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# ON THE DYNAMICS OF A NEUTRALLY-BUOYANT FLOAT IN A NON-LINEARLY STRATIFIED FLUID

by

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Abstract

The transient properties of a neutrally-buoyant float in a non-linearly stratified fluid is studied both theoretically and experimentally. Numerical experiments from a theoretical model based on a quadratic drag law are in agreement with data from field experiments.

## 1. *Introduction*

The oscillations of a neutrally buoyant sphere in a linearly stratified fluid has been studied theoretically by both LARSEN, 1969 and WINANT, 1974, and experimentally by CAIRNS, MUNK and WINANT, 1979. Their studies were partly prompted by the use of neutrally buoyant floats, *i.e.* the so-called Swallow floats, as Lagrangian current meters in the deep oceans (see *e.g.* SWALLOW, 1955). In later years moored pycnocline riders have been used rather frequently in the exploration of the Baltic (SHAFFER, 1982). However, in this area, the density stratification deviates considerably from the linearized case, in fact it normally approaches a two-layer state. The transient properties of a neutrally buoyant float in a non-linearly stratified fluid is therefore of interest, theoretically as a hydrodynamical problem in itself, and because of its practical applications, especially as a preliminary study for a future free-floating device for coastal waters. The results of the following model, which is based on a classical quadratic drag law, is in agreement with field experiments also undertaken by the authors.

The governing equations presented in section 2 are applied to a continuous »two-layer case» in section 3 whereafter a comparison with experimental data takes place in section 4.

## 2. *The governing equations*

Consider a float of density $\rho_s$, falling through a stably stratified fluid at rest, whose density $\rho$ is given by $\rho = \rho(\zeta)$, where $\zeta$ is the vertical position (positive upwards) measured relative to the level of neutral buoyancy.

The force balance on the float is given by

$$(\rho_s + \rho c_m)\, V \frac{d^2\zeta}{dt^2} = -D + B \tag{1}$$

where $D$ is the drag force, $B$ the buoyancy force and $c_m$ the virtual mass coefficient (see LAMB, 1930), added to the inertia term. This addition is equivalent to assuming the ambient fluid at rest, instead of treating the complicated interaction between the float and the surrounding fluid. The drag force is determined by the quadratic drag law

$$D = \frac{1}{2} c_D A \rho \frac{d\zeta}{dt} \left|\frac{d\zeta}{dt}\right| \tag{2}$$

where $c_D$ is the drag coefficient (assumed to be constant) and $A$ the cross-sectional area of the float. The buoyancy force is given by

$$B = gV(\rho(\zeta) - \rho_s) \tag{3}$$

where $g$ is the gravitational acceleration and $V$ the volume of the float. The Boussinesq approximation will be used throughout this work. Equation (1) can now be written as

$$(1 + c_m)\rho_0 V \frac{d^2\zeta}{dt^2} = -\frac{1}{2} c_D A \rho_0 \frac{d\zeta}{dt} \left|\frac{d\zeta}{dt}\right| + gV(\rho(\zeta) - \rho_0) \tag{4}$$

where $\rho_0 = \rho_s$ is a reference density of the fluid. This equation can be expressed as a set of first-order differential equations,

$$\begin{cases} \dfrac{d}{dt}\zeta = \eta \\ \dfrac{d}{dt}\eta = -\dfrac{c_D A}{2(1 + c_m)V}\,\eta|\eta| + \dfrac{g}{(1 + c_m)\rho_0}(\rho(\zeta) - \rho_0) \end{cases} \tag{5}$$

3. *The two-layer case*

The solutions to equation (5) are obtained numerically by using a fourth-order Kutta-Merson scheme and are shown in phase-planes. Solutions for one specific case, where the stratification is given by a continuous »two-layer» structure with a sharp interface, are shown in figure 1. This choice of density profile is due to its obvious geophysical applications.

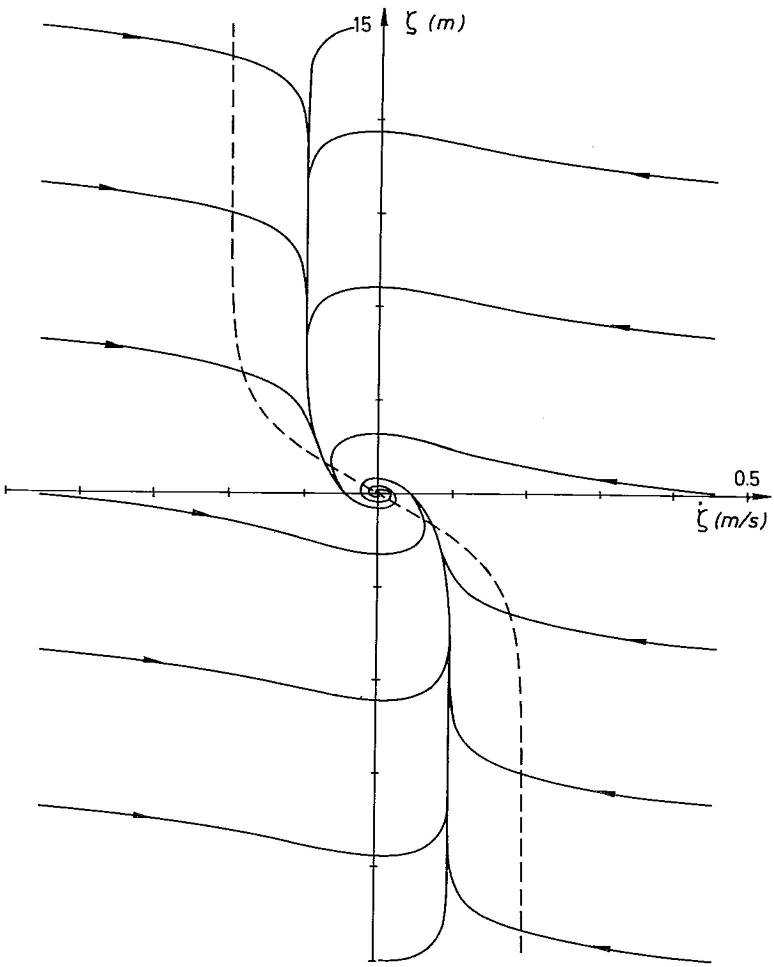


Figure 1. The »two-layer case». Trajectories of solutions of the equations for different initial values shown in the phase-plane. The arrows indicate the evolution of time. The density profile is shown as a dashed line.

Equations (5) can now be written as

$$\begin{cases} \dfrac{d}{dt}\zeta = \eta \\ \dfrac{d}{dt}\eta = -a\eta|\eta| - b\tanh\left(\dfrac{\zeta}{c}\right) \end{cases} \tag{6}$$

where the two-layer case has been modelled by the tanh function. The parameter values chosen ($a$ = 1.0, $b$ = 0.01, $c$ = 2.5) are typical for a realistic case. The appropriate density profile is plotted in the same figure as the phase-plane, to illustrate the dynamical response of the float for some different initial values. Despite the initial values chosen, in the more or less homogeneous layers, a rapid adjustment to the terminal speed of the float takes place. This speed is the result of a balance between the buoyancy force and the drag on the float. An order of magnitude calculations reveals the characteristic terminal speed $V_T$,

$$V_T \sim \left\{\frac{2gV}{c_D A}\frac{\Delta\rho}{\rho_0}\right\}^{1/2} \tag{7}$$

where $\Delta\rho$ is a typical density difference between the float and the ambient fluid.

Another reflection of the quadratic drag law, which results in relatively light damping at low velocities, is noticeable in the phase-plane as an increase in the trajectory density in the neighbourhood of the origin where the buoyancy decreases.

Due to the inertia of the sphere overshoots occur at the level of neutral buoyancy and a damped oscillation around that level takes place. In this phase of the descent, there is a balance between all three terms in equation (4), making possible the calculation of the scale of the overshoot $\delta$;

$$\delta \sim \frac{2V}{c_D A} \tag{8}$$

In the case of a sphere,

$$\delta \sim \frac{8r}{3c_D}$$

where $r$ is the radius of the sphere, the typical amplitude is a few diameters (see WINANT, 1974 who experimentally verified this calculation). The frequency of the damped oscillations is slightly lower than the Brunt-Väisälä frequency. In the pycnocline with sufficiently low velocities, to lowest order, there is a balance between the inertia and buoyancy terms in equation (4). This yields the following frequency:

$$\omega \sim \left\{ \frac{gk}{\rho_0} \right\}^{1/2} \tag{9}$$

where $k$ is the linearized density gradient of the pycnocline.

## 4. *Field work*

Field experiments were conducted in the Gullmaren fjord on the Swedish west-coast, with varying stratification conditions. An example of the stratification is shown in figure 2.

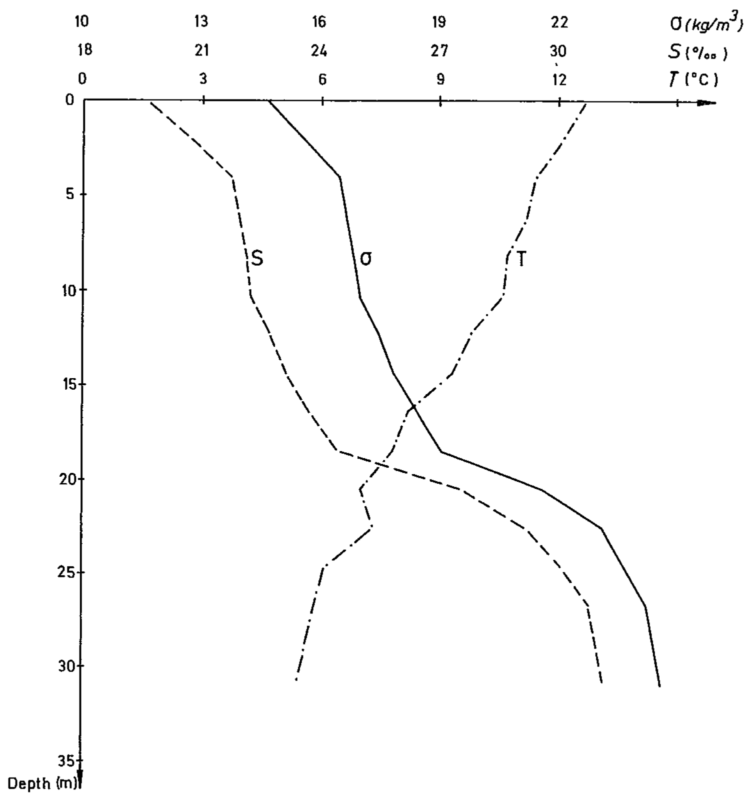


Figure 2. Example of the stratification observed during the 2-nd run.

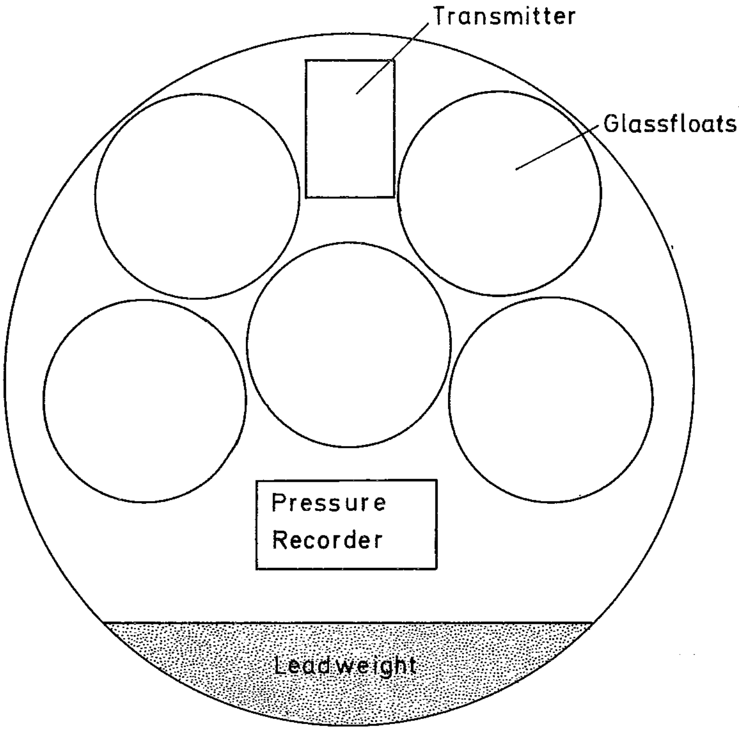


Figure 3. Principal sketch of the sphere.

The float consisted of an outer spherical plastic shell of radius 0.20 m. The geometry was chosen because drag coefficients for spheres have been determined with great accuracy (see *e.g.* BATCHELOR, 1967). The assumption of a constant drag coefficient turns out to be a reasonable approximation in the velocity range present, where the Reynolds number becomes $10^3 - 10^4$. The sphere contained a pressure recorder, lead ballast and some small air-filled glass floats. The sphere was filled with surface-water before the descent in order to avoid pressure-strains on the outer shell. Furthermore the sphere contained an ultra-sonic transmitter to facilitate location. (See figure 3). Although the float was not completely closed (in order to simplify the de-airing), the time scale of the water exchange was long compared to the time-scale of the experiment. Moreover the effect of a changing temperature of the contained water was of even less significance. When the density of the surface-water was determined the weight of the float was adjusted with small leadshot to make it neutrally buoyant at a desired isopycnal. The sphere was then released and allowed to sink. After a quick descent, the

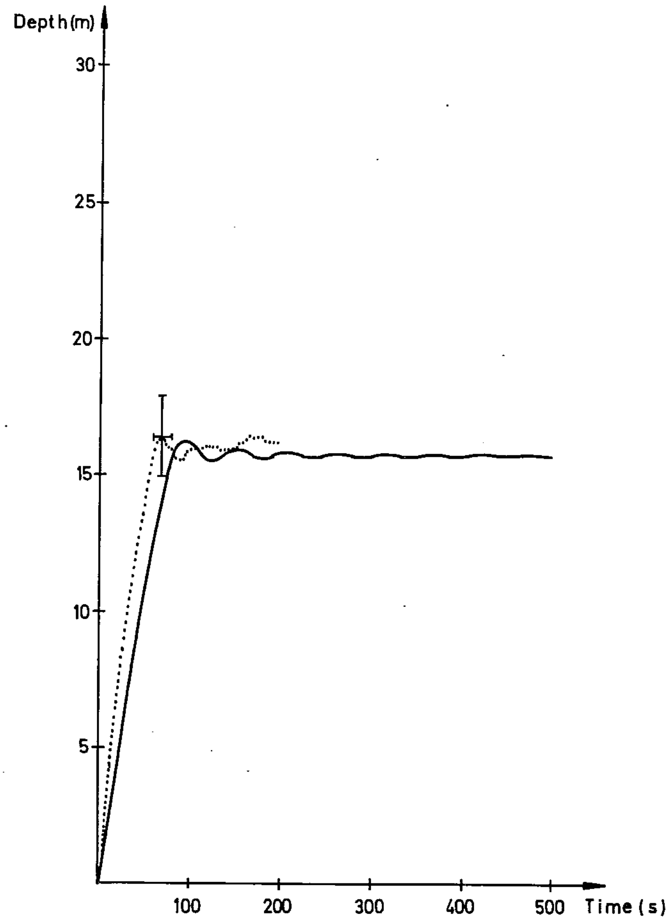


Figure 4. Run No 1. The graph shows the predicted (solid line) and observed (dotted line) pressure values versus time during one descent. The error-bars on the observed curve shows the errors caused by the uncertainties of the pressure recorder.

float asymptotically approached the level of neutral buoyancy by a damped oscillation. After a sufficient time some of the lead ballast was released by a time-controlled mechanism, and the free-floating sphere returned to the surface where it was recovered.

Three runs with different stratifications were made. The density was calculated from temperature and salinity, measured at every two meters, using a portable CTD. Continuous density profiles were obtained by utilizing a cubic-spline scheme. By inserting these profiles into equations (5), the theoretically predicted motions

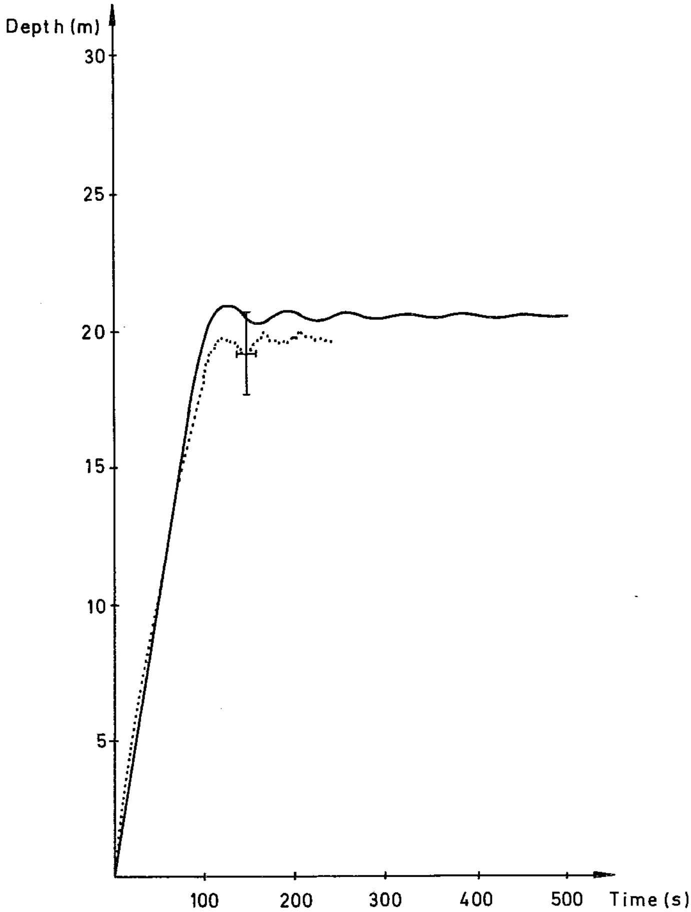


Figure 5. Run No 2.

of the sphere were obtained. Data from the pressure recorder and the corresponding data of density are shown in figures 4–6. The descent of the capsule agreed well with the predicted behaviour, based upon $c_m = 0.0$. Thus the assumed quadratic drag law seems to represent the actual drag on the float very well during the descent. It is obvious that the sphere performs damped oscillations around the level of neutral buoyancy. The amplitude of the first overshoots are of the same order as the diameter of the sphere, a result which further verifies the estimated scale (8). The observed frequency and amplitude of the oscillations were of the same order as the predicted ones. A comparison of the effect of a varying virtual mass coefficient, $c_m$, during one experiment (Run 2) is shown in

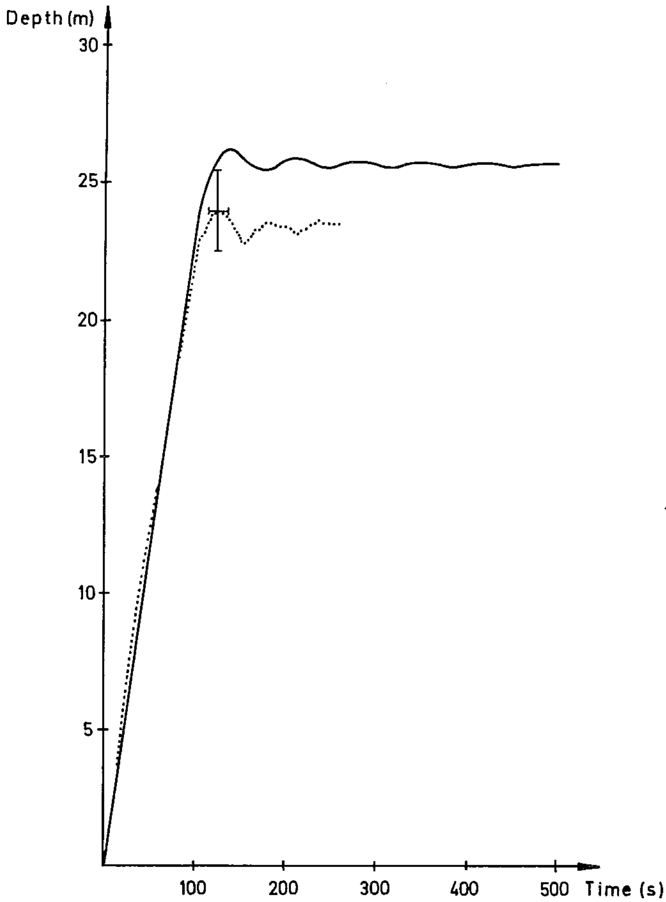


Figure 6. Run No 3.

figure 7. It is obvious that the variation of $c_m$ has no effect upon the actual descent, but that it effects the frequency and amplitude of the damped oscillations. A classical theory for non-viscous fluids (LAMB, 1930) yields a theoretical value $c_m = 0.5$. However WINANT, 1974 reports a value of $c_m = 0.21$ deduced from experiments in a viscous fluid, a value adopted by CAIRNS *et al.*, 1979 for field experiments in Lake Tahoe. As the proper virtual mass coefficient is unknown for this experiment and the effect on the frequency of the damped oscillation is approximately 10 %, the use of $c_m = 0.0$ throughout the calculations is motivated by the uncertainty upon the observations. The discrepancy between experiments and theory can partly be explained by the relatively sparse registrations of salinity and temperature, partly by the insufficient resolution of the pressure recorder.

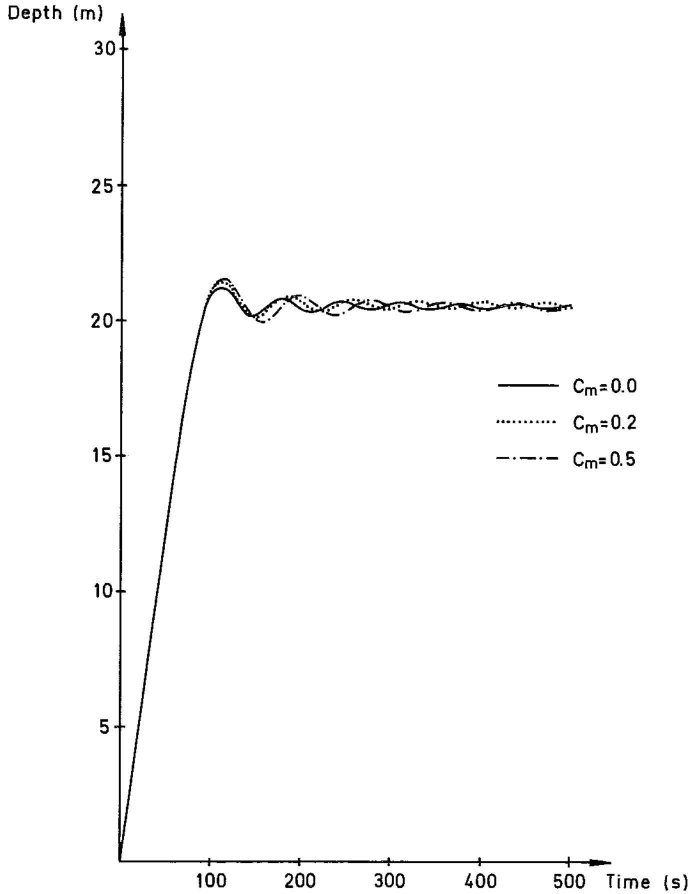


Figure 7. The predicted behaviour for varying virtual mass coefficients ($c_m$ = 0.0, 0.2 and 0.5) during one descent with hydrographic conditions equal to run 2.

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## REFERENCES

BATCHELOR, G.K., 1967: *An introduction to fluid mechanics.* Cambridge University Press.
CAIRNS, J., MUNK, W. and C. WINANT, 1979: On the dynamics of neutrally buoyant capsules, an experimental drop in Lake Tahoe. *Deep-Sea Res.*, 26, 369–381.
LAMB, H., 1930: *Hydrodynamics.* Cambridge University Press.

LARSEN, L.H., 1969: Oscillation of a neutrally buoyant sphere in a stratified fluid. *Deep-Sea Res.*, 16, 587–603.
SHAFFER, G., 1982: Personal communication.
SWALLOW, J.C., 1955: A neutral-buoyancy float for measuring deep currents. *Deep-Sea Res.*, 3, 74–81.
WINANT, C.D., 1974: The descent of neutrally buoyant floats. *Ibid.*, **21**, 445–453.