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DETERMINATION OF THE ABSOLUTE SEA LEVEL DIFFERENCE BETWEEN VARBERG AND FREDRIKSHAVN

by

LARS ROMELL and ANDERS STIGEBRANDT

Department of Oceanography
University of Gothenburg
Box 4038, S–40040 Gothenburg, Sweden

Abstract

The mean sea level difference between Varberg and Fredrikshavn is determined in two independent ways. The first way is to use 'geodetic' information and the result is 97 ± 30 mm. The second way is to use a hydrodynamic model for the Kattegat to compute the difference. The Kattegat – Belt Sea model by STIGEBRANDT (1983) was run for a 18 month long period. The computed monthly mean sea level difference is compared to the measured monthly mean sea level difference. The best fit is obtained for a mean sea level difference amounting to 100 mm. Thus the 'geodetic' and the hydrodynamic methods agree quite well and the uncertainty bounds may be reduced to ±15 mm.

1. *Introduction*

In this paper we will concentrate upon the east–west slope of the sea surface in the northern Kattegat. Registrations, covering an eighteen months long period, from the sea level gauges in Varberg and Fredrikshavn (see map in Fig. 1) have been used. To be able to calculate the absolute sea level difference between the two locations the sea level gauges have to be related to one common height system (parallel to the geoid). The first national height systems in Sweden and Denmark were established nearly a century ago. When established these systems were intended to be parallel to the geoid. Since then landrise/landfall has carried away the systems from their original positions. These motions in the solid earth may also have led to slight changes of the geoid. In addition the Swedish and Danish

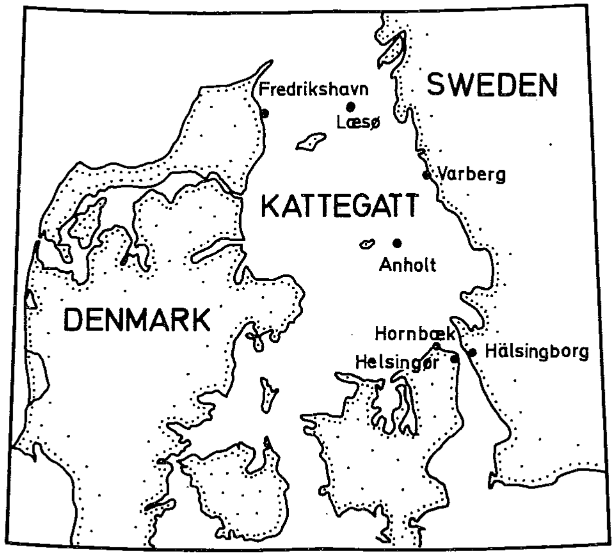

Fig. 1. Map over the Kattegat.

systems were already when established displaced a bit and thus not exactly on the same level.

Here we will relate the sea level gauges in Varberg and Fredrikshavn to one common height system. The establishment of a common height system might be quite important *e.g.* for future testing and running of hydrodynamic models for the Kattegat. At the end of this paper we compare the sea level difference between Varberg and Fredrikshavn, estimated from the sea level gauges, with the prediction from the Kattegat – Belt Sea model developed by STIGEBRANDT (1983).

2. *National height systems*

The Swedish and Danish height systems, denoted by RHOO and DNN respectively, were intended to be parallel to the geoid when established in 1900 and 1890 respectively. The height systems were defined by marks in the rock base on a number of locations. This means that the national height systems, as time passes, deviate more and more from the original surfaces as the marks are carried away by landrise (-fall). Here we introduce the height system SNO which was identical to RHOO in 1900. If we neglect the changes of the geoid SNO remains parallel to the geoid. In the same manner we introduce the height system DNO which thus was identical with DNN when this was established.

The sea level on a certain location is usually measured relative to the national height system. However, we are interested in the sea level relative to the geoid-parallel systems SNO and DNO. Therefore we have to correct the measurements for the difference.

3. *Sea level measurements in Varberg, Sweden*

The sea level in Varberg is measured from a level *below* the RHOO-level. The distance between the RHOO-level and the zero-level of the gauge is 2599 mm. However, a relatively recent revision of the Swedish height system showed that the RHOO-level in Varberg was low by 5 mm. The new system is called RH70. Thus the distance between the RH70-level and the zero-level of the gauge is 2604 mm.

The sea level in Varberg measured relative to the SNO-level is denoted by Sl_a. Because of the landrise both the RHOO (RH70)-level and the zero-level of the gauge move upwards. This is corrected for by the subtraction of the local landrise which is 0.637 mm/year. Thus

$$Sl_a = Sl - (2604 - (A - 1900) \cdot 0.0637)$$

where Sl is the sea level measured relative to the gauge zero-level in the year A. The mean sea level in 1900 was 25 mm below the (corrected) RHOO-level in Varberg. Thus the mean sea level is 25 mm below SNO.

4. *Sea level measurements in Fredrikshavn, Denmark*

In Denmark sea levels are measured relative to the DNN-level. Because of the landrise in the Fredrikshavn area (0.25 mm/year, pers. comm. Danish Geodetic Institute) the DNN-level moves upwards relative to the DNO-level. An additional fact is that there was a land subsidence of about 50 mm in Fredrikshavn in 1942. In order to obtain the level relative to DNO in Fredrikshavn, Sl_a, we thus have to use the following equation

$$Sl_a = Sl - 50 + (A - 1890) \cdot 0.25$$

where Sl is the sea level relative to DNN in the year A. In 1976 the mean sea level was 21 mm below DNN. Thus the mean sea level in Fredrikshavn is 50 mm below DNO.

5. *The height difference between SNO and DNO*

In 1939 the height difference between RHOO and DNN was estimated to 78 mm from hydrostatic levelling across the Öresund (between Helsingborg and Helsingör) (pers. comm., National Land Survey, Gävle). Helsingborg has subsided slightly, about 1.5 mm (0.04 mm/year) during the period 1900–1939 while Helsingör has subsided a little more, about 5 mm (0.10 mm/year from the sea level record in Hornbeak, DMI (1970)). Consequently the SNO-level is about 75 mm above the DNO-level. In a recent letter the National Land Survey, Gävle (Leif Eliasson) makes a preliminary estimate, based on recent trigonometric levelling across the Öresund, amounting to 72 mm. We believe that this is the best available estimate.

Finally we calculate the sea level difference between Varberg and Fredrikshavn. The mean sea level in Varberg is 25 mm below the SNO-level which is 72 mm above the DNO-level. Thus the mean sea level in Varberg is 47 mm above the DNO-level. The mean sea level in Fredrikshavn is 50 mm below the DNO-level. Hence the mean sea level should stand 97 mm higher in Varberg than in Fredrikshavn.

There are a number of uncertainties in the estimate of the level difference between SNO in Varberg and DNO in Fredrikshavn. The largest of these should be the following:

First, there are uncertainties in the determination of the SNO-level in Varberg relative to that in Helsingborg and the DNO-level in Fredrikshavn relative to that in Helsingör. These uncertainties are both around ± 20 mm.

Second, the uncertainty of the estimate of the level difference between the SNO and DNO-levels. Leif Eliasson (pers. comm.) estimates this uncertainty to about ± 5 mm.

Third, there is an uncertainty in the estimate of the level difference between the DNO and DNN-levels in Fredrikshavn. The uncertainty of the magnitude of the land subsidence is estimated to be ± 7 mm (pers. comm. Dansk Geodetic Institute). The uncertainty of the accumulated land rise up to 1976 is perhaps ± 5 mm.

Fourth, the uncertainty of the accumulated land rise in Varberg is perhaps ± 8 mm (Leif Eliasson, pers. comm.).

It seems reasonable to assume that the uncertainties listed above are independent of each other. Then the total uncertainty in the estimate of the level difference between SNO in Varberg and DNO in Fredrikshavn should amount to approximately ± 30 mm.

6. *Computation of the sea level difference between the Swedish and Danish coasts in the northern Kattegat by a hydrodynamical model*

The vertical stratification in the Kattegat can be roughly considered as two-layered. The lower layer consists of Skagerrak water of salinity S2k. Because of the outflow of fresher water from the Baltic the upper layer is less salty. It has the salinity S1k and the thickness H1k. STIGEBRANDT (1983) gives the mean values for H1k, S1k and S2k to 15 m, 23 and 33 o/oo respectively.

A front, which separates Kattegat surface water from Skagerrak water, exists in the northern Kattegat (the Northern Kattegat front). The front is oriented essentiall in the northeast–southwest direction and is known to move in the north–south direction. Near the Swedish coast the front becomes parallel to the coast and between the coast and the front a coastal current runs northward (the so called Baltic current with volume transport *Qs*), draining the Kattegat surface layer.

The steric height difference of the sea levels across the front is denoted by Δhs and is related to $\Delta\rho/\rho$ (the relative density difference between the two layers) and H1k in the following way

$$\Delta hs \approx \Delta\rho/\rho \, \mathrm{H1k}$$

where it has been assumed that the horizontal pressure gradient vanishes at the depth H1k.

Superposed upon the baroclinic flow is a barotropic flow, into/out of the Baltic (denoted by *Qb* and taken positive when directed into the Baltic). There is also a small component of barotropic flow caused by volume changes in the Kattegat itself. The geostrophic effect of the barotropic flow upon the sea level in the Kattegat was studied by STIGEBRANDT (1984) using a long sea level record from Varberg and the atmospheric pressure over the Kattegat. He found that the geostrophic effect is of importance for periods up to about one month. There is also an inflow of water to the Kattegat caused by the vertical entrainment of 'deep water' into the surface layer. This is approximately equal to *Qs*. The inflow to the Kattegat from the Skagerrak is then approximately $Qb + Qs$. This quantity may be negative for certain periods. The geostrophically induced sea level difference is denoted by Δhg and is approximated by

$$\Delta hg = -f(Qb + Qs)/(gH)$$

where H is the mean depth of the Kattegat between Varberg and Fredrikshavn ($\approx$ 30 m) and f is the Coriolis parameter.

Assuming that Fredrikshavn is situated to the north of the Northern Kattegat front, and Varberg to the south of it, the whole steric effect should be seen in

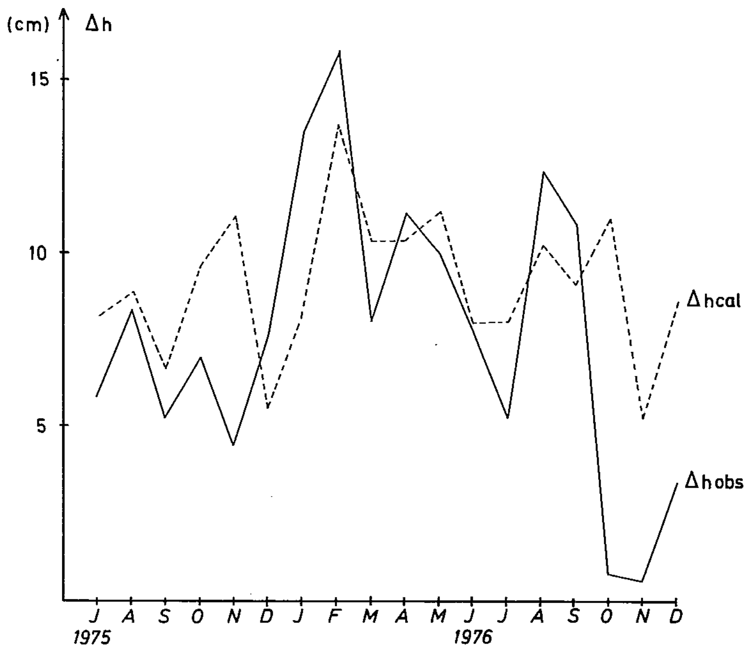

Fig. 2. Observed ($\Delta hobs$) and computed ($\Delta hcal$) sea level difference between Varberg and Fredrikshavn.

the sea level difference between Varberg and Fredrikshavn. The 'total' (steric plus geostrophic) sea level difference, Δh, between Varberg and Fredrikshavn is then

$$\Delta h = \Delta\rho/\rho \cdot \mathrm{H1k} - f(Qb + Qs)/(gH)$$

The model in STIGEBRANDT (1983) has been used to compute monthly means of Δh. The result is shown in Fig. 2 ($\Delta hcal$). The monthly mean sea level difference between Varberg and Fredrikshavn calculated from sea level measurements is also shown ($\Delta hobs$).

7. *Discussion*

1) The front may occasionally be located north of Fredrikshavn. We suspect this to occur when the measured sea level difference is much less than the computed one. From the Danish lightvessels Läsö Trindel and Anholt Knob (see Fig. 1) hydrographic measurements are conducted daily (published by Dansk Meteorologisk Institut). From the monthly means of the salinity, $S(z)$, we calculate the 'fresh-

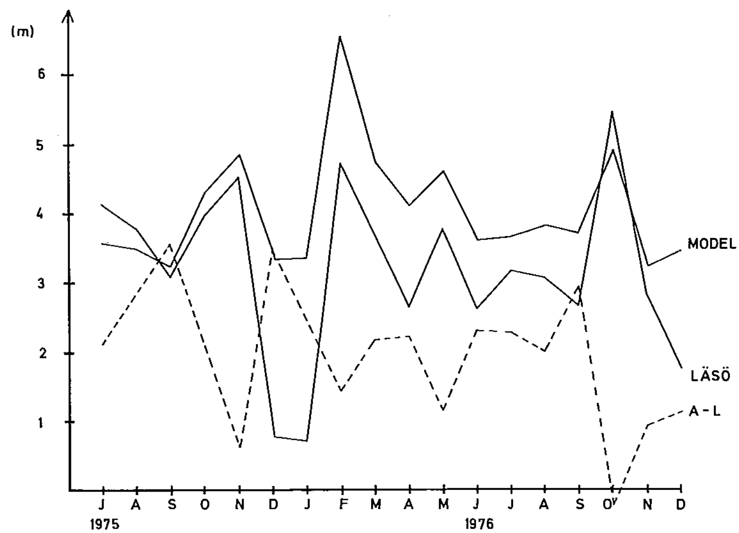

Fig. 3. The freshwater height calculated from hydrographic measurements at Läsö (solid line) and the difference in freshwater height at Anholt and Läsö (dotted line). Also shown is the freshwater height computed by the hydrodynamic model.

water height' defined by $\int(33-S(z))/33 \cdot dz$ where the integration is performed from the sea surface down to 30 m depth or to the depth where S = 33 o/oo (if this depth is less than 30 m). In Fig. 3 we have plotted the freshwater heigh at Läsö and the difference in freshwater height between Anholt and Läsö (A –. We have also plotted the freshwater height calculated by the model. When A –. is small the front should be situated in a northern position in Kattegat. This occurs in November 1975 and in October, November and December 1976. Froı Fig. 2 we can see that during these months the measured sea level difference is much smaller than the calculated sea level difference. During December 1975 and January 1976 the front is in a southerly position as the freshwater content at Läsö is quite small.

2) The steric effect in Varberg must not necessarily be equal to the mean steric effect in Kattegat (which is what the model predicts). Especially during periods with strong outflows from the Belt Sea/Baltic and weak southeasterly winds the fresher water may propagate up along the Swedish coast and create extra large steric effects there. The very large difference between measured and computed sea levels in January 1976 appears to be of this origin.

3) The model might be too crude to capture all kinds of events. In order to calculate mean values of the measured and the computed sea level differences for the test period we exclude the 'anomalous months' mentioned under 1) and 2) above (Nov. 1975, Jan., Oct., Nov. and Dec. 1976). We then obtain 90.3 for the measured and 92.8 mm for the computed mean sea level difference for the test period. Thus the results from the hydrodynamical model suggest that the long term mean sea level difference between Varberg and Fredrikshavn is 99.5 mm (recall that the 'geodetic' estimate is 97 mm).
4) The uncertainty of the hydrodynamic estimate is hard to estimate. The best way to determine this would be to extend the computations to a longer period. However, as the hydrodynamic and the 'geodetic' methods are truly independent we suggest that the mean sea level difference between Varberg and Fredrikshavn is 98 mm with an uncertainty marginal which is at most half of the uncertainty we estimated before. Thus the mean sea level should stand 98 mm higher in Varberg than in Fredrikshavn.

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