Geophysica (2000), 36(1–2), 129–139

Influence of the Depth-Dependence of the PAR Diffuse Attenuation Coefficient on the Computation of Downward Irradiance in Different Water Bodies

H. Arst, A. Reinart, A. Erm and M. Hussainov

Estonian Marine Institute, Paldiski Road 1, 10137 Tallinn, Estonia

(Received: November 1999; Accepted: March 2000)

Abstract

Three methods of determination of underwater irradiance in PAR region of the spectrum are considered: (1) measurements in situ *(alternative possibility is theoretical calculations using spectral values of radiation) (2) computations using the depth-averaged diffuse attenuation coefficient (mean value for PAR region), (3) computations using depth-dependent diffuse attenuation coefficient. The results by methods (1) and (2) are compared for optically homogeneous water column (model calculations), the methods (1), (2) and (3) are compared using* in situ *measurements in Estonian and Finnish lakes and corresponding calculations. The depth averaged diffuse attenuation coefficient (determined by means of a semilog plot of irradiance vs. depth) describes rather well the optical contrasts between the different water bodies and allows satisfactory estimation of 1 % depth, but it is not suitable for determination of the real vertical profiles of the underwater irradiance. Essentially better results give the computations of underwater PAR using the depth-dependent values of respective diffuse attenuation coefficient.*

Key words: underwater irradiance, diffuse attenuation coefficient of light, lake optics

1. Introduction

The apparent optical properties of the aquatic environment are characteristics widely used in studies of the water bodies. From these properties most essential is the diffuse attenuation coefficient of the downward irradiance in the water, $K_{d,\lambda}(z)$, defined in the following way (*Dera*, 1992):

$$K_{d,\lambda}(z) = -\frac{1}{E_{d,\lambda}(z)}\frac{dE_{d,\lambda}(z)}{dz}. \tag{1}$$

Here λ is the wavelength of light, z is depth, $E_{d,\lambda}(z)$ is the downward vector irradiance at the wavelength λ and depth z. If Eq. (1) is treated as a simple differential equation a wellknown form of the solution of this equation is the downward irradiance as a function of depth:

$$E_{d,\lambda}(z) = E_{d,\lambda}(z=-0)\exp\left[-\int_0^z K_{d,\lambda}(\xi)d\xi\right]. \quad (2)$$

Here $E_{d,\lambda}(z{=}{-0})$ is the downward irradiance just below the water surface (after refraction). If the diffuse attenuation coefficient does not depend on depth Eq. (2) can be replaced by

$$E_{d,\lambda}(z) = E_{d,\lambda}(z=-0)\exp(-K_{d,\lambda}z)\,. \quad (3)$$

Eq. (3) can be used also considering $K_{d,\lambda}$ as a diffuse attenuation coefficient averaged over depth.

From Eq. (3) follows that diffuse attenuation coefficient is possible to determine for any layer in the water body:

$$K_{d,\lambda}(z_1,z_2) = \frac{1}{z_2-z_1}\ln\left[\frac{E_{d,\lambda}(z_2)}{E_{d,\lambda}(z_1)}\right]. \quad (4)$$

Here $K_{d,\lambda}$ is the mean diffuse attenuation coefficient for the water layer with upper border at depth z_1 and lower border at z_2.

As can be seen, all formulae discussed above refer to a particular wavelength of light. However, we are often interested in the irradiance transmittance of a wider range of wavelengths, from λ_1 to λ_2 . For this there are three main possibilities. The first is simply to measure the values of $E_d(z, \lambda_1, \lambda_2)$ by an underwater spectroradiometer. The second is to calculate the spectral irradiance by Eq. (2) or (3) and to integrate these values over wavelength:

$$E_d(z,\lambda_1,\lambda_2) = \int_{\lambda_1}^{\lambda_2} E_{d,\lambda}(z)d\lambda\,. \quad (5)$$

From this equation we obtain the value of E_d for PAR region ($E_{d,PAR}$), taking λ_1 =350 and λ_2 =700 nm (*Dera,* 1992). However, since the ultraviolet is strongly absorbed in the water, most often the range of PAR is considered to be from 400 to 700 nm. The respective irradiance will be in W/m^2 . The other possibility is to measure $E_{d,PAR}$ in μmol m^{-2} s^{-1}, which corresponds to the formula

$$E_{d,PAR}(z) = \int_{400}^{700} \frac{\lambda}{c_0 h} E_{d,\lambda}(z)d\lambda\,, \quad (6)$$

where h (Plank's constant) is 6.6255x10^{-34} J s and c_0 (velocity of light in vacuum) is 2.9979x10^8 m s^{-1}. The ratio between $E_{d,PAR}$ measured in W m^{-2} and μmol m^{-2} s^{-1} is not constant but depends on the depth and the water properties (*Reinart et al.*, 1998).

There is also a possibility to use the weighted average of the diffuse attenuation coefficient

$$\overline{K}_d(z,\lambda_1,\lambda_2) = \frac{\int_{\lambda_1}^{\lambda_2} K_{d,\lambda}(z)E_{d,\lambda}(z)d\lambda}{\int_{\lambda_1}^{\lambda_2} E_{d,\lambda}(z)d\lambda} \tag{7}$$

and Eq. (3).

Widely used in practice is the estimation of diffuse attenuation coefficient, averaged both over the depth and wavelength, on the basis of measured vertical profiles of integral downward irradiance. Note, that the integration over wavelength is often accomplished by the sensor design (e.g. LI-COR PAR sensors). For depth averaging, a semilog plot of radiation results *vs.* depth are applied, where the mean attenuation coefficient is found as the slope of the straight regression line through these points (*Dera,* 1992; *Arst et al.*, 1996). Of course, this method brings about certain errors in K_d (and afterwards in corresponding $E_d(z)$ determined using this K_d). The K_d determined in this way cannot describe properly the water column, the depth dependence of K_d is lost. As known, the values of K_d for wide spectral interval depend on depth even in optically homogeneous water column. As an example, let's consider the PAR region. The spectral composition of PAR is remarkably changing with depth, due to strong absorption in violet, blue and red regions of the PAR. In deeper layers only yellow-green radiation is remaining. This leads to the decrease of $K_{d,PAR}$ with depth in optically homogeneous water column. The other reasons of the vertical change of $K_{d,PAR}$ are: 1) the variation of the optical properties of the water with depth (due to vertical change of the water constituents), 2) the differences in angular distribution of radiance in different depths. The resulting effect of these two factors can be a decrease or increase of $K_{d,PAR}$ with depth, and also irregular $K_{d,PAR}$ profiles are possible i.e. *Erm et al.* (1999). In the last case there must be significant vertical variations in the transparency of the water column.

When estimating $E_{d,PAR}(z)$ using depth averaged $K_{d,PAR}$ it is assumed that Eq. (3) is applicable over the PAR band. An alternative possibility is to determine the values of $K_{d,PAR}$ for all layers in the water column by Eq. (4) and compute the values of $E_{d,PAR}(z)$ by a formula, analogous to Eq. (2) (instead of an integral there will be a sum), which is assumed to be valid for PAR region.

It is of interest how much the results by these two methods differ from each other and also from $E_{d,PAR}(z)$ values actually measured in the water body. The main objective of the present study is to assess the errors due to computing the values of $E_{d,PAR}(z)$ not from the respective spectral values (by Eq. (5)), but by using the depth-averaged or depth dependent values of $K_{d,PAR}$ in different types of water bodies.

2. Measurements and methods

In the present study we used the data of measurements carried out in 1997–98. The values of the downward vector irradiance in PAR region were measured by an underwater radiation sensor LI-192 SA (firm LI-COR). These measurements were performed in six Estonian and seven Finnish lakes, and resulted in more than 50 separate measurement series. From the values of $E_{d,PAR}(z)$ obtained the depth averaged diffuse attenuation coefficient $(K_{d,PAR})$ was determined by a semilog plot of $E_{d,PAR}(z)$ *vs.* depth. Additionally, the values of $K_{d,PAR}$ for separate water layers were estimated according to Eq. (4). To avoid the influence of fluctuations of incident solar radiation on the measurements a special system was used. Two LI-192 SA sensors were fixed on a frame at a distance of 0.5 m from each other. These sensors measured simultaneously, eliminating the errors due to different light conditions at different measurement times. By lowering the frame with the sensors, the vertical profile of $K_{d,PAR}$ can be obtained at a depth interval of 0.5 m over the whole water column.

Some examples of the determination of depth-averaged $K_{d,PAR}$ are shown in Fig. 1 (a,b). The values of $E_{d,PAR}$, presented in this figure are measured with the PAR sensor LI 192 SA. The results of the measurements were corrected taking into account the possible change of illumination conditions during the measurement procedure (the method by *Virta and Blanco-Sequeiros* (1995) was used). For this the simultaneous recording of the incident irradiance by an integral radiation sensor LI200 SA was performed. If the change of the incident irradiance exceeded 20% the corresponding results were left out from our data base. Some vertical profiles of $K_{d,PAR}$ (for 0.5 m thick layers) are presented in Table 1. The values for the first layer (0–0.5 m) is in brackets, because there were difficulties for fixing the device with its upper sensor just below the water surface. The profiles presented in Table 1 are typical: the monotonous decrease of $K_{d,PAR}$ was observed only in a few cases, and maxima and minima in all measured $K_{d,PAR}$ profiles were mostly in the range of those shown in Table 1. Consequently, the waters sampled can be considered vertically inhomogeneous, since the vertical profiles of $K_{d,PAR}$ are not decreasing monotonously with increasing depth.

The values of $K_{d,PAR}$ averaged over depth can be useful for investigating the underwater light climate: on the basis of $K_{d,PAR}$(av) it is possible to estimate the diurnal variation of underwater light field from recording of the incident PAR during the day (or even during a week) and calculating $E_{d,PAR}$(z) analogously to Eq. (3). If we have measured the vertical profile of $K_{d,PAR}$ we can determine $E_{d,PAR}$(z) by Eq. (2). The temporal and spatial dependence of $K_{d,PAR}$ in some water body is essentially slower than the time dependence of irradiance (diurnal and synoptic variations). Planning these studies it is of interest to estimate the errors caused by applying the depth-averaged $K_{d,PAR}$ instead of its vertical profiles or spectral calculations.

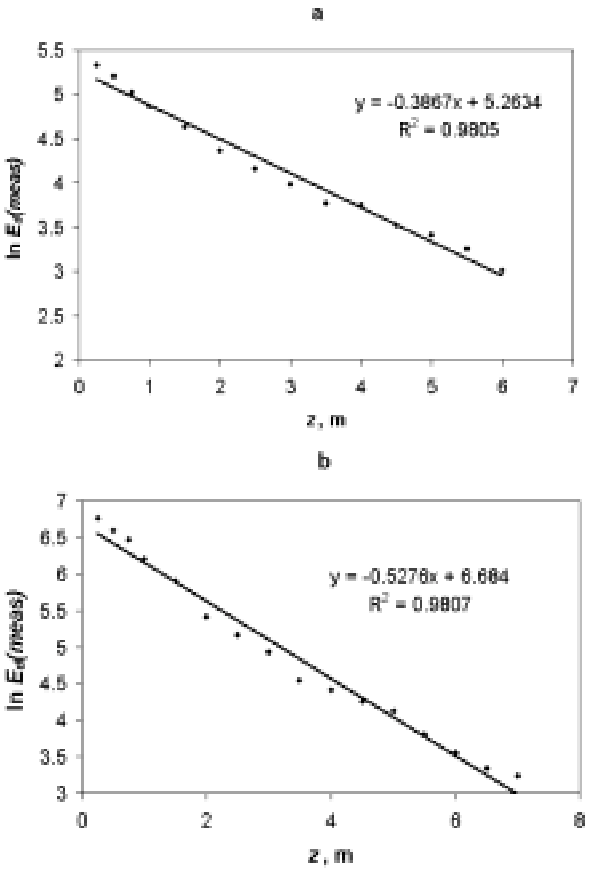

Fig. 1. Determination of the depth-averaged value of $K_{d,PAR}$ as the slope of the regression line of $ln[\ E_{d.PAR}(z)]$ *vs*. z: (a) Lake Koorküla Valgjärv, 17.06.97, $K_{d,PAR}(av)$=0.387 m^{-1}, (b) Lake Nohipalu Valgjärv, 13.08.98, $K_{d,PAR}(av)$=0.528 m^{-1} (both lakes are in Estonia).

Table 1. Values of $K_{d,PAR}$ (in m^{-1}) for layers with thickness of 0.5 m for lakes Koorküla Valgjärv, Paukjärv (both in Estonia), Lammi Pääjärvi and Lohjanjärvi (in Finland).

Layer Δz (m)	Lake Koorküla Valgjärv	Lake Pääjärvi Station 2	Lake Lohjanjärvi Station 5	Lake Paukjärv
	17.06.97	13.08.97	13.05.98	11.08.98
0–0.5	(0.89)	(1.53)	(2.30)	(0.71)
0.5–1.0	0.76	1.45	1.94	0.60
1.0–1.5	0.63	1.00	1.72	0.52
1.5–2.0	0.53	0.90	1.14	0.39
2.0–2.5	0.44	0.63	0.92	0.48
2.5–3.0	0.33	0.64	1.15	0.42
3.0–3.5	0.33	0.56	1.18	0.44
3.5–4.0	0.27	0.48	1.09	0.35
4.0–4.5	0.42	0.39	–	0.31
4.5–5.0	0.46	0.20	–	0.28

The first step is to perform these estimations for optically homogeneous water column, which is possible by model calculations. Thus, we considered here three Jerlov's water types (*Jerlov,* 1976) and three lakes. The initial data were the values of $K_{d,\lambda}$

for 10 nm-width spectral intervals ($\Delta\lambda$), determined by data taken from *Jerlov* (1976) and *Reinart and Herlevi* (1999). For each spectral interval the values of $E_{d,\Delta\lambda}(z)$ were calculated by Eq. (3) and then integrated by Eq. (5) from 400 to 700 nm (instead of integral there was a sum). The results described $E_{d,PAR}(z)$ profiles for different water types mentioned above. Then using these data the depth-averaged $K_{d,PAR}$ was determined by semilog plot of $E_{d,PAR}(z)$ *vs*. depth and $E_{d,PAR}(z)$ was determined again applying Eq. (3) for PAR region.

3. *Results and discussion*

The comparison of the vertical profiles of $E_{d,PAR}(z)$ determined by spectral data and by $K_{d,PAR}(av)$ is presented in Fig. 2 (a,b,c,d). From the results obtained the relative error, *r*, was determined:

$$r = \frac{100\left[E_{d,PAR}(K_{d,av}) - E_{d,PAR}(calc)\right]}{E_{d,PAR}(calc)}. \tag{8}$$

Here $E_{d,PAR}(K_{d,av})$ is the underwater irradiance estimated through $K_{d,PAR}(av)$ and $E_{d,PAR}(calc)$ is that obtained from spectral (10 nm width intervals) data.

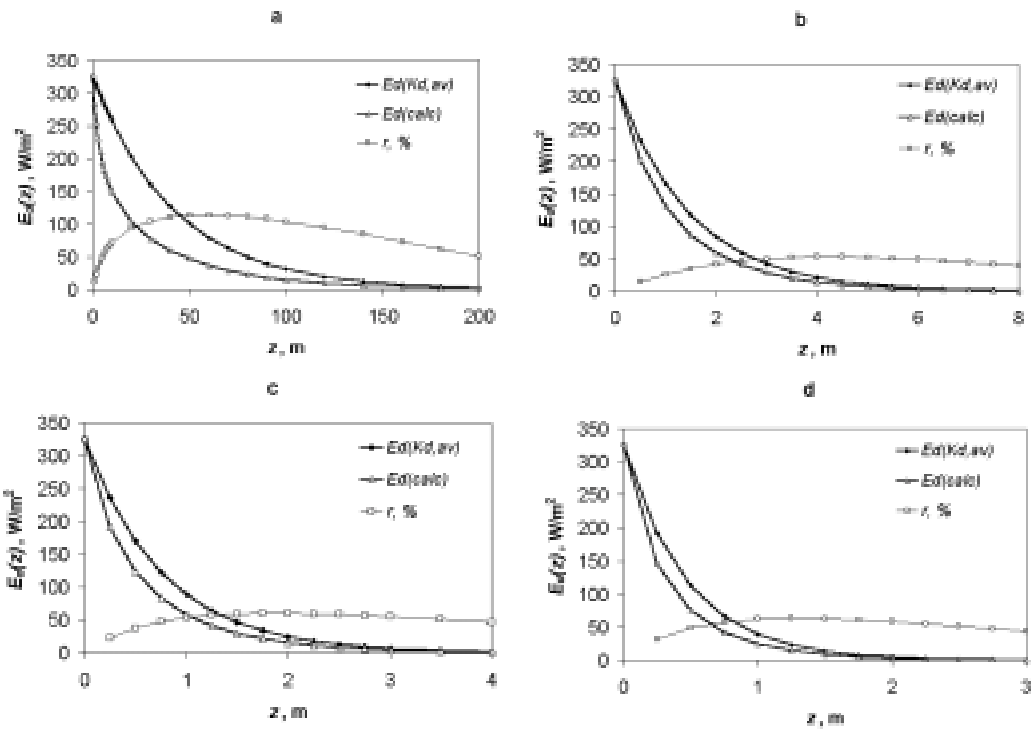

Fig. 2. Vertical profiles of PAR estimated on the basis of $K_{d,PAR}(av)$ and calculated from spectral data (respectively $E_{d,PAR}(K_{d,av})$ and $E_{d,PAR}(calc)$) for the following waters: (a) Jerlov's oceanic water type I ($K_{d,PAR}(av)$=0.0235 m^{-1}), (b) Jerlov's coastal water type 9 ($K_{d,PAR}(av)$=0.680 m^{-1}), (c) Lake Pääjärvi, Finland ($K_{d,PAR}(av)$=1.3 m^{-1}), (d) Lake Võrtsjärv, Estonia ($K_{d,PAR}(av)$=2.07 m^{-1}). Additionally the profiles of the error computed by Eq. 8 are shown (the scale of r (%) is the same as for E_d.

As seen from these results, the absolute errors are maximal not in the very upper layer and not in deeper layers, but somewhere between these layers, at depths where the PAR is attenuated to 25–60 % from its subsurface value. The value of the depth of the maximal absolute error depends essentially on the transparency of the water. Some estimations of maximal errors (both absolute and relative) and the respective depths at which these errors occur are shown in Table 2. In this table the data for three Jerlov's water types (I, III, 9) and three lakes (Paukjärv and Võrtsjärv in Estonia, Lammi Pääjärvi in Finland) are presented. These lakes differ by their properties: the typical values of Secchi depth are for Lake Paukjärv 5 m, Lake Lammi Pääjärvi 2 m and Lake Võrtsjärv 0.8 m, the respective effective concentrations of yellow substance (*Arst et al.,* 1996) are 2, 20 and 13 mg L^{-1}.

Table 2. Values of maximal errors calculated from Eq. 8 and the corresponding depths in the water bodies.

Water type or lake	Jerlov I	Jerlov III	Jerlov 9	Lake Paukjärv	Lake L.Pääjärvi	Lake Võrtsjärv
$K_{d,PAR}$ (m^{-1})	0.0235	0.135	0.680	0.455	1.30	2.13
max absolute error (W/m^2)	108	43	34	25	47	46
max relative error (%)	115	41	53	39	60	63
depth of max absolute error (m)	10	8	2	3	2	0.5
depth of max relative error (m)	60	12	4.5	6.5	2	1.25

It appears that the depth of the maximal relative error correlates well with the values of thediffuse attenuation coefficient $K_{d,PAR}$ (Fig. 3). The regression formula is:

$$z_{max} = 2.67\, K_{d,PAR}^{-0.833}. \quad (9)$$

The correlation coefficient R=0.984. However this formula is obtained on the basis of only six points, consequently, the numerical values of the constants are approximate, but the general conclusion – dependence of z_{max} on $K_{d,PAR}$ in the form of a power function is probably valid.

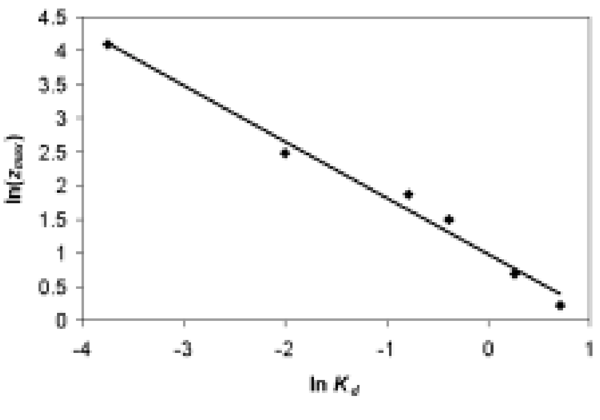

Fig. 3. Relationship between the depth of maximal relative error (z_{max}) and diffuse attenuation coefficient ($K_{d,PAR}$).

The next step is to assess the errors of $E_{d,PAR}(z)$ calculated by Eq. (2) or (3) in a real water environment, which can be vertically inhomogeneous. To accomplish this, the computations were carried out choosing the initial data from different types of lakes. Some of the results obtained are presented in Fig. 4 (a,b,c,d). Lakes Paukjärv and Nohipalu Valgjärv are moderately clear water (Secchi disk depth respectively 5 and 4 m). The maximal differences between $E,_{d,PAR}(meas)$ and $E_{d,PAR}(K_{d,av})$ occur at the depths 1–2.5 m, whereas $E_{d,PAR}(K_{d,av})$ exceeds $K_{d,PAR}(meas)$ about 50–100%. Lake Lammi Pääjärvi is characterized by high amount of yellow substance (2–2.5 times that of other lakes), the Secchi disk depth is 2 m, Lake Ülemiste is strongly eutrophic, Secchi disk depth ~1 m. In these lakes of low transparency the maximal differences between $E_{d,PAR}(meas)$ and $E_{d,PAR}(K_{d,av})$ occur between the depths 0.3–1.5 m and the error can be more than 100 %. In all cases the values of $E_{d,PAR}$ computed using depth-dependent $K_{d,PAR}$ are rather close to $E_{d,PAR}(meas)$, differences being in the range of natural measurement errors.

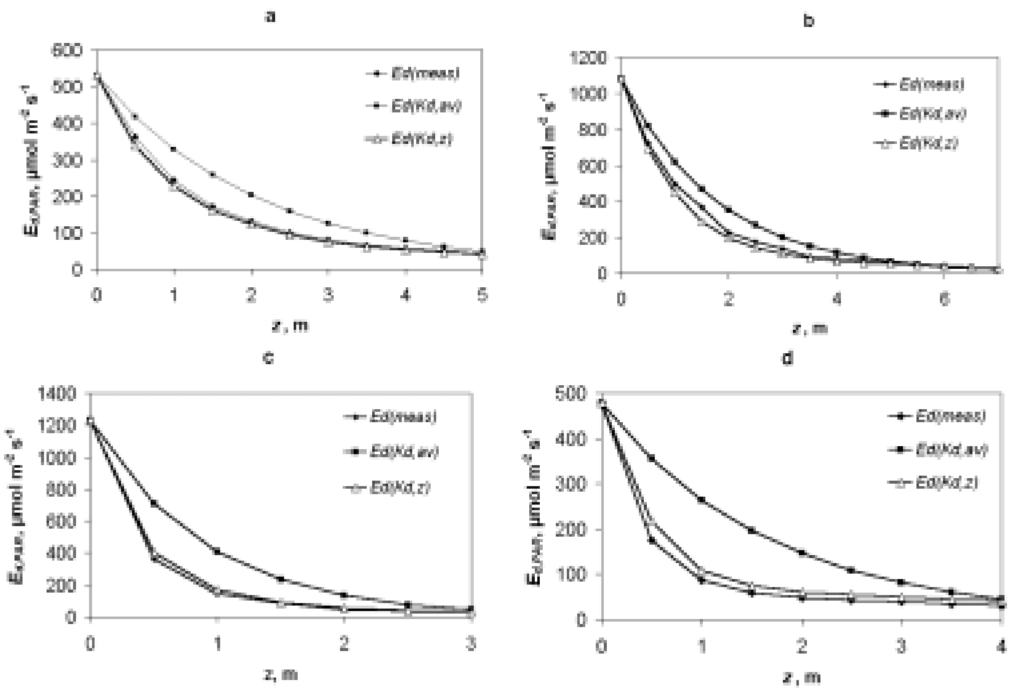

Fig. 4. Vertical profiles of PAR obtained by three methods: (1) measurements *in situ* ($E_{d.PAR}(meas)$), (2) using the values $K_{d,av}(PAR)$ ($E_{d,PAR}(K_{d,av})$), (3) using the depth-dependent values of $K_d(PAR)$ ($E_{d,PAR}(K_{d,z})$). The data for the following lakes were used: (a) Paukjärv 05.05.97, (b) Nohipalu Valgjärv 13.08.98, (c) Ülemiste, station Plant, 03.06.98 (all in Estonia), (d) Lammi Pääjärvi 13.08.97 (Finland).

For all of our lake data the comparison of $E_{d,PAR}(meas)$ and $E_{d,PAR}(K_{d,av})$ shows greater differences than were found in conditions of optically homogeneous waters (compare the data shown in Figs. 2 and 4). This can be explained by changes in the transparency with depth ($K_{d,PAR}$ changes more quickly in comparison to the homogeneous water column).

Earlier results (*Arst et al.,* 1996, 1999; *Reinart and Herlevi,* 1999) and those in the present study show that the depth averaged $K_{d,PAR}$ describes well the optical contrasts between different water bodies, but it is not suitable for computing the accurate vertical profiles of underwater irradiance. Better results are obtained here from computations using the depth-dependent $K_{d,PAR}$.

The different ways of determination of $E_{d,PAR}$ (Eqs. 5 and 6) bring about the dependence of $K_{d,PAR}$ on the units used for measuring the PAR. However, by our estimation, this dependence has practically no influence on the relative errors of computation of $E_{d.PAR}$ profiles by depth averaged $K_{d..}$ Note, that the spectral values of K_d (Eq. 4) do not depend on the units used for the measurements of $E_{d,\lambda}$.

Special attention has to be paid to the values of $E_{d,PAR}(z)$ which are less than 10 % of its value just below surface. In this region the relative errors are big (Fig. 2) but absolute errors are comparable with the errors of measurements. This has importance in regards to the estimation of the 1 %-depth (widely used as the lower border of euphotic zone) and leads to a conclusion, that the 1 %-depth can be successfully determined using the depth-averaged values of $K_{d,PAR}$.

There are two main causes for the errors in the depth-dependent diffuse attenuation coefficient estimations of PAR (curves $E_d(K_{d,z})$ in Fig. 4): (1) The non-correspondence of irradiances measured in the air and under the water (quick change of the cloud conditions); (2) difficulties of the measuring of $K_{d,PAR}$ value in the first (subsurface) layer and the corresponding errors in the results. Difficulties in measuring the diffuse attenuation coefficient in the layer just below the water surface arise also in the spectral measurements. One way to solve this problem is to use not the value of E_d just below the surface (practically at the depths of 1–10 cm), but to derive this value from the measurement data of incident irradiance (taking into account the albedo of the water) and to measure simultaneously the irradiance at the lower border of the subsurface layer.

Our data base is rather large but not uniform: there is only a little data for lakes with high concentrations of yellow substance, and the measurements in subsurface layer of strongly eutrophic lakes were performed with too large a depth interval. For this reason our conclusions on the connections between errors of different computation methods and bio-optical type of water body, made by experimental data, have to be considered as preliminary. The additional measurements by special programme are needed.

4. Conclusions

1. In optically homogeneous water column the diffuse attenuation coefficient for PAR region of the solar spectrum decreases with depth. This decrease is mainly due to the changes in the spectral composition of light with depth, but some influence may also come from the differences in the angular distribution of radiation. Less transparent layers in the water column cause the vertical

stability or increase in $K_{d,PAR}$ values, increased transparency causes quicker decrease of $K_{d,PAR}$.

2. The depth averaged diffuse attenuation coefficient, determined by means of a semilog plot of irradiance vs. depth, describes rather well the optical contrasts between different water bodies, but it is not suitable for determining accurate vertical profiles of underwater irradiance. Maximum absolute errors occurred at the depths 1–2.5 m in clear-water lakes and 0.3–1.5 m in turbid and “yellow” lakes. The relative errors at these depths can be 100% and more.
3. More accurate results can be obtained by computations of underwater irradiance profiles using depth-dependent values of $K_{d,PAR}$. In this case the differences between measured and calculated irradiances are mostly in the limits of the natural measurement errors.
4. At the depths where the underwater irradiance is less than 10% of its value just below the surface calculations using averaged and depth-dependent $K_{d,PAR}$ yield similar quantities: the absolute differences between measured and calculated irradiances are small. The respective relative differences can be high, but the relative error of measurements also increases with decreasing of the irradiance, and we can assume that they are comparable with the differences. Therefore, for determination of the 1%-depth all three methods considered above are suitable.

Acknowledgements

The authors are indebted to the Estonian Science Foundation (grants 751 and 1804) and to the Academy of Finland for financial support to this investigation.

References

Arst, H., S. Mäekivi, T. Kutser, A. Reinart, A. Blanco-Seqeiros, J. Virta and P. Nõges, 1996. Optical investigations of Estonian and Finnish lakes, *Lakes & Reservoirs: Res. and Manag.* **Vol. 2**, 187–198.

Arst, H., A. Erm, K. Kallaste, S. Mäekivi, A. Reinart, A. Herlevi, P. Nõges and T. Nõges, 1999. Investigation of Estonian and Finnish lakes by optical measurements in 1992–97, *Proc. Estonian Acad. Sci., Biology, Ecology,* **Vol. 48**, No. 1, 5–24.

Dera, J., 1992. *Marine Physics*, PWN, Warszawa, and Elsevier, Amsterdam, 516 pp.

Erm, A., H. Arst, M. Hussainov, T. Kurser and A. Reinart, 1999. Optical measurements in Lake Ülemiste, *Proc. Estonian Acad. Sci., Biology, Ecology,* **Vol. 48**, No 1, 63–74.

Jerlov, N.G., 1976. *Marine Optics*, Elsevier, Amsterdam.

Reinart, A., H. Arst, A. Blanco-Sequeiros and A. Herlevi, 1998. Relation between underwater irradiance and quantum irradiance in dependence on water transparency at different depths in water bodies, *J. Geophys. Res.*, **Vol. 103**, No. C4, 7749–7752.

Reinart, A. and A. Herlevi, 1999. Diffuse attenuation coefficient in some Estonian and Finnish lakes, *Proc. Estonian Acad. Sci., Biology, Ecology,* **Vol, 48**, No. 4, 267–283.

Virta, J. and A. Blanco-Sequeiros, 1995. Correction of measured underwater spectral irradiance to the variation of some external effects with examples, *Rep. Ser. Geophys.*, **No. 32**, University of Helsinki, 93–100.