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GAUSSIAN CURVATURE OF POSTGLACIAL REBOUND AND THE DISCOVERY OF CAVES CREATED BY MAJOR EARTHQUAKES IN FENNOSCANDIA

by

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Abstract

The Gaussian curvature of the postglacial rebound of Fennoscandia at the end of the deglaciation is computed. The result is used for investigating the origin of boulder caves, where the bed-rock has been split into caves and accumulations of huge boulders. It is concluded that the three large boulder caves, concentrated at the Swedish coast of the southern part of the Gulf of Bothnia, most probably are created by major earthquakes produced by the Gaussian postglacial curvature about ten thousand years ago. The Gaussian curvature theory explains both the location and the character of the boulder accumulations and caves. Furthermore, the curvature seems to be great enough to allow major earthquakes to occur, the curvature of one century of rebound being 10^9 times the curvature of the earth tide. On the other hand the theory as applied here fails to explain the distribution of the many small boulder caves. Finally, an earlier conclusion of the author on the origin of today's minor earthquakes in central Fennoscandia is here considered erroneous.

1. *Introduction*

In recent years attention has been drawn to certain types of caves in Sweden. Their common characteristic is that they are formed within accumulations of huge boulders. SJÖBERG (1987) has shown these caves to be of postglacial origin. His main arguments for this are that the boulders are glacially striated, proving that they accumulated after the advance of the inland ice, and that scarcely any weathering products are found on the floors of the caves. Sjöberg suggests that the boulder caves were created by major earthquakes occurring at the end of the deglaciation, when the isostatic rebound was very rapid; *cf.* MÖRNER (1985).

The caves are more or less labyrinthine. Most of them have a length of about 100 m or less, but three caves are considerably larger having a length of the order of 1 km. These three large caves are situated within a common area. Since there seems to be no other reason for their size they may be suspected to represent the strongest postglacial earthquakes.

A mathematical method using the concept of curvature was developed by EKMAN (1985) for trying to study postglacial uplift as a possible origin of earthquakes. This curvature theory will now be applied to the boulder caves. We first give an outline of the theory, then apply it to the land uplift at the end of the deglaciation as revealed by ancient shore lines, and finally compare the result with the location and character of the caves and with the curvature of the earth tide.

2. *Postglacial curvature theory*

Postglacial land uplift, like any function on a sphere, may be expanded in a series of surface spherical harmonics,

$$U = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (\bar{a}_{nm} \cos m\lambda + \bar{b}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\varphi) \tag{1}$$

(see *e.g.* HEISKANEN & MORITZ, 1967). The coefficients are given by

$$\begin{aligned} \bar{a}_{nm} &= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} U(\varphi,\lambda)\, \bar{P}_{nm}(\sin\varphi) \cos\varphi \cos m\lambda \, \mathrm{d}\varphi \mathrm{d}\lambda \\ \bar{b}_{nm} &= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} U(\varphi,\lambda)\, \bar{P}_{nm}(\sin\varphi) \cos\varphi \sin m\lambda \, \mathrm{d}\varphi \mathrm{d}\lambda \end{aligned} \tag{2}$$

Here φ denotes latitude, λ longitude, and $\bar{P}_{nm}$ the normalized Legendre functions.

The uplift U to be used here is the absolute uplift, which is the apparent uplift – according to the shore line observations – corrected for the change of the geoid and for eustatic changes of the water level. The uplift may be thought of either as a vertical velocity or as a height change between different epochs; it will be used in both senses in the following.

Now, postglacial uplift causes the Earth's crust not only to rise (and to tilt) but also to curve. The postglacial curvature, especially the Gaussian curvature, serves as a kind of measure of the amount of postglacial stress and strain produced in the crust. This may be realized through the geometric interpretation of the Gaussian curvature, or through the »Theorema egregium» of Gauss, stating that a surface which is bent without being strained does not change its Gaussian curvature (*cf. e.g.* LIPSCHUTZ, 1969).

A formula for the Gaussian postglacial curvature has been derived, starting form (1), by EKMAN (1985):

$$
\begin{aligned}
K = {} & \frac{1}{R^4} \sum_{n=0}^{N} \sum_{m=0}^{n} (\bar{a}_{nm} \cos m\lambda + \bar{b}_{nm} \sin m\lambda) \\
& \left[\left(-n(n+1) + (n+1) \tan^2\varphi + \frac{m^2}{\cos^2\varphi} \right) \bar{P}_{nm}(\sin\varphi) \right. \\
& \left. - \frac{\sqrt{(2n+1)(n+m+1)(n-m+1)} \sin\varphi}{\sqrt{2n+3} \cos^2\varphi} \bar{P}_{n+1,m}(\sin\varphi) \right] \\
& \sum_{n=0}^{N} \sum_{m=0}^{n} (\bar{a}_{nm} \cos m\lambda + \bar{b}_{nm} \sin m\lambda) \\
& \left[\left(-(n+1) \tan^2\varphi - \frac{m^2}{\cos^2\varphi} \right) \bar{P}_{nm}(\sin\varphi) \right. \\
& \left. + \frac{\sqrt{(2n+1)(n+m+1)(n-m+1)} \sin\varphi}{\sqrt{2n+3} \cos^2\varphi} \bar{P}_{n+1,m}(\sin\varphi) \right] \\
& - \frac{1}{R^4} \left(\sum_{n=0}^{N} \sum_{m=0}^{n} (\bar{a}_{nm} \sin m\lambda - \bar{b}_{nm} \cos m\lambda) \right. \\
& \left[- \frac{(n+2) m \sin\varphi}{\cos^2\varphi} \bar{P}_{nm}(\sin\varphi) \right. \\
& \left. \left. + \frac{\sqrt{(2n+1)(n+m+1)(n-m+1)}\, m}{\sqrt{2n+3} \cos^2\varphi} \bar{P}_{n+1,m}(\sin\varphi) \right] \right)^2
\end{aligned}
\tag{3}
$$

Here R is the radius of the Earth; the other symbols have already been explained. For obvious reasons the series expansion is cut off at some large number $n = N$, the coefficients (2) being then calculated by summation over a lot of surface elements (squares) of a corresponding size.

3. *Gaussian curvature at the end of the deglaciation*

Let us now apply the formula (3) to the postglacial rebound of Fennoscandia some eight thousand years ago, when the ice had recently melted away.

The first step is to determine the apparent uplift at that time. This can be accomplished for the Baltic Sea area by using two maps of ERONEN (1983), showing the heights above present sea level of the shore lines at about 7000 B.C. and 5500 B.C. The difference between them represents the apparent uplift between the mentioned years, turning out to be 170 m at its maximum. For the Atlantic coast area we use the shore displacement curves compiled in HAFSTEN (1983), together with a few additional curves (MÖRNER, 1980; DONNER, 1980).

The apparent uplift should then be corrected for changes in the amount of water. In the Baltic Sea area the eustatic change of the water level is composed of two parts: the lowering of the Ancylus Lake between approximately 7000 B.C. and 6500 B.C., c. 15 m (BJÖRCK, 1987), and the rise of the sea approximately from 6500 B.C. to 5500 B.C., c. 10 m (MÖRNER, 1980). The two parts nearly cancel out. The influence of the rise of the geoid, being slightly reduced by the existence of the Ancylus Lake, does nowhere exceed 10 m (*cf.* EKMAN, 1988). These corrections may be considered insignificant for our further computations. In the Atlantic coast area, on the other hand, we should correct for the eustatic rise of sea level during the whole period; it amounts to c. 25 m (MÖRNER, 1980).

Applying the correction above we obtain the absolute uplift between 7000 B.C. and 5500 B.C. From this we easily find the absolute uplift rate at about 6000 B.C., the map of which is shown in Figure 1. Its maximum value is more than 80 mm/year, nearly 10 times the value of today (*cf.* EKMAN, 1988).

From Figure 1 we next determine mean values of the uplift rate within squares of $1^\circ \times 1^\circ$ (1° latitude x 2° longitude). Using these values the coefficients $\bar{a}_{nm}$ and $\bar{b}_{nm}$ can now be calculated according to (2).

Finally we compute the Gaussian postglacial curvature K applying the formula (3). Thereby the maximum possible value of N is 180, corresponding to the minimum wave-length 1°. To examine the stability of the solutions for various values of N some computations of K for $180 \geqslant N \geqslant 120$ were made, leading to quite similar results. The result for $N = 150$ is shown as a Gaussian curvature map in Figure 2.

From the map we see that the Gaussian curvature of one century of postglacial uplift is great $-K > 5 \cdot 10^{-27}\ \mathrm{mm}^{-2}$ – within a rather small area located at the Swedish coast of the southern part of the Gulf of Bothnia. The maximum is $K = 20 \cdot 10^{-27}\ \mathrm{mm}^{-2}$.

Now, Figure 2 shows the situation at about 6000 B.C. when the ice did no-

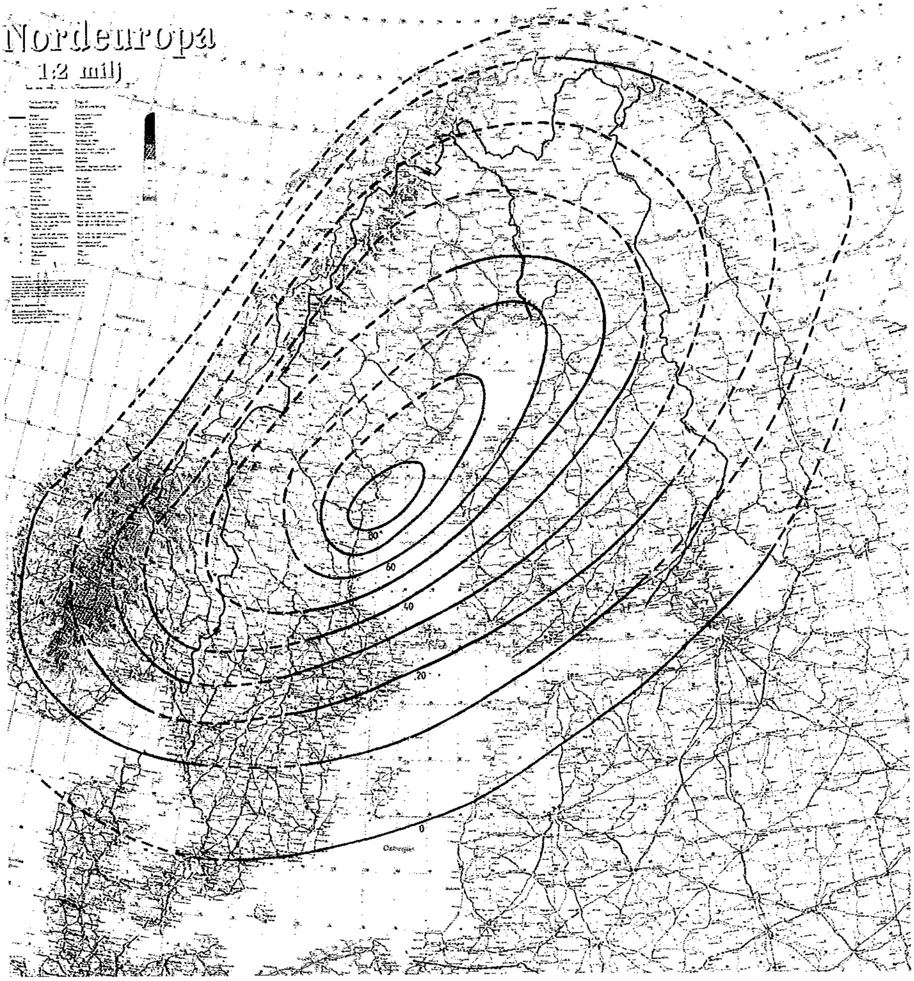

Figure 1. Absolute uplift of Fennoscandia in mm/year at about 6000 B.C.

longer exist. However, the peak rate of land uplift here was probably reached about 2000 years earlier, before the ice had disappeared completely (MÖRNER, 1980). The Gaussian curvature at that time can only be estimated through an attempt to extrapolate back to 8000 B.C.: First, the uplift rate then was at least three times larger (MÖRNER, 1980), making the Gaussian curvature one order of magnitude larger. Second, although the general pattern of the land uplift has kept remarkably unchanged up till today the maximum point has slowly

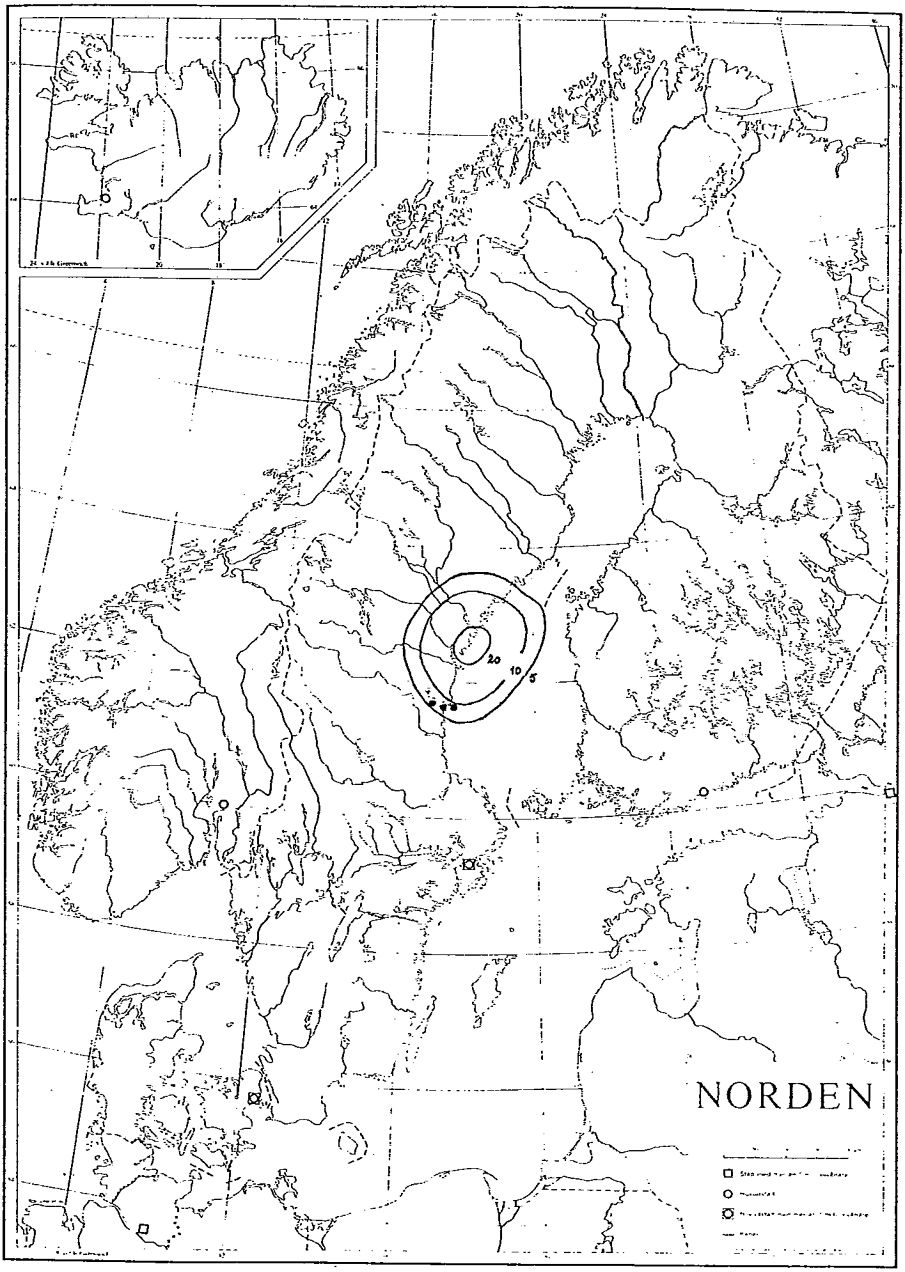

Figure 2. Gaussian curvature K of the postglacial uplift of Fennoscandia at about 6000 B.C. ($N = 150$). Unit of K for one century of uplift: 10^{-27} mm^{-2}. Dots denote large boulder caves.

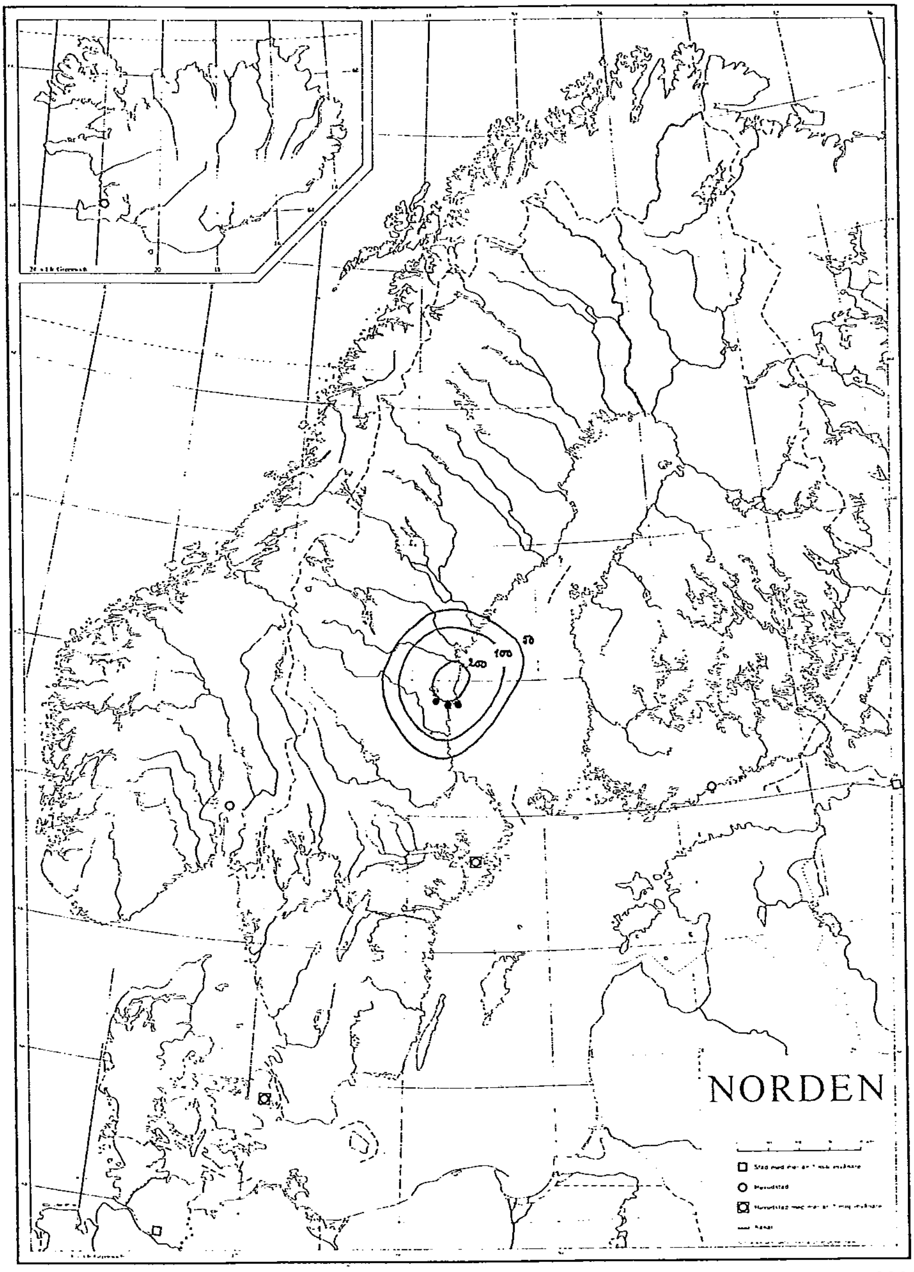

Figure 3. Estimated Gaussian curvature K of the postglacial uplift of Fennoscandia at about 8000 B.C. (N=150). Unit of K for one century of uplift: $10^{-27}\mathrm{mm}^{-2}$. Dots denote large boulder caves.

migrated to the north-northeast by nearly half a degree per millenium. This can be seen by studying *e.g.* ERONEN's (1983) map of the uplift since the Ancylus Lake, his map of the uplift since the early Litorina Sea, and a geodetic map of the present uplift. Assuming that this process had started not later than 8000 B.C. the area with large K values was at that time located nearly one degree further to the south-southwest, as illustrated in Figure 3.

4. *Gaussian curvature, earthquakes and boulder caves*

The character of the boulder caves is in good accordance with what one should expect from a positive Gaussian postglacial curvature (as we have in Figures 2 and 3). Positive values of K, *i.e.* elliptic curvature, should here correspond to tension in all directions. According to SJÖBERG (1987) the accumulation of boulders forming the caves clearly show that the bed-rock was split by tensional forces in all directions.

If postglacial rebound is the origin of the earthquakes that seem to have created the boulder caves, then the largest boulder accumulations should be found where the Gaussian postglacial curvature is great. As was mentioned in the Introduction there are three large boulder caves, all of them situated within a common area. They are marked by dots in Figures 2 and 3. In Figure 2, representing 6000 B.C., we find the large boulder caves close to the Gaussian curvature maximum. In Figure 3, representing the more realistic 8000 B.C., we find the large boulder caves almost coinciding with the Gaussian curvature maximum. We must, however, bear in mind that the extrapolation makes Figure 3 uncertain. In any case, the large caves are situated very close to where predicted by the Gaussian postglacial curvature.

One would now also expect to find some correlation between the Gaussian curvature and the small boulder caves, but this is not the case. Most of the twenty small caves are scattered in the whole eastern part of Sweden. Our mathematical method fails to explain this.

Finally we note that the maximum Gaussian curvature of one century of postglacial rebound amounts to $K = 10^{-25}\ \text{mm}^{-2}$ or more. This may be compared with the Gaussian curvature of the earth tide which is $K = 10^{-34}\ \text{mm}^{-2}$ (EKMAN, 1985). The secular postglacial curvature is thus of the order of 10^9 times the tidal curvature, *i.e.* the secular postglacial strain may be roughly estimated to $10^4 - 10^5$ times the tidal strain. This should allow major postglacial earthquakes to occur (*cf.* HEATON, 1975 and JOHNSTON, 1987).

Before summarizing the conclusions a few words ought to be said about the long postglacial fault in northernmost Sweden, the Pärvie fault (LUNDQVIST & LAGERBÄCK, 1976). This fault might have been formed by a zone of great mean curvature of postglacial rebound. However, whether this is so cannot be determined since no shore lines exist to give information on the postglacial curvature there. A more plausible explanation is recently given in terms of glacially stored plate tectonic stresses by TALBOT & SLUNGA (1989).

Conclusions: The three large boulder caves, concentrated at the Swedish coast of the southern part of the Gulf of Bothnia, most probably are created by major earthquakes produced by the Gaussian curvature of the postglacial rebound of Fennoscandia. The Gaussian curvature theory explains both the location and the character of the boulder accumulations and caves. Furthermore, the curvature seems to be great enough to allow major earthquakes to occur. On the other hand the theory as applied here fails to explain the distribution of the small boulder caves. In the light of this the conclusion on the origin of today's minor earthquakes in central Fennoscandia drawn by the author a few years ago (EKMAN, 1985) must be considered erroneous.

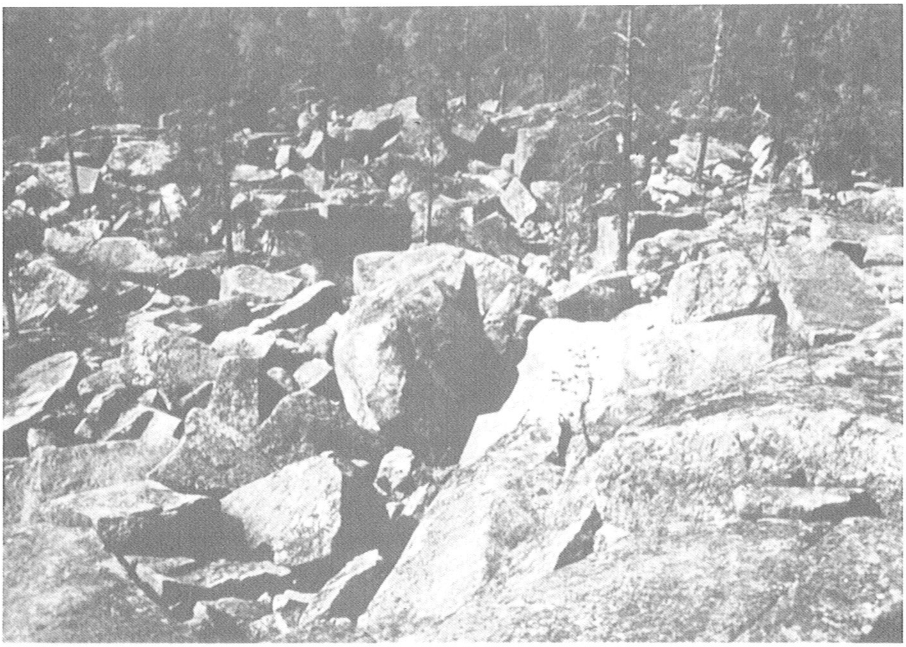

Figure 4. Split bed-rock above the largest boulder cave (Boda).

A picture of the split bed-rock above the largest cave system, the Boda caves, is shown in Figure 4. According to our curvature calculations the picture could be mathematically described as Nature's own illustration of the Theorema egregium of Gauss!

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model. In an examination of several cases, Doswell found good agreement in the patterns of QG forcing and model vertical motions in some regions, but poor agreement in others. Terrain variations and moist convective processes were implicated in producing many of the disagreements. Where there were disagreements, the primitive equation model's vertical velocities were better correlated with precipitation than with QG forcing.

3. *Concluding remarks*

Judging from the feedback received, the symposium was highly successful. During the symposium great progress was reported in our understanding of cyclone behaviour, and in our ability to predict cyclone development. It is sobering to note, however, that during two days of the symposium it rained even though no rain was forecast. I remember Palmén once saying that if it were possible to accurately forecast tomorrow's weather, he would not be interested in meteorology any more. With those two unforecast rainy days during the symposium one can say that we still have a lot to do, and that Erik Palmén, if alive, would still be interested in the weather!

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