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APPLICATION OF TRANSFER FUNCTION AND CONCEPTUAL PULSE MODELS TO THE STUDY OF GROUNDWATER LEVEL FLUCTUATIONS

by

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Abstract

This paper discusses the applications of transfer function (TF) and PULSE models to simulate the fluctuations in groundwater level in a glaciofluvial delta. The TF model can use both the measured and calculated groundwater levels during the previous time interval as an input value whereas the PULSE model requires only the initial groundwater level value and some meteorological data.

The semi-monthly TF model gave a satisfactory simulation of the groundwater fluctuations as a function of lysimeter outflow and groundwater level during the previous time interval. The correlation coefficient (r = 0.979) was 0.025 units higher than for the monthly TF model. During the snowmelt period, the semi-monthly model could not follow the rapid increase in groundwater levels because the time step, two weeks, was still too long.

On a daily basis, the PULSE model could be fitted well to the groundwater fluctuations in the one year used for calibration. The results of the PULSE model were generally comparable to those of the TF model. During the snowmelt period the PULSE model was even more effective than the TF model used to simulate the groundwater fluctuations.

Key words: variation of groundwater level, sandy aquifer, transfer function model, conceptual model

1. *Introduction*

A time series of groundwater levels reflects all natural hydrological processes as well as any human influence on the aquifer. The geological environment plays a prominent role in groundwater level fluctuations, determining flow velocities within and outflow from the aquifer.

The primary source of recharge for an aquifer is precipitation. Under Finland's climatic conditions, most of the annual recharge occurs in spring after the snowmelt period. This implies a clear annual periodicity in the time series of groundwater levels.

Analysis of a groundwater level time series provides information about future states of the aquifer. This information has become increasingly important with the growing utilization, changes in quality and artificial recharge of groundwater. As a result, a considerable number of applications of time series methods have been made in groundwater studies, *e.g.* ERIKSSON (1970), KRIZ (1972), GOTTSCHALK, LINDBERG and NORDBERG (1974), JACKSON (1974). These applications were recently widely reviewed by GANOULIS and MOREL-SEYTOUX (1985).

In the present study, the discrete time series of groundwater levels in a sandy aquifer at Hyrylä (60° 23′ N, 25° 02′ E) were analysed. This glaciofluvial delta formation covers about 3 km^2, and has a mean height of 60 m a.s.l. The slope of the groundwater table is 1:2 000. The clay areas surrounding the delta formation are at 42 – 47 m a.s.l.

The experimental hydrological station is in the middle of the groundwater divide. There are 14 groundwater measuring wells in the delta formation, but analysis of only the weekly time series gathered from one well at the experimental station was sufficient to represent the whole area (LEMMELÄ & KUUSISTO 1986).

The mean groundwater depth in this well in the period 1968–84 was 719 cm. The extremes were 644 cm (Jan. 1975) and 781 cm (Mar. 1977). The average annual amplitude of groundwater levels was 58 cm. In two years, the amplitude exceeded 100 cm, but in three years it was less than 30 cm.

The average annual amplitude of the soil moisture storage in the uppermost 300 cm layer was 185 mm (LEMMELÄ & TATTARI 1986). In September the storage averaged 520 mm, in November 590 mm and in April 610 mm. At greater depths, the variation in soil moisture was much smaller.

A cubical lysimeter with a volume of 1.0 m^3 was installed at the experimental field. The cumulative percolation from the lysimeter during 1969–1984 averaged 220 mm in December–May, 40 mm in June–August and 123 mm in September–November. Annually, percolation amounted up to 54 per cent of the corrected precipitation.

On the basis of the lysimeter and soil moisture measurements, the average annual groundwater recharge in the period 1972–85 was 405 mm. The aquifer response method gave a slightly lower value, 396 mm (LEMMELÄ 1989). However, the agreement between the results of these two methods can be considered satisfactory.

2. *Transfer function model*

2.1. Theory of the model

Owing to snow accumulation, snowmelt and evapotranspiration, the effect of monthly precipitation values on groundwater recharge differs sharply from month to month. In contrast, the relationship between the outflow from the lysimeter at a depth of 100 cm and the fluctuations in the groundwater table are more stable throughout the year. This relationship was studied with a transfer function model.

The theory behind the TF models is well established in the literature (*e.g.* BOX and JENKINS 1976). The stages of the iterative approach to the TF model used are presented in Fig. 1a. According to the TF model chart, the main assumption is that there are two time series data, $x(t)$ and $y(t)$, which are stationary, *i.e.* the means, variances and autocorrelation are constant throughout the observation period. Autocorrelation ($r(k)$) (Eq. 1a) and cross-correlation ($rxy(k)$) (Eq. 1b) are then used to determine the TF model parameters.

$$c(k) = 1/N \sum_{t=1}^{N-k} (z(t) - zm)\,(z(t+k) - zm) \qquad (1a)$$

$$r(k) = c(k)/c(0) \qquad k = 1,...,L$$

where z = input vector of length N containing the time series
N = input length of z
L = input number of autocovariances and autocorrelations to be computed
xm, zm = means
$c(0)$ = variance
$c(k)$ = autocovariance

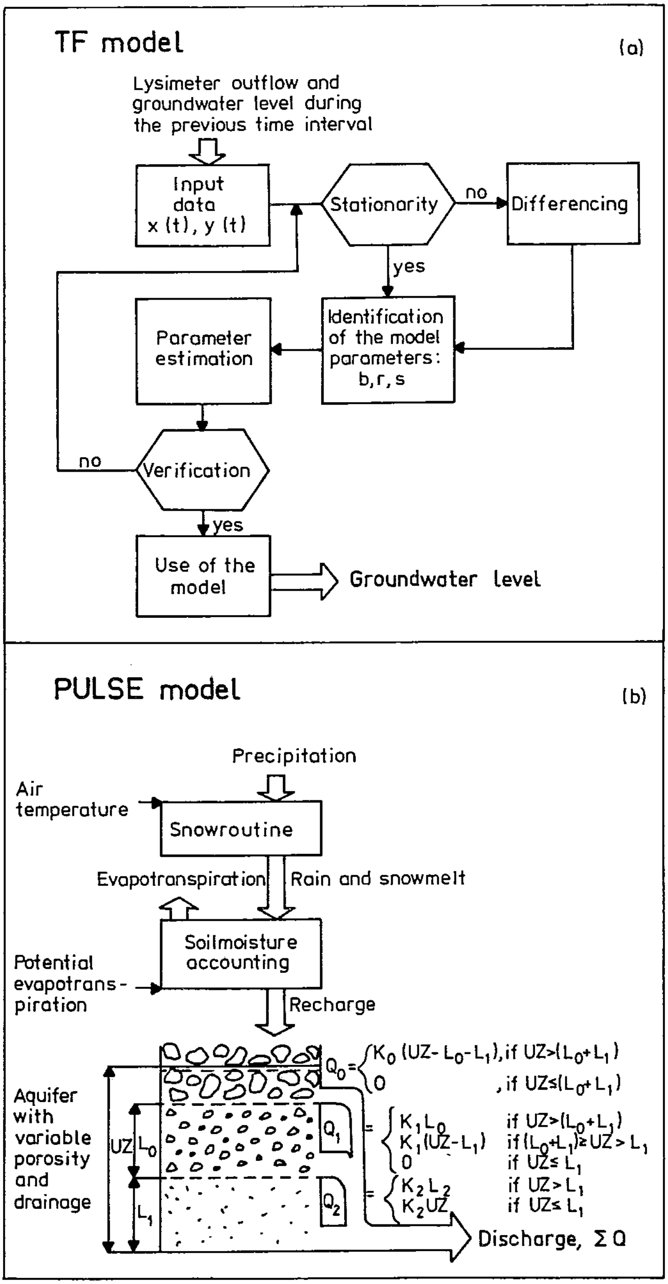


Fig. 1. Stages in the iterative approach to the transfer function model (a) and a schematic presentation of the structure of the conceptual PULSE model (b).

$$cxy(k) = \begin{cases} (1/N) \sum_{t=1}^{N-k} (x(t)-xm)\,(y(t+k)-ym), & k = 0,\dots,L \\ (1/N) \sum_{t=1}^{N+k} (y(t)-ym)\,(x(t-k)-xm), & k = -1,\dots,-L \end{cases} \tag{1b}$$

$$rxy(k) = cxy(k)/sx/sy$$

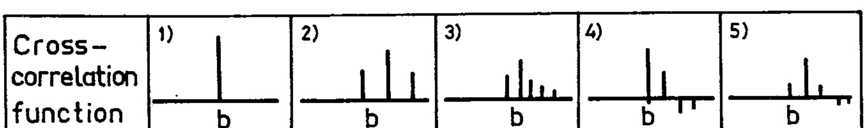
where sx, sy = standard deviations
cxy = cross covariance

The general equation of the transfer function model can be written as:

$$y(t) = f(1)y(t-1) + f(2)y(t-2) + \dots + f(r)y(t-r) + g(0)x(t-b) + g(1)x(t-b-1) + \dots + g(s)x(t-b-s) + v \tag{2}$$

where $y(t)$ = discrete observation series of output variable at time t
$x(t)$ = discrete observation series of input variable at time t
$f_1,\dots,f_r$ = parameters
$g_0,\dots,g_r$ = parameters
v = constant
b = delay parameter
r and s = orders of TF model

The identification process of model parameters involves the determination of tentative values of f and g as well as values for b, r and s. The determination of b is straightforward, b being equal to the time lag of the first significant cross-correlation between $x(t)$ and $y(t)$. In our case, b is the time from lysimeter out-



	1)	2)	3)	4)	5)
r	0	0	1	2	2
s	0	2	1	0	2

Fig. 2. Examples of the identification of TF model parameters with the cross-correlation function. Parameter b has a constant value in all cases.

flow to the occurrence of the groundwater response. The determination of parameters r and s is not so concise. These parameters, r and s, can be identified through association of the cross-correlation pattern with the pattern of an autocorrelation function (VANDAELE 1983). In Fig. 2, examples of the values of r and s are given with the corresponding cross-correlation function, column 3 representing the situation in this context. Once the parameters have been identified, the model can be calibrated with standard library software programs (IMSL 1987).

2.2. The semi-monthly TF model

Because of the lag between the observed and calculated groundwater levels of the monthly TF model during the snowmelt period (LEMMELÄ and KUUSISTO 1986), a semi-monthly model was also calculated. The best model was found to be:

$$W_i = 0.965\ W_{i-1} - 0.101\ Q_{i-1} + 27 \qquad (3)$$

where W_i = groundwater depth at time i, cm
i = time step, two weeks
Q_i = lysimeter outflow at time i, mm

The correlation coefficient was 0.979, which is 0.025 units higher than the correlation coefficient for the monthly TF model. The average of the maximum annual deviations between observed and modelled groundwater levels was reduced from 19 cm to 11 cm.

However, in some years with relatively large and rapid variations in groundwater level even this model (Eq. 3) performed less satisfactorily. For example, in 1981 the correlation coefficient remained at 0.84 (Fig. 3). The reason for this may be the still excessive length of the time step.

The TF model (Eq. 3) was also tested using the modelled semi-monthly averages of groundwater depth rather than the measured ones (Fig. 4). The correlation coefficient was 0.851, that is, 0.080 units smaller than when the measured groundwater levels (W_{i-1}) were used. In this case the model also markedly smoothed the groundwater level fluctuations compared with the measured ones.

One application of the TF model is for evaluating or calculating the effect of precipitation on the groundwater levels during the different seasons of the year. According to previous studies in this area (LEMMELÄ and TATTARI 1986), 22 per cent of the rain infiltrated into the soil in summer. The corresponding figures for the autumn were 61 per cent and for winter-spring 94 per cent. The TF model indicates that a 100 mm rain event during the summer period would have little

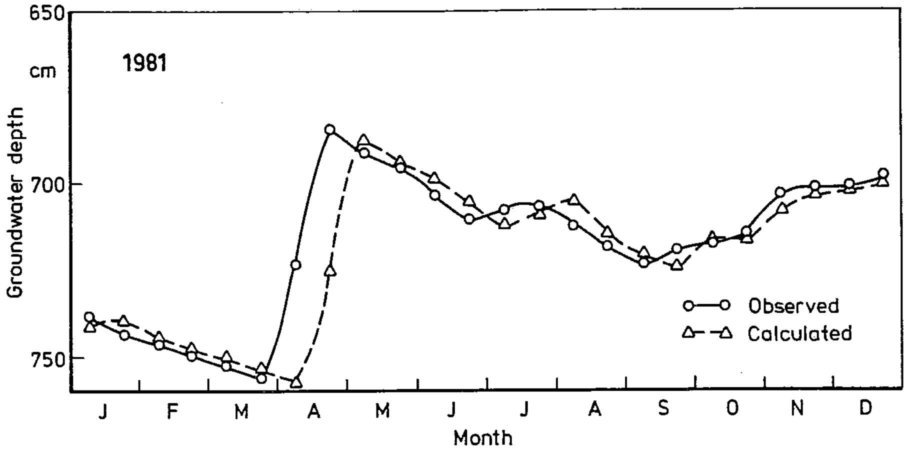


Fig. 3. Observed and calculated (with Eq. 3) groundwater depths at Hyrylä in 1981. This year is an example of extreme variations in groundwater level.

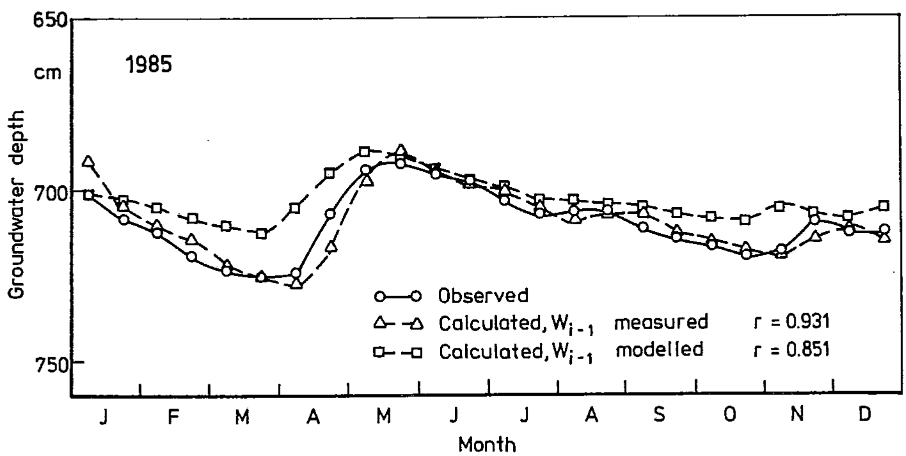


Fig. 4. Observed and calculated (TF model, Eq. 3) groundwater depths at Hyrylä in 1985. The TF model uses the calculated and measured semi-monthly groundwater levels as an input value.

impact on groundwater level (3 mm). In autumn the groundwater level would rise by about 40 mm and in spring by 70 mm. The above values represent an average rise in groundwater level, because the model was evaluated for the whole year, not separately for different seasons.

The influence of the length of the time step used in the TF model is obviously

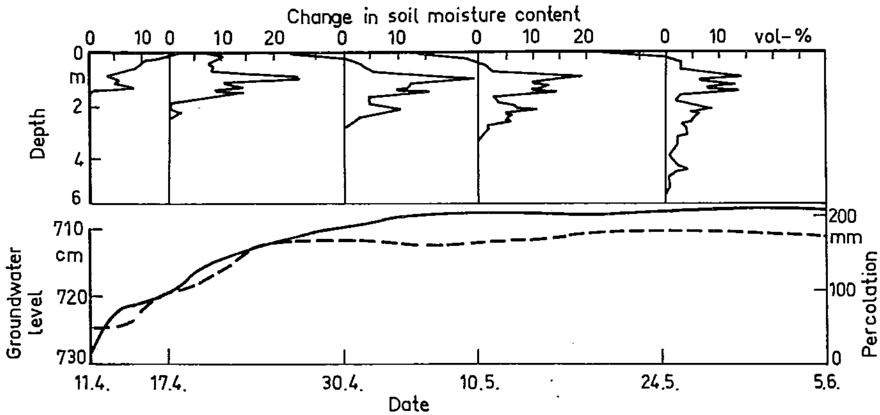


Fig. 5. Change in soil moisture content, increase in groundwater level (——) and percolation from the lysimeter (- - -) in spring 1969.

dependent on the lag in percolation from lysimeter outflow level to groundwater table. Fig. 5 presents an example of a change in soil moisture storage, amount of percolation and rise of groundwater level. The wetting pulse reaches groundwater level very slowly, the average wetting front velocity (year 1969) being 8 cm/day (LEMMELÄ & TATTARI 1986). Despite this, the groundwater level begins to rise almost immediately the snow starts to melt. Thus the two-week time step is too long in modelling the exact date for the start of snowmelt.

The significance of the length of the calibration period on the correlation coefficient between observed and calculated groundwater levels was also studied. The results are shown in Table 1. The years used in the analyses were chosen randomly.

Table 1. The dependence of the length of the calibration period on the correlation coefficient of the two years verification period.

Time, years	*r*
1	0.866
3	0.833
5	0.841
10	0.835
15	0.868

The results indicate that in this case the TF model was not so sensitive to the length of the calibration period and thus permits the model to be calibrated even with short observation series for operative groundwater inventories.

3. *PULSE model*

3.1. Theory of the model

The Swedish PULSE model (Bergström & Sandberg 1983 and Carlsson *et al.* 1987) was also applied to simulate the variations in groundwater level (Fig. 1b). The name of the model derives from the rain and meltwater pulses percolating through the soil layers. The input variables of the model were daily precipitation and mean air temperature together with monthly potential evapotranspiration calculated with Penman's equation (Penman 1948). The output consisted of daily groundwater storages, which were converted to groundwater levels by the measured effective porosity value. This value was measured by taking samples from the research area and by making in situ measurements with a neutron method.

In the model the snow routine is a degree-day approach with a melt factor which gives the amount of snow melt in mm $^{\circ}C^{-1}$ d^{-1} and with a liquid water holding capacity of dry snow delaying meltwaters. A correction factor, a different one for solid and liquid precipitation, is also used to account for aerodynamic losses of gauge precipitation and winter evaporation from snow cover.

The soil moisture accounting procedure is based on three parameters: the first controls the increase in soil moisture storage from precipitation and snowmelt; second is the threshold value of evapotranspiration reaching its potential value; and the third is the maximum soil moisture storage value.

The principle of the groundwater drainage subroutine is based on the assumption that the rise in groundwater level will increase groundwater outflow. The outflow occurs in the saturated layer, which is divided into three sublayers. Each layer has a recession coefficient (K) which gives the proportion of respective storage which will empty in 24 hours. In addition, the model gives the estimates for snow storage, soil moisture deficit and actual evapotranspiration.

3.2. Application of the PULSE model

The PULSE model was calibrated for 1969, which is taken to represent typical groundwater fluctuation behaviour in the area. The value of the goodness-of-fit criterion (Nash and Sutcliffe 1970) was 0.897. The model was verified for 1984 and the criterion had a value of 0.889. In the verification period the fit

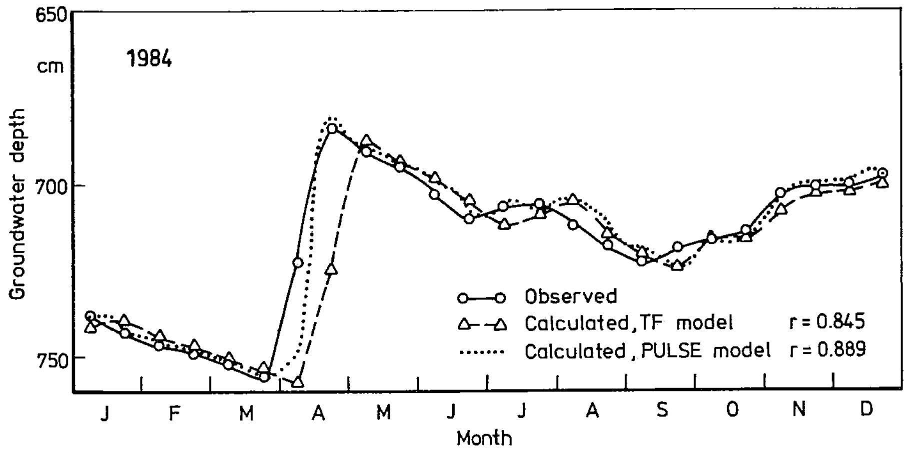


Fig. 6. Observed and calculated groundwater depths at Hyrylä in 1984 with the PULSE model with the TF model. The time step of the TF model is two weeks whereas the PULSE model uses daily input values.

criterion for the first six months of the year was better (0.942) than for the calibration period *i.e.* in 1969.

Compared with the verification of the TF model (Eq. 3) for 1984 (r = 0.845), the PULSE model simulated the snowmelt period more accurately even though the start of the groundwater level rise was delayed (Fig. 6). Outside the melting period the PULSE model is comparable to the TF model.

The daily time step of the PULSE model would obviously be too short for predicting groundwater level variations in practice, but because of the model structure, longer time steps cannot be used. A semi-monthly or monthly TF model is more relevant in an area of this kind. According to Figs. 4 and 6 it is evident that groundwater level predictions could be further improved, if these two models were combined. The method could be analogous to the method used in real time flood forecasting (Malve 1986).

In the combination, the PULSE model would be used to calculate the outflow from the soil moisture storage to the groundwater storage. Then the TF model could be used to calculate the effect of this inflow to groundwater level.

4. *Conclusions*

In this study a semi-monthly transfer function model and a conceptual PULSE model were used to estimate variations in groundwater level. The following conclusions can be drawn:

1. The effect of lysimeter outflow values on the groundwater level could be satisfactorily simulated with a semi-monthly transfer function model. The correlation coefficient of the semi-monthly TF model was 0.025 units higher than that for the monthly model. The average of maximum annual deviation between observed and modelled groundwater levels was reduced from 19 cm to 11 cm. During the snowmelt period neither the monthly nor the semi-monthly model could follow the rapid increase in groundwater levels.
2. The use of measured instead of modelled groundwater levels in the semi-monthly TF model gives a correlation coefficient higher by 0.080 units. The use of modelled groundwater values smoothed the groundwater fluctuations to a considerable extent.
3. According to the TF model, a 100 mm precipitation event would increase the groundwater level by 40 mm in autumn and by 70 mm in spring. The increase in summer (3 mm) only stops the lowering of the groundwater level.
4. The PULSE model, calibrated with data collected daily during one year simulated well the groundwater fluctuations determined in another year with calibrated parameter values. The use of the PULSE model is recommended for real time forecasting during the snowmelt period, but the TF model is more practical in water economy studies, *e.g.* in estimating aquifer recharge or when forecasting the changes in groundwater storage. If the two models were combined, the groundwater level predictions could be further improved.

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