A Contribution to the Further Investigation of McKenzie's Rifting Model

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Abstract

We have continued the investigation of generalized geodynamic McKenzie's model of rifting which was not developed exhaustively. Filling this want, we have examined the rift basin and heat flow evolution under different regimes of spreading. Within the mathematical simulation, three model versions of finite extension of the lithosphere were studied and compared: instantaneous, accelerated and with a constant rate. In case of the lithosphere extending with an exponential acceleration, the solution was obtained through confluent hypergeometrical functions; for the case of the constant-rate extension the solution was obtained by the factorizing method. Under the same initial parameter values and the same spreading stage duration, the subsidence dynamics appeared to be manifested more intensely at the end of the stage, in the case of accelerated lithosphere extension, and at the beginning, in the case of the constant rate extension. The deviations of results were diminished with decreasing duration of the spreading stage. Beyond this period, the solutions for these two variants approached the solution for the simplified version of instantaneous extension. The same may be concluded for the evolution of heat flow values. As follows from the model calculations, slower spreading rate led to substantial deviations of the subsidence dynamics and evolution of heat flow. However, inasmuch as the real spreading rates were probably no less than 0.1 cm/yr, the original simple McKenzie's (1978) version of instantaneous finite extension of the lithosphere may be used for rough but sufficiently correct estimates. The reason is that after the spreading stage, the long subsidence of trough floor and heat flow evolution proceeded independently of the extension mode during the initial period. The applicability of model versions is confirmed by the close agreement between the simulation results and the observed dynamics of tectonic subsidence inferred from the structure of sedimentary sequences in well-studied rift hasins

Key words: rifting model, mathematical simulation

1. Introduction

Modern dynamic models of rifting processes combine two main mechanisms: lithospheric extension and thermal contraction. The founding ideas for lithospheric extension go back to the work of *Vening Meinesz* (1950), *Artemjev* and *Artyushkov* (1971), *Bott* (1971), *Fuchs* (1974) and *Whiteman et al.* (1975), while the role of

thermal contraction in respect to the lithosphere state was emphasized, for example, by *Sleep* (1971), *Sleep* and *Snell* (1976), *Haxby et al.* (1976) and *Parsons* and *Sclater* (1977). A generalized geodynamical model for lithospheric extension involving rifting was developed by *D. McKenzie* (1978). This model involves lithospheric extension and thinning, with ascent of hot asthenosphere followed by restoration of isostatic balance, posterior cooling and thermal contraction of the upwelled asthenosphere. As a consequence of these processes associated graben-like basins and troughs are formed and filled with compacting sediments.

Suggested in the year 1978 by McKenzie a simple model with rift basin generation was based on "instantaneous" finite extention (stretching) of a double-layer lithosphere. Later, *Jarvis* and *McKenzie* (1980) modified the model by including the duration parameter of finite extension of the lithosphere. Here, it was assumed that lithospheric extension accelerated exponentially and then stopped instantaneously after attainment of finite stretching. A third version of this model involving finite extension as a matter of fact exhaust possible principal cases of one-act dynamics of rift spreading if the complications connected with their combinations are not considered.

2. The model

The relation between rate of extension - v_2 and that of decrease in lithospheric thickness - v_1 can be represented in the following way. Let us assume that *a* is the initial thickness of the lithospheric plate, *b* - its length, and *t* is time. Then,

$$v_1 = da / dt , \tag{1}$$

$$v_2 = db / dt . (2)$$

Balance of the matter is:

$$b \cdot v_1 = a \cdot v_2 \tag{3}$$

If v_1 is constant, then

$$db / dt = b \cdot v_1 / a$$

$$t = 0; \quad b = b_0$$
(4)

On integration:

$$b = b_0 \exp\left(\frac{v_1}{a_0}t\right) \tag{5}$$

Integrating eq. (2) when v_2 is constant and substituting for eq. (3), gives the conservation condition:

$$v_1 \cdot (b + v_2 \cdot t) = a \cdot v_2 \tag{6}$$

The equation of heat transfer from the asthenosphere into lithosphere extending at a constant rate is written in the form (*McKenzie* 1978, *Jarvis* and *McKenzie* 1980):

$$\partial T / \partial t = k \cdot \partial^2 T / \partial^2 z - v_1(t) \cdot (1 - z / a) \cdot \partial T / \partial z \quad , \tag{7}$$

with the following initial and boundary conditions:

$$t = 0; \quad T = T_1 \cdot (1 - z / a),$$
 (8)

$$t > 0, \quad z = 0: \quad T = T_1$$
, (9)

$$z = a: \quad T = 0 \quad , \tag{10}$$

where *z*- the vertical coordinate, T(z) - temperature of the lithosphere, T_1 - temperature of the asthenosphere (upper mantle), *k* - thermal diffusivity coefficient.

Let denote

$$x = (1 - z / a), \theta = T / T_1, \tau = k \cdot t / a^2, G' = v_2 \cdot a^2 / b \cdot k, \nu(\tau) = G' / (1 + G' \cdot \tau), (11)$$

$$\beta = 1 + G' \cdot k \cdot \Delta t / a^2,$$

here, G' - dimensionless rate of spreading, β - coefficient of spreading. Then the eqs. (7-10) can be rearranged as follows:

$$\partial \theta / \partial \tau = \partial^2 \theta / \partial x^2 + v(\tau) \cdot x \cdot \partial \theta / \partial x, \qquad (12)$$

$$\tau = 0: \theta(x) = x \quad , \tag{13}$$

$$\tau > 0, x = 0; \, \theta = 0 \ , \tag{14}$$

$$x = 1: \theta = 1 \tag{15}$$

Solution of the eq. (12) under the conditions (eqs. 13-15) was obtained by the finite difference method with factorizing of 3-diagonal matrix using the value of $v(\tau)$ from eq. (11).

In the case of lithospheric extension occuring at an exponential rate with acceleration, when $v(\tau)$ =constant, a solution was derived as a series on the eigen functions of the Sturm-Liouville problem which were found by numerical integration of the boundary value problem (*Jarvis* and *McKenzie* 1980). However, the solution may also be obtained through confluent hypergeometrical functions. In the manner of *McKenzie* (1978) and *Jarvis* and *McKenzie* (1980) we sought a solution of eqs. (11-15) as sum:

$$\theta(\tau, x) = \theta_c(x) + \overline{\theta}(\tau, x) \tag{16}$$

where

$$\theta_c = \frac{erf(\alpha x)}{erf(\alpha)}, \quad \alpha = \sqrt{\nu/2}$$
(17)

is stationary solution satisfying the non-uniform boundary conditions; $\overline{\theta}$ is the solution of the problem:

$$\partial \overline{\Theta} / \partial \tau = \partial^2 \overline{\Theta} / \partial x^2 + 2 \cdot \alpha^2 \cdot x \cdot \partial \overline{\Theta} / \partial x \quad , \tag{18}$$

$$\tau = 0:\overline{\Theta} = x - \Theta_c(x) , \qquad (19)$$

$$\tau > 0, x = 0; \overline{\theta} = 0 \quad , \tag{20}$$

$$x = 1:\overline{\Theta} = 0 \tag{21}$$

Representing $\overline{\theta}$ as product $\overline{\theta} = T(\tau) y(x)$, we separate variables in eq. (18) and obtain the solution for $T(\tau)$:

$$T(\tau) = \exp\left(-2\alpha^2 \nu_n \tau\right),\tag{22}$$

and the Hermite equation for the function y(x):

$$y'' + 2 \cdot \alpha^2 x \cdot y' + 2\alpha^2 v_k \cdot y = 0 , \qquad (23)$$

with the boundary conditions:

$$x = 0; y = 0 \tag{24}$$

$$x = 1: y = 0$$
 . (25)

The solutions of eq. (23) are the Hermite functions orthogonal on [0 - 1] with a weight of $exp(\alpha^2 x^2)$. The boundary condition (eq. 24) is satisfied by some linear combination of these functions which is expressed through the confluent hypergeometric function $\Phi(a,b,x)$:

$$y = x \cdot \Phi\left(\frac{1+v}{2}, \frac{3}{2}, -\alpha^2 x^2\right).$$
 (26)

With the help of the Kummer formula (*Abramowitz* and *Stegun*, 1964) eq. (26) can be transformed into:

$$y = x \cdot \exp\left(-\alpha^2 x^2\right) \cdot \Phi\left(1 - \frac{v_k}{2}, \frac{3}{2}, \alpha^2 x^2\right).$$
(27)

Applying boundary conditions (eq. 25) we find the equation for eigen values v_k :

$$\Phi\left(\frac{1+v_k}{2}, \frac{3}{2}, -\alpha^2\right) = 0$$
(28)

or

$$\Phi\left(1 - \frac{\mathbf{v}_k}{2}, \frac{3}{2}, \alpha^2\right) = 0 \quad . \tag{29}$$

The norm of eigen functions is determined by the quadrature:

$$d_k^2 = \int_0^1 x^2 \exp(-\alpha^2 x^2) \cdot \Phi\left(1 - \frac{\nu_k}{2}, \frac{3}{2}, \alpha^2 x^2\right) \cdot dx.$$
(30)

The expansion coefficients of the initial condition with respect to eigen functions:

$$a_k = \frac{1}{d_k} \int_0^1 \left[x - \frac{erf(\alpha x)}{erf(\alpha)} \right] \cdot \Phi\left(1 - \frac{v_k}{2}, \frac{3}{2}, \alpha^2 x^2\right) \cdot dx \,. \tag{31}$$

By gathering obtained expressions we come to the solution (eqs. 12-15) in the form:

$$\theta(\tau, x) = \frac{erf(\alpha x)}{erf(\alpha)} + x \cdot \exp\left(-\alpha^2 x^2\right) \sum_{k=1}^{\infty} a_k \exp\left(-2\alpha^2 \nu_k \tau\right) \cdot \Phi\left(1 - \frac{\nu_k}{2}, \frac{3}{2}, \alpha^2 x^2\right).$$
(32)

The heat flow on the earth's surface can be calculated by using of eq. (32):

$$J = \partial \theta / \partial x = \frac{2\alpha}{\sqrt{\pi \cdot erf(\alpha)}} + \sum_{k=1}^{\infty} a_k \exp(-2\alpha^2 v_k \tau) \quad .$$
(33)

Several first eigen functions and eigen values are given in Fig. 1.



Fig. 1. The first eigen functions in equation (26): a) G' = 10; the first eigen values: 2.128, 5.245, 10.20, 41.32; b) G' = 100; the first eigen values: 2.0, 4.0, 6.0, 8.0.

Table 1. Values of parameters used for the calculations.

$$a=125 \text{ km}$$

$$\rho_0=3.33 \text{ g/cm}^3$$

$$\rho_c=2.8 \text{ g/cm}^3$$

$$\rho_w=1.0 \text{ g/cm}^3$$

$$K=3.3 10^{-5} 1/{}^{0}\text{ C}$$

$$k=0.008 \text{ cm}^2/\text{s}$$

$$T_{I}=1333{}^{0}\text{C}$$

There are some difficulties when calculating the function $\Phi(a,b,x)$ in the case of large values of parameters and/or arguments. Let us take occasion the parameter *b* is small in this instance. Then the function Φ with small value of argument is calculated by the expansion into series:

$$\Phi(a,b,x) = \sum_{k=0}^{\infty} \frac{(a)_k x^k}{(b)_k k!}$$
(34)

or by using a asymptotic formula in the case of large argument value:

$$\frac{\Phi(a,b,x)}{\Gamma(b)} = \frac{(-x)^{-a}}{\Gamma(b-a)} \sum_{k=0}^{\infty} \frac{(-1)^k (a)_k (a-b+1)_k}{k! x^k} + \frac{e^x x^{a-b}}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(1-a)_k (b-a)_k}{x^k k!}$$
(35)

If the value of parameter *a* is large, the recurrent formula is applied:

$$(b-a) \cdot \Phi(a-1,b,x) + (2a-b+x) \cdot \Phi(a,b,x) - a \cdot \Phi(a+1,b,x) = 0$$
(36)

Since the parameter *a* is negatve in eq. (27) we need to add a sufficiently large number to the value *a* so that the sum $a + n = \overline{a}$ falls into inteval [0 - 1]. Calculating the values of function $\Phi(\overline{a}, b, x)$ and $\Phi(\overline{a} - 1, b, x)$ by eqs. (34 or 35) and using the recurrent eq. (36) (n-1) times we can find an unknown quantity of function $\Phi(a, b, x)$.

The lithosphere plate subsidence was calculated taking into account: 1) lithospheric extension and thinning during stretching stage and isostatic compensation during asthenosphere ascent, and 2) thermal contraction during cooling. The calculations were performed at each iteration using the factual present-time temperature profiles in the lithosphere. Appropriate formulae used for calculations was deduced by *McKenzie* (1978) and *Jarvis* and *McKenzie* (1980). The subsidence depth, $S(\tau)$, was computed at each iteration with allowance for the weitht of the lithosphere which included the crust and uplifted part of the asthenosphere. Approximately, it can be expressed by:

$$S(\tau) \approx \frac{P(\tau) - P(0)}{\rho_0 (1 - K \cdot T_1) - \rho_w}$$
(37)

where $P(\tau)$ is the lithosphere weight at the moment τ , P(0) - initial weight of the lithosphere, $\rho(0)$ - density of the asthenosphere, T_1 - temperature of the asthenosphere, $\rho(w)$ - density of water, K - coefficient of thermal expansion. In the limit of $\tau \rightarrow \infty$, the temperature profile becomes linear and the subsidence depth approaches the value (*Jarvis and McKenzie* 1980):

$$\frac{S_{\infty}}{a} = \frac{C \cdot (1 - \rho_c / \rho_0) \cdot (1 - 1 / \beta) \cdot (1 - C \cdot K \cdot T_1 \cdot (1 + 1 / \beta) / 2)}{1 - K \cdot T_1 - \rho_w / \rho_0}$$
(38)

where C - the part of lithosphere thickeness that was occupied by the crust at the initial moment, ρ_c - density of the crust. The dimensionless heat flow was calculated numerically with the the help of eq.(12).

3. Results for different rates of lithospheric extension

Some interesting results of the model calculations involving subsidence dynamics and heat flow evolution with different values of parameters β and G' are shown in Figs. 2-4. Calculations for "instantaneous" finite extension of the lithospheric plate (*McKenzie* 1978) are also performed for comparison. As it turned out for the same of initial parameters and period of spreading (extension) duration, the subsidence dynamics for the accelerating and constant-rate solutions are closely similar. But it appears to be manifested more intensely at the end of spreading period, in the case of the accelerated lithospheric extension (*Jarvis* and *McKenzie* 1980), and at the beginning, - in the case of the constant-rate extension. The divergence in subsidence dynamics rises with the increasing duration (Fig. 4a) and is diminished with decreasing one (Fig. 2a). The subsidence values are markedly lower for some (Figs. 2a and 3a) or many (Fig. 4a) million years of spreading period than in the case of a instantaneous extension. After starting period the subsidence dynamics are the same for all these solutions approaching the instantaneous case. The same relationships also apply to the thermal flux evolution associated with the extension and beyond this period (Figs. 2b, 3b, 4b).

At the extension rate of about 0.5 cm/yr, the rifts of 25 km or 67.5 km wide could be formed for 5 or 13.5 m.y., respectively. This is close to the upper and lower limits of known graben widths (for example, Bott 1976). The sides of rift troughs probably pulled apart at an average rate of 0.5 cm/yr in the Danish basin (Vejbaek 1989), and at a rate of 0.2 - 0.4 cm/yr in the Pannonian basin (Jarvis and McKenzie 1978). As appears from the model calculations, slower spreading entails considerable differences in subsidence dynamics and heat flow evolution (see Fig. 4). However, inasmuch as a real rate of spreading was no less than 0.1 cm/yr (for example, (Le Pichon 1968), the first version of McKenzie's (1978) model assuming "instantaneous" extension of the lithosphere plate may be successfully used for rough estimates. It is conditioned by the fact that after stretching stage the subsidence of rift basement and the heat flow evolution proceeded independently of the extension mode during the initial period. The effectiveness of "the instantaneous stretching model to be used to calculate the subsidenc history" had been noticed before by Jarvis and McKenzie (1980, p. 50). That circumstance was explained by sufficiently short extension period in the real rift basins. We corroborate this conclusion not only by means of aforecited theoretical computations but also the close agreement between the model calculation results and the observed dynamics of tectonic subsidence inferred from the structure of sedimentary sequences in well-studed rift basins. The data on sedimentary successions and their ages served as a source of independent information on the dynamics of crust subsidence (Royden and Keen 1980, Guidish et al. 1985, Ungerer et al. 1990). With the help of special sofware package (Friedinger 1988, Friedinger et al. 1991), the comparison of simulated and observed evolution of several rift basins was carried out. The software package performs the follows: the simulation of dynamics of basin bottom subsidence on the basis of McKenzie's model taking into account an available geophisical information and the numerical modelling of the lithosphere sinking and sediment accumulation on the ground of stratigraphic (including thicknesses and ages of sediment layers) and lithologic data for sedimentary sequences using the backstripping method (Royden and Keen 1980, Sclater and Christie 1980, Guidish et al. 1985, Ungerer et al. 1990). The comparison of these two computation results allows to estimate the divergence between them. It is performed by using the least square method



Fig. 2. Subsidence depth, "a", (S, km), and reduced value of surface heat flow, "b", (J/J_0) , for different regimes of finite extension of the lithosphere: 1 - instantaneous (*McKenzie* 1978), 2 - accelerated (*Javris & McKenzie* 1980) and 3 - constant-rate (this paper). It was taken: β =1.5, G[/]=50, a=125 km, t - in millions of years.



Fig. 3. The same, for $\beta=3$, G[/]=50.



Fig. 4. The same, for $\beta=3$, G'=2.

to determine the difference between observed and simulated positions of the basement in sediment stripped basin (*Reverdatto et al.* 1997). This comparison demonstrates an excellent coinsidence in the cases of such well-studied rift basins as the Central North Sea graben (*Friedinger* 1988, *Sclater* and *Christie* 1980), the Danish basin (*Vejbaek* 1989, *Reverdatto et al.* 1993), the Dnieper-Donets aulacogen (*Reverdatto et al.* 1993, *Stephenson et al.* 1993, *Chekunov et al.* 1992), etc.: the difference model/observation does not exceed one or some per cent. We are going to consider this question in the special paper.

4. Conclusion

To continue the investigation of McKenzie's model the evolution of rift basins and variations of heat flow were studied by us with respect to the following cases: 1) when finite extension of the lithosphere happened instantly, 2) when lithosphere was extended with acceleration, and 3) when spreading rate was constant. It was found with the same values of initial parameters and extension duration the subsidence dynamics concerning two latter versions differed insignifically. The subsidence dynamics during the period after stretching, i.e. in the course of cooling and thermic contraction, approximated very much to the calculation results obtained when finite instantaneous extending the lithosphere. A similar conclusion can be drawn relative to heat flow evolution. Hence, it follows that *McKenzie's* (1978) model involving the instantaneous finite expansion of the lithosphere is valid for rough but sufficiently correct estimates of rift subsidence and heat flow evolution. The good agreement between the model results and dynamics of tectonic subsudence inferred from the observed structure of sedimentary section with layers of known ages points to the evident plausibility and reality of McKenzie's rifting model.

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