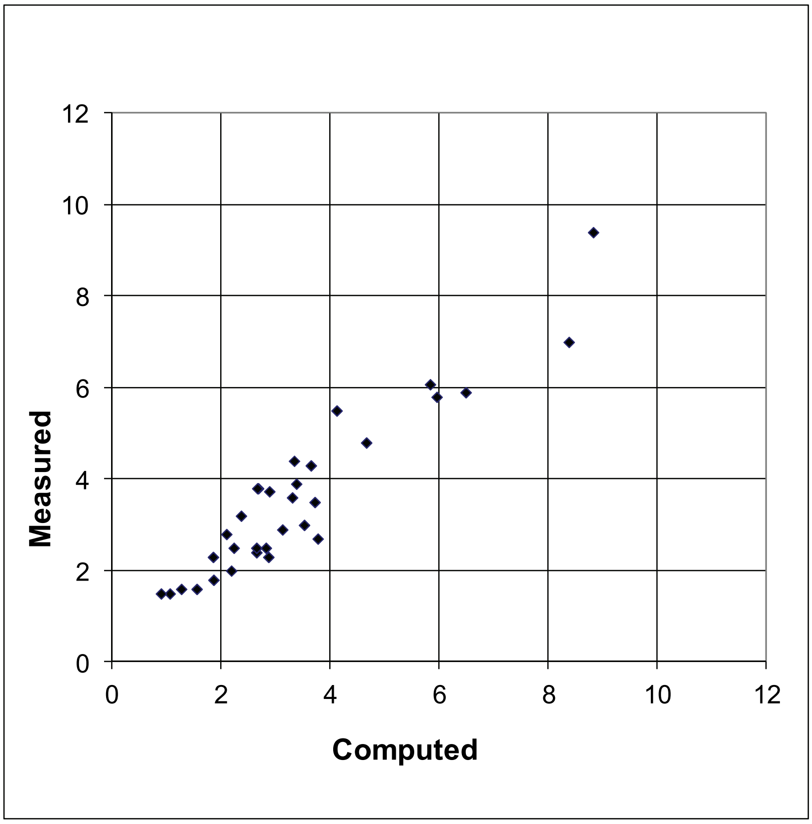

Fig. 3. Measured and computed depth (m) of the thermocline of the calibration lakes.

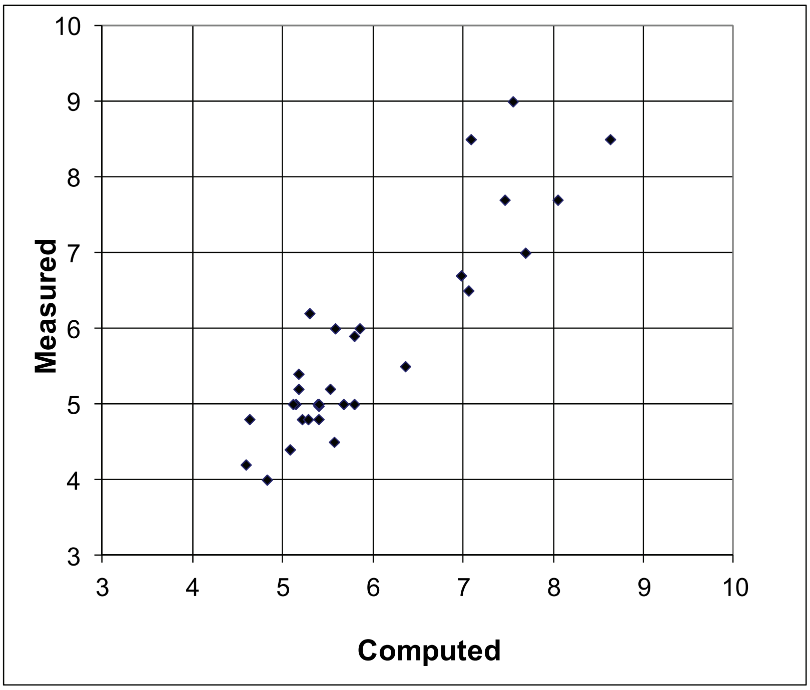

Fig. 4. Measured and computed temperature (°C) of the calibration lakes at the depth of 0.8*H* (corresponding to the hypolimnetic temperature).

A discussion of the parameter values is presented below. The empirical calibration does not give “exact” values for individual parameters, but they ought to be treated as a whole. So the following discussion may be considered only as indicative.

- C_1 and C_2 are very near their respective values 0.13 and 0.5 given in *Imberger and Patterson* (1981). Also C_4 is near its initial guess 1.0.
- b_1 is the scaling factor of oscillating velocity, which ought to be less than unity but in our results it is ten-fold. It includes effects of some simplifying assumptions of the model, such as the way to estimate the friction velocity. Its value is difficult to estimate beforehand. One cause to this discrepancy may lay in possible other causes of mixing which are difficult to take into account as their physics may be complicated (*Imberger and Patterson*, 1981).
- b_2 is greater than its initial guess. It corresponds to the mixing efficiency γ_{mix}, which has a typical value of 0.15 (*Wüest and Lorke*, 2003).

4.2 Test results

Calibration of the thermocline depth and temperature profile models represents a smoothed situation, because the input data were averaged over some years. The model can be tested for single years using daily values of wind velocity, incoming radiation and surface temperature. We used three test lakes, each having two or three years of measurements. Except in Pääjärvi in 1974, we used Aanderaa´s recording thermistor chains to measure the temperature profiles. In 1974 we measured the temperature manually at 5–7 days intervals. Tables below give the following information: the correction coefficient for wind C_C, optimal K and m (optimal and that of based on the depth contour map). It was observed in early test runs that for each lake m did not vary too much in different years and thus the values were combined. Optimization was based on comparison of modelled thermocline depth and near bottom temperature to measurements in July.

Lake Pääjärvi (Pa)

The area of the lake is 13.6 km^2, its greatest length is 7.5 km, its greatest depth is 85 m, and its mean depth is 14 m. The catchment area is 225 km^2. The total area of the three largest catchments flowing into the lake is about 74% of the whole catchment area. Lake Pääjärvi was used in the calibration and also as a test lake, but for different years. The test data had been gathered in summer 1974 in the frame of an IHP (International Hydrological Programme) project (1969–1974) and of a SILMU project in 1992 (Finnish project for research of effects of climate change). Both wind velocity and surface temperature were measured on the lake. Vertical temperature profiles were measured manually at about 5-day intervals in 1974 and using recording instruments in 1992. Global radiation was measured in both years in Jokioinen observatory of the Finnish Meteorological Institute (FMI, Climate Service Centre), 90 km west of Lake Pääjärvi. Depth information of Pääjärvi was obtained from the depth map by *I. Hakala* (LBS). Table 3 gives the results of the model optimization.

Table 3. The results of parameter optimization for Lake Pääjärvi (C_C is correction coeffient for wind K is diffuse attenuation coefficient, m is form exponent.

Year	C_C	K, 1/m	m (optimized)	m (from depth-square root of area)	m (from depth length relation)
1974	0.75	4.0	2.0	2.7	3.1
1992	0.60	4.0	2.0	2.7	3.1

The computed average vertical turbulent diffusion coefficient from June 10 to July 28 was 1.5×10^{-5} m^2 s^{-1} in 1974 and 4.7×10^{-6} m^2 s^{-1} in 1992, taken as the mean below the thermocline. Figures 5 and 6 show comparisons with measured and computed results.

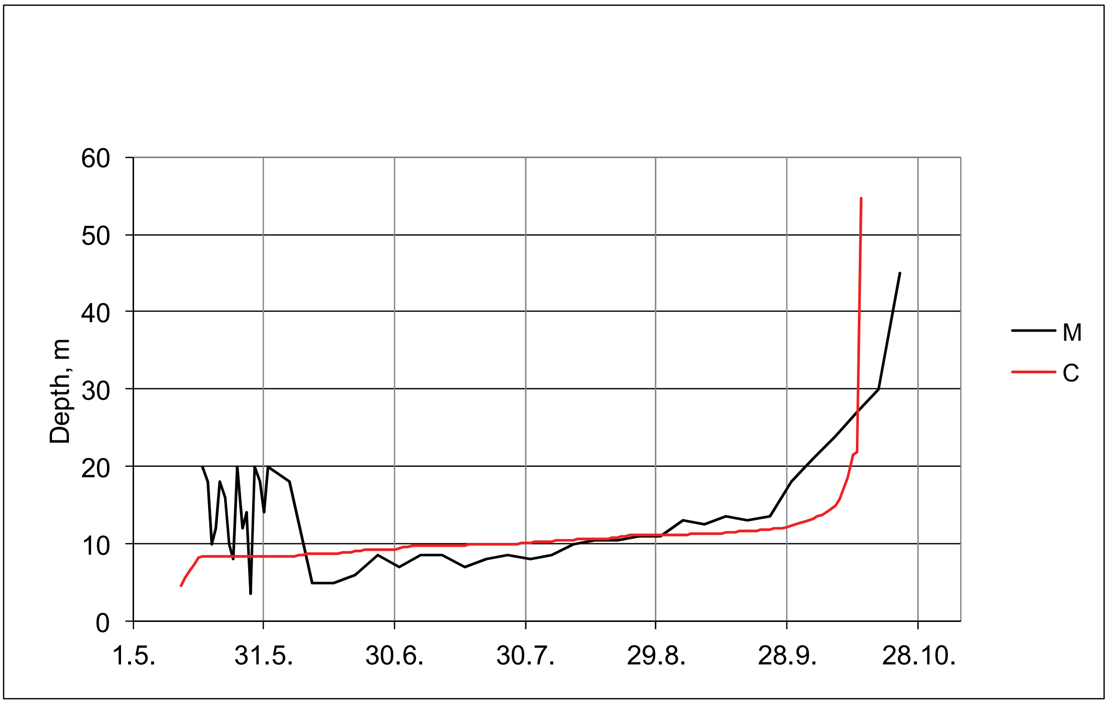

Fig. 5. Measured (M) and computed (C) depth of thermocline of Lake Pääjärvi in 1992.

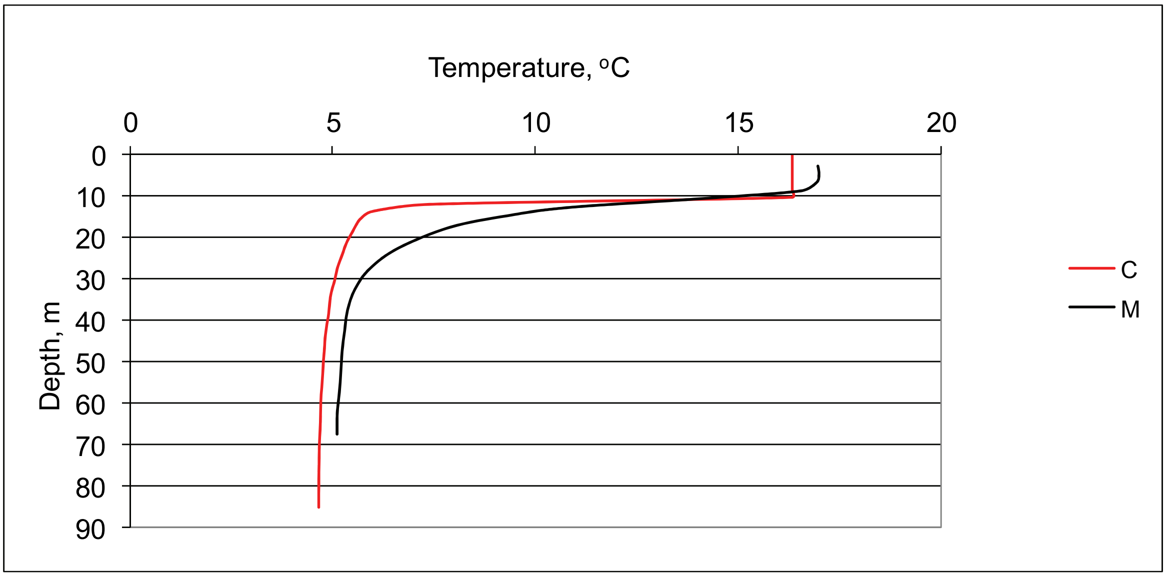

Fig. 6. Measured (M) and computed (C) 24-h average of the vertical temperature profile of Lake Pääjärvi on the 15th of August 1992.

Lake Valkea-Mustajärvi (Va)

The surface area of Lake Valkea-Mustajärvi is 0.140 km^2 and maximum depth is 10.5 m. We had thermistor chains there recording continuously in 1979–1981 (with some interruptions) in the frame of an IHP-project. Wind and surface temperature were measured on the lake. Global radiation were obtained from Jokioinen observatory (90 km WSW from Valkea-Mustajärvi). (FMI, Climate Service Center). In 1981 we had own measurements of global radiation on the lake. Table 4 gives the optimization results.

Table 4. Results of optimization for Lake Valkea-Mustajärvi (C_C is wind velocity correction coefficient, K is diffuse attenuation coefficient, m is form exponent).

Year	C_C	K, 1/m	m (optimized)	m (from depth-square root of area)
1979	1.1	0.7	0.7	0.9
1980	1.0	0.6	0.7	0.9
1981	1.0	1.0	0.7	0.9

The computed average vertical turbulent diffusion coefficient from June 10 to July 28 was 1.4×10^{-7} m^2 s^{-1} in 1979 and 1980, and 4.8×10^{-7} m^2 s^{-1} in 1981, taken as the mean below the thermocline. Figures 7 and 8 show comparisons with measured and computed results.

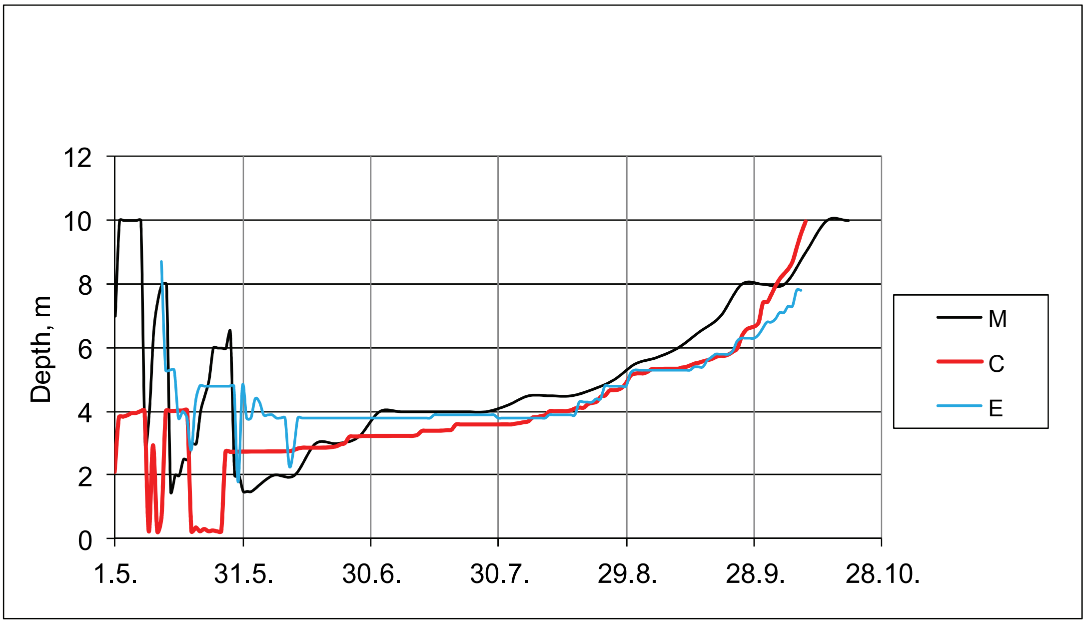

Fig. 7. Measured (M) and computed (C) depth of thermocline of Lake Valkea-Mustajärvi in 1980. (E) means computation of *Elo* (2005) with the k-ε model.

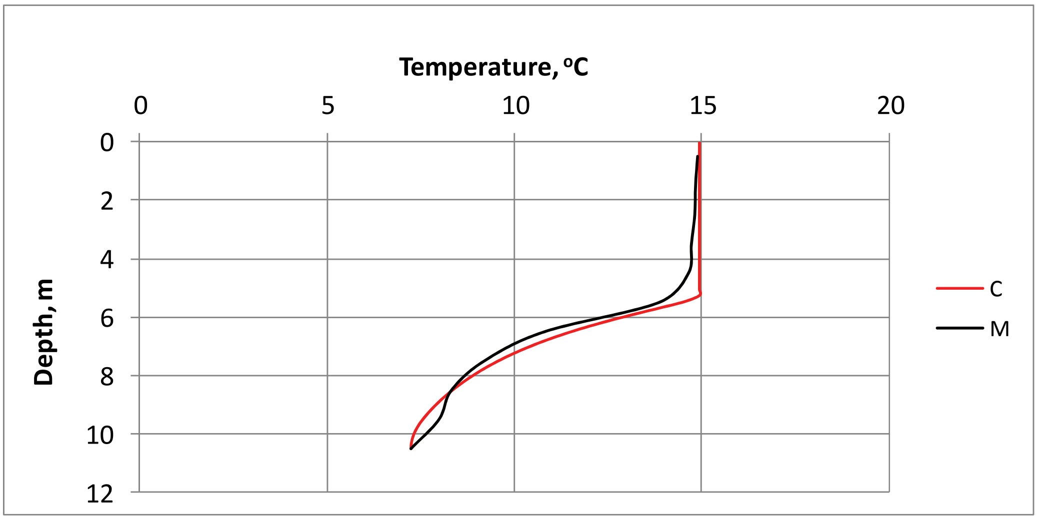

Fig. 8. Measured (M) and computed (C) 2-h average of vertical temperature profile of Lake Valkea-Mustajärvi on the 6th September 1980.

Lake Jääsjärvi (Ja)

Lake Jääsjärvi is a complicated lake with a total surface area 81 km^2. The north-western sub-basin of the lake is called Harjunselkä, used here as the test basin; its surface area is 11.4 km^2, maximum depth is 28 m, length is 6 km, and breadth is 2 km. The Secchi depths of 3–4 m were observed in 1978 in the southern sub-basin corresponding to the diffuse light attenuation coefficient of 0.7–0.9 m^{-1}. Because the sound connecting Harjunselkä to the main basin is broad and shallow, water exchange may affect the thermocline depth. The mean throughflow is about 10 m^3 s^{-1}. Inflow velocity is small, because the channel is broad. The temperature of inflowing water is essentially the same as the surface temperature, and so the direct influence of the inflowing water on hypolimnion is small. For testing, we could use the data from years 1980 and 1981. We used same meteorological measurements as on Lake Valkea-Mustajärvi (85 km SW from the lake). Vertical temperature profiles were recorded in the frame of an IHP-project. Surface temperature was obtained from records of the Finnish Environment Institute (SYKE) as well as the bathymetric map of Lake Jääsjärvi. Table 5 gives the optimization results.

Table 5. Results of optimization for Lake Jääsjärvi. (C_C is wind velocity correction coefficient, *K* is diffuse attenuation coefficient, *m* is form exponent).

Year	C_C	*K*, 1/m	*m* (optimized)	*m* (from depth-square root of area relation)	*m* (from depth length relation)
1980	1.2	0.8	0.15	1.5	0.5
1981	1.05	1.5	0.15	1.5	0.5

The computed vertical average turbulent diffusion coefficient from June 10 to July 28 was 1.7×10^{-3} m^2 s^{-1} in 1980 and 2.3×10^{-3} m^2 s^{-1} in 1981, taken as the mean below the thermocline. Figures 9 and 10 show comparison with measured and computed results.

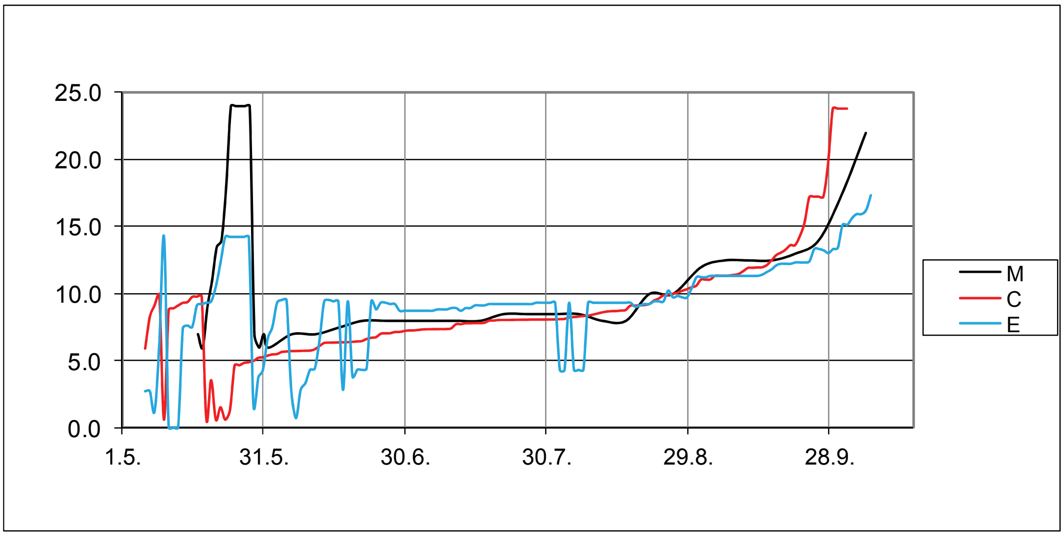

Fig. 9. Measured (M) and computed (C) depth of thermocline of Lake Jääsjärvi in 1980. (E) means computation of *Elo* (2005) with the k-ε model.

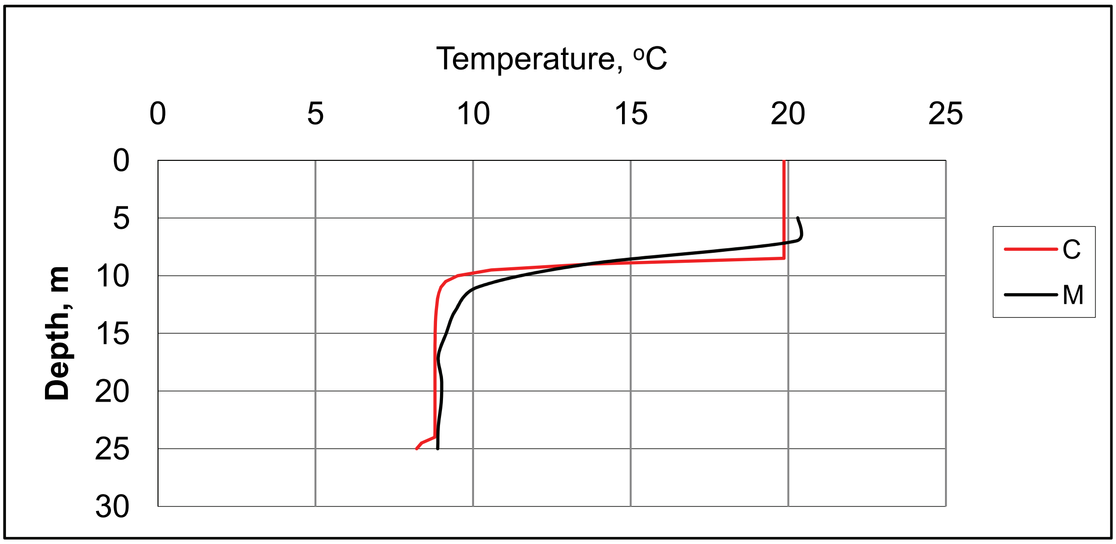

Fig. 10. Measured (M) and computed (C) 24-h average of vertical temperature profile of Lake Jääsjärvi on the 16th of August 1980.

In order to get some ideas of the quality of the model, Table 6 gives comparison of the averages of measured and computed thermocline depths and hypolimnetic temperatures.

Table 6. The measured and computed near bottom temperature (TM and TC) and the depth of thermocline (hM and hC). Means over the period July 1 to August 28.

		Temperature		Depth of thermocline	
Lake and year	Depth m	TM, °C	TC, °C	hM, m	hC, m
Pa74	50	4.7	5.5	7.8	7.8
Pa92	50	5.1	4.8	8.4	10.0
Va79	8.5	6.5	7.0	5.2	4.8
Va80	8.5	7.9	8.1	4.2	3.5
Va81	8.5	5.6	5.6	4.3	4.6
Ja80	19	8.8	8.6	8.2	7.9
Ja81	19	7.9	8.4	7.0	6.8

5. *Conclusions and discussion*

In spite of the fact that the present models include rather coarse approximations, especially the calibrated parameters of thermocline depth seem to agree generally with the corresponding values in the literature. The calibrated parameters for the test lakes do not vary too much in different years. The value of coefficients C_c varies more, the reason for this may be in the fact that the wind velocity was not always measured at the site. The computed and measured average bottom temperature and the depth of the thermocline were in good agreement.

The results with the test lakes show that the concept of thermocline depth is unclear in spring and early summer. This is because the temperature stratification is weak and may have day-to-day variations.

The calculated results were compared with results obtained with Elo's (2005 and 2007) k-ε turbulence model (Figures 7 and 9). The computations showed that her results follow the measurements better than those of the present model, especially in spring. One reason may lie in that we used the time step of 24 h, which causes too smooth results.

According to earlier experiments (e.g., *Fee et al.*, 1996), temperature rise in hypolimnion and thermocline deepening are dependent in smaller lakes mainly on the incoming global radiation and in large lakes mainly on the wind. This is also seen in our model computations. In large Lake Pääjärvi, wind velocity is a very important factor and incoming radiation or diffuse attenuation coefficient have less influence. This is opposite to the small lake Valkea-Mustajärvi. Weather conditions in spring seem to be a very important factor for midsummer hypolimnetic temperature at least in Lake Pääjärvi. Lake Jääsjärvi has shown to behave somewhere between Pääjärvi and Valkea-Mustajärvi. Model results are not so good in Pääjärvi than in other lakes. This lake is much larger than the other test lakes.

The results of this study support possibilities to calculate conditions in several small lakes in an area, when there are no possibilities to apply a more detailed data and model. On the other hand, as the model describes seasonal development including the effects caused by the limited area of the basin, it is reasonable to believe that the model can further be used as a research tool in analysing these effects. The k-ε-model still tends to give too abrupt profile in small lakes (*Elo,* 2005, 2007), therefore the possibility to study further shapes of the profiles and the blocking of heat in the epilimnion is important.

Model tests can be made concerning differences caused by weather variations and using more data for adjusting. The shape of the temperature profile in hypolimnion can further be studied in lakes with different morphology. Although the limited amount of data restricts possibilities to use the k-ε-model, several processes and their importance can be studied and tested with the models of this study, because the needed data are much more abundant: temperature profiles at the end of summer, describing seasonal stratification developed during summer.

Acknowledgements

We thank Professor Lauri Arvola, Dr. Ants Erm and Dr. Marko Järvinen for their support in the early phase of this research. Also we like to thank Dr. Arkady Terzhevik, Professor Timo Huttula and Mr Petri Kiuru for their reading of the manuscript and making valuable comments.

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List of symbols

A	Surface area of a basin
B	Breadth of a basin
b_1, b_2	Calibration coefficients for the hypolimnetic temperature model
C_1, C_2, C_3, C_4	Calibration coefficients of the thermocline depth model
C_C	Wind velocity correction coefficient for a single lake
C_p	Specific heat of water
C_w	Drag coefficient for wind
d	Height of the surrounding forest
$\bar{d}$	Mean depth of the hypolimnion
D	Wind shading coefficient
h	Depth of thermocline
H	Maximum depth of a basin
$\widetilde{H}$	Down ward flux of heat from the surface
k	von Kárman constant
K	Diffuse light attenuation coefficient
K_t	Turbulent diffusion coefficient of heat

K_m	Molecular diffusion coefficient
K_{mt}	Combined molecular–turbulent diffusion coefficient
L	Length of a basin (square root of area); subscript 0 refers to half-length, *h* refers to half-length at the depth of *h,* and *y* refers to half-length at the depth of y
m	Power in the depth-length relation of the basin
m	Subscript for the depth $(H+h)/2$
n_h	Ordinal of the uppermost grid point of hypolimnion
N	Väisälä frequency
Q	Discharge per breadth of the basin in the hypolimnion
q	Turbulent energy
R	Correlation coefficient
S_h	Deflection of thermocline at the distance of L_h
s	Deflection angle of thermocline
t	Time
T	Temperature; subscript s0 refers to the surface (0.5 m), *s* to the mean of epilimnion, *p* to the former time step, *m* to the depth of $0.8H$ and *h* to the temperature at the depth of $(H+h)/2$
T_i	Period of internal seiches
u_f	Vertical convective velocity
u_*	Friction velocity of wind
w	Wind velocity
$v(x,z)$	Velocity at point (x,z)
V	Cross sectional mean velocity in the hypolimnion
V^*	Friction velocity of hypolimnetic flow
x	Horizontal co-ordinate, $x = 0$ in the middle of the lake (max depth)
y	Vertical co-ordinate, positive downwards, zero at the surface
Y	Local depth, $Y(x) = H[1 - (x/L_0)^{1/m}]$
z	Distance from the local bottom
z_c	Thickness of the lowest part of the hypolimnion
z_0	Roughness parameter
Z	Local distance between the bottom and the thermocline, $Z(x) = Y - h$
α_w	Thermal expansion coefficient of water
γ_{mix}	Mixing efficiency
γ	Parameter of the wind shadowing function
η	Coefficient of the stirring efficiency of wind
ρ	Density, subscript *a* refers to air, *w* to water and *h* to depth of $(H+h)/2$
Δu	Horizontal velocity difference between epilimnion and hypolimnion
$\Delta\rho_{hs}$	Density difference between epilimnion and hypolimnion