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ESTIMATION OF LAKE EVAPORATION BY USING DIFFERENT AERODYNAMICAL EQUATIONS

by

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Abstract

In this paper the applicability of three different aerodynamical equations for estimation of the evaporation from lake Tuusulanjärvi is discussed. One of these equations has been widely used previously. To determine the coefficients in the two other equations the method of least squares was used. Calibration of the equations was performed with the evaporation values measured daily with a GGI-3000 pan mounted on a float. These measurements have been made continuously since 1973 during the ice-free season. The years 1973 to 1976 were used for calibration and 1977 for verification.

1. *Introduction*

The first measurements to determine the lake evaporation in Finland were made already at the end of the last century (HOMÉN 1894). In 1913 and 1914 evaporation measurements were carried out by the Hydrographic Office on lake Pyhäjärvi at Tampere, with evaporation pans on land as well as on the water (BLOMQVIST 1917).

At the end of the 1930-ies the evaporation from lake Puujärvi was studied, using a.o. observation floats (FRANSSILA 1940). During the international hydrological decade (IHD) the lake evaporation was studied more intensively in Finland. The evaporation studies were organized by The Hydrological Office and they were started in southern Finland, on lakes Pääjärvi and Pyhäjärvi (HYVÄRINEN *et al.* 1973). In 1972 the programme was extended to the Lokka storage basin in northern Finland and in 1973 detailed investigations on Tuusulanjärvi were started (JÄRVINEN 1978). This paper deals with measurements from Tuusulanjärvi.

The object of the programme at The Hydrological Office is to study the evaporation from a watercourse, with its geographical and temporal variations. The amount of water evaporating is determined with the water budget method, GGI-3000 floating pans and bulk aerodynamical methods. An important part of the study programme is the determination of the numerical values of the coefficients of the aerodynamical evaporation equations for lakes of varying nature. In this study the applicability of various evaporation equations to lake Tuusulanjärvi is discussed.

2. *The lake area*

Lake Tuusulanjärvi (A = 6 km^2, F = 92 km^2) is situated in southern Finland, about 30 km from the coast and north of Helsinki (Fig. 1). The lake is narrow and elongated

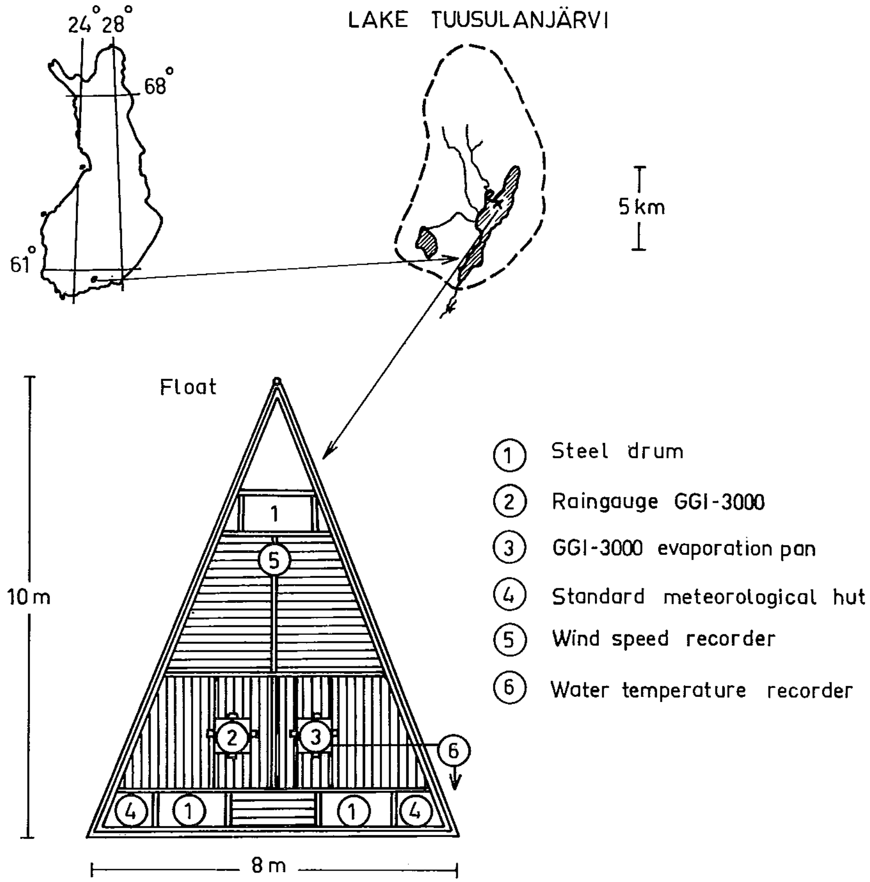

Fig. 1. The research lake and the float.

from the SW to the NE. It is shallow (mean depth 3.1 m, maximum depth 10 m) and without islands. The water level is regulated and varies in summertime a few decimeters only. The theoretical residence time of the lake is about 240 days. The mean annual run-off from the area is about 9.5 l/skm^2 ($0.88\ m^3/s$).

The main meteorological properties of the area are presented in table 1. The ice cover on the lake lasts for 150 days on the average.

Table 1. The mean air temperature and precipitation for the period 1931 to 1960 in the Tuusulanjärvi area.

Month	Air temperature/°C	Precipitation/mm
May	8.8	46
June	14.2	52
July	17.0	76
Aug.	15.4	77
Sept.	10.2	67
Oct.	4.8	70
Whole year	4.2	700

3. *Methods*

The lake evaporation was evaluated with the aerodynamical method and measured directly using a GGI-3000 pan mounted on a float (Fig. 1). The parameters necessary for the calculations were measured and registered on the float. The water budget method could not be used here because of the uncertainty in the amount of inflow. The GGI-3000 pan values were used as standards for calculating the coefficients of the different aerodynamic equations. In this paper the main emphasis was put on the investigation of the equation of SHULIAKOVSKI (1969), when applied to Tuusulanjärvi. This equation had been used when calculating evaporation aerodynamically on Tuusulanjärvi and some other Finnish lakes (JÄRVINEN 1978).

When calculating the coefficients of the different aerodynamic equations the period 1973 to 1976 was used for calibration and 1977 for verification. The daily evaporation from 21 May to 20 October was taken for every year from 1973 to 1977.

3.1 The floating GGI-3000 evaporimeter

On a float which freely turns with the wind (Fig. 1), the evaporation and precipitation were measured twice a day. The pans were sunk into the lake water. The meteorological parameters were measured with a standard meteorological station mounted on the float. The surface temperature of the lake and in the GGI-3000 pan were at first measured manually twice a day, later these values were registered automatically on magnetic tape with 30 minutes intervals.

The effect of the difference in surface temperature in the pan and in the lake was corrected for with the equation (1) (KUSNETSOV 1970):

$$E = 0.88\, E_0 \frac{e_o - e_2}{e_o' - e_2} \tag{1}$$

where

E = actual evaporation from the lake in mm/d

E_0 = evaporation from the GGI-3000 pan in mm/d

e_0 = the saturation vapour pressure in mb which corresponds to the surface temperature of the lake water

e_o' = the same as e_o but for the GGI-3000 pan

e_2 = the water vapour pressure in mb measured at a height of 2 m above the water surface

3.2 The bulk aerodynamical method

This method is based on varying transformations of Dalton's law. There are many such transformations, proposed by different authors. In principle, the method is based on the assumption that the evaporation is primarily proportional to the wind speed and the difference of water vapour pressure at the water surface and in the air mass.

In this paper three different aerodynamical equations are compared. Equation (2) is the Shuliakovski equation. In equations (3) and (4) the numerical values of the coefficients have been determined with the method of least squares, using the measured values for the period 1973 to 1976.

$$E = (0.15 + 0.108\, V_2)(e_o - e_2) \tag{2}$$

$$E = (a + b\, V_2)\,(e_o - e_2) \tag{3}$$

$$E = (a + b\, V_2^c)(e_o - e_2) \tag{4}$$

where

E = evaporation from the lake surface in mm/d

V_2 = wind velocity in m/s at a height of 2 m above the water surface

e_o and e_2 are the same as in equation (1)

a, b and c are empirical constants; $c < 1$

4. *The representativity of the period 1973 to 1980*

The representativity of climatological circumstances of the study period was investigated by comparing the hydrometeorological factors for the period 1973 to 1980 to those for the period of 1961 to 1980. For the comparison the precipitation, wind and air temperature values of the Helsinki–Vantaa airport and the Class A pan evaporation values of Tikkurila were used. These stations are situated about 10 km south of Tuusulanjärvi.

The greatest deviations from the mean values for the period 1961 to 1980 occur in precipitation and evaporation. In table 2 the differences between the Class A pan evaporation and the precipitation for the separate months of the period 1973 to 1980 are compared with the corresponding mean differences for the period 1961 to 1980. This gives information on when the weather has been on the dry side, when on the wet. It is known that evaporation on land differs rather much from the lake evaporation especially in early and late summer, but for a representativity study like this one this type of comparison is acceptable. The precipitation values in table 2 are uncorrected. Because freezing often prevents the measurement of evaporation in October, no October values are given in table 2.

Notably dryer than on an average were the open water periods 1973 and 1975, and especially so in July and August of 1973. The month of May in 1978 was also dry, with a precipitation of 3 mm only. Wet seasons of open water occurred in 1974, in the autumn, and in 1977, when the July precipitation of 159 mm was more than twice the average of 70 mm.

Table 2. The differences between Class A pan evaporation on land and the precipitation for the separate months of the study period 1973 to 1980, in mm, with the corresponding averages for the period 1961 to 1980.

Month	The difference between pan evaporation and precipitation in mm								
	1973	1974	1975	1976	1977	1978	1979	1980	1961–80
V	72	74	87	115	73	131	87	42	81
VI	141	91	134	82	91	103	101	134	114
VII	185	29	109	64	–67	37	–27	86	61
VIII	93	10	70	61	49	–44	33	–13	26
IX	–69	–84	9	–8	–26	–57	–28	–26	–29
V–IX	422	120	409	314	120	170	166	223	247

5. *Results and discussion*

In Figure 2 the distribution of the daily evaporation is shown, as measured with the GGI-3000 pan on the float. The distribution is shown separately for each of the summer months, for the whole period 1973 to 1977. The number of observations n, the mean values $\bar{x}$ and the mean deviations $\bar{s}$ are given in the figure. The class intervals are 0.5 mm.

For May the distribution is calculated from the values of the last ten days of the month only. If measurements always could have been started immediately after the lake was ice free, the distribution would have had quite another form, with many days of small evaporation values. The October distribution, too, would change somewhat if the measurements could have been continued up to the freezing of the lake.

The percentage of the days with large daily evaporation should be larger than shown in Fig. 2, as measurements are sometimes lost because of excessive wave build-up on days with a strong wind.

Figure 3 shows an example of the registrations of water and air temperatures on Tuusulanjärvi from 16 to 24 July 1981. The air temperature is measured on the float at a height of 2 m, the surface water temperatures are measured in the lake as well as in the evaporation pan. The interval of registration is 30 minutes.

The curves show that the surface temperature in the evaporation pan follows the air temperature somewhat more closely than does the surface temperature of the lake. The daily maximum of the air temperature occurs at around 18 hours and the minimum at around 06 hours. The maximum surface temperature in the pan occurs somewhat earlier and the maximum surface temperature in the lake somewhat later than the maximum air temperature. The daily minimum temperatures in the pan as well as in the lake occurred somewhat after the air temperature minimum.

When the daily evaporation values measured with the GGI-3000 pan in the period 1973 to 1976 are used for calibration the method of least squares leads to the following equations for lake Tuusulanjärvi:

$$E = (0.236 + 0.068\ V_2)(e_o - e_2) \tag{5}$$

$$E = (0.127 + 0.16\ V_2^{0.62})(e_o - e_2) \tag{6}$$

The symbols are the same as in equation (2).

During the calibration period 1973 to 1976 503 reliable daily evaporation values were measured with the GGI-3000 pan. As the observation seasons were of varying lengths, the study was restricted to the period of 21 May to 20 October. During the

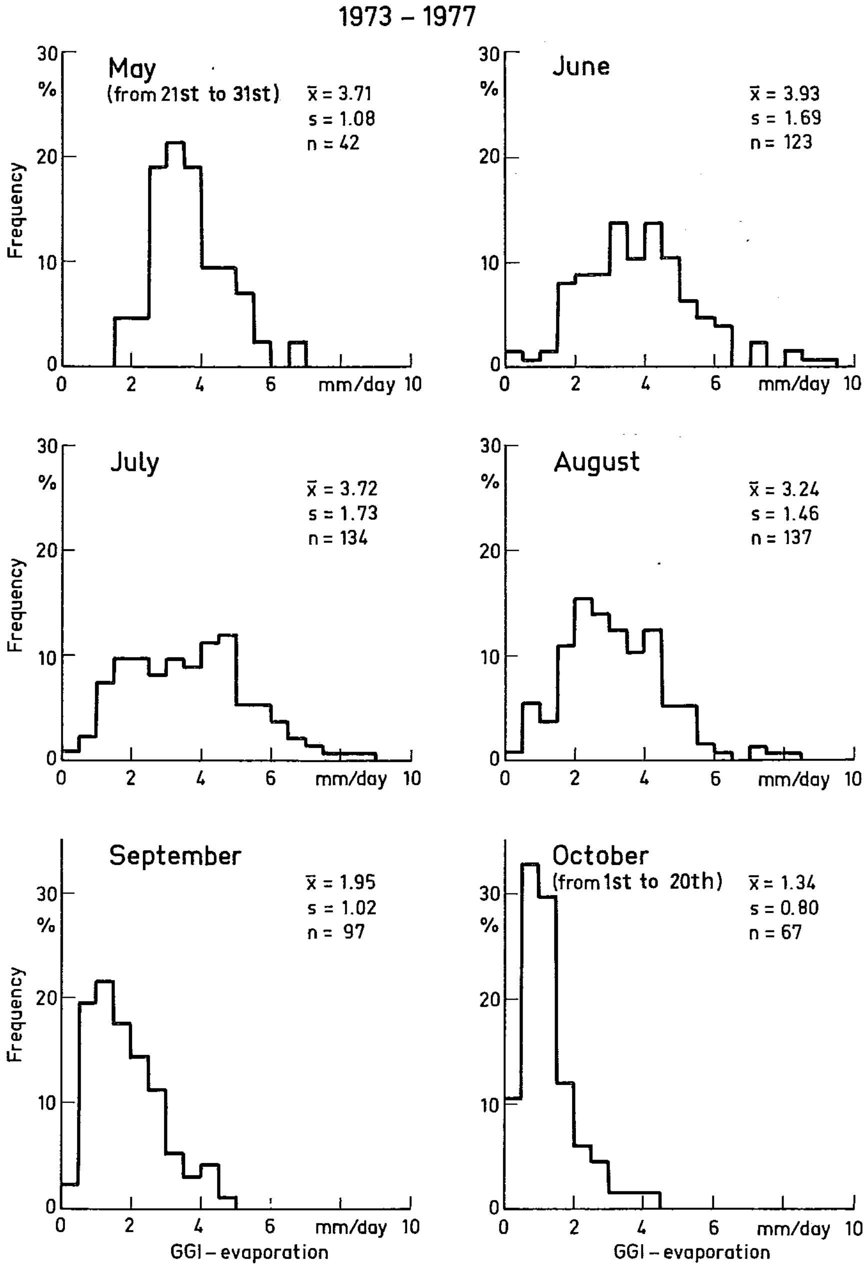

Fig. 2. The monthly frequency of the floating GGI-3000 pan evaporation values in the period 1973 to 1977.

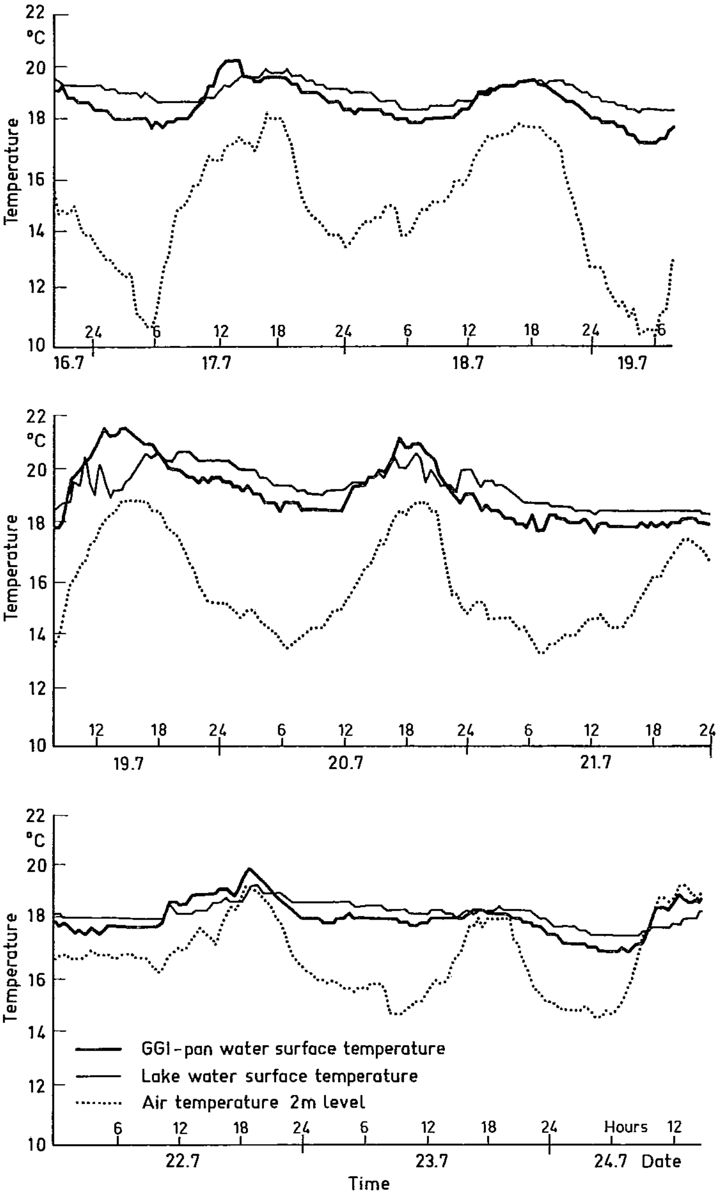

Fig. 3. The water surface temperature in the floating GGI-3000 pan and in the lake together with the air temperature at 2 m height during 16–24 July 1981.

calibration period the mean error of the equation of the SHULIAKOVSKI form (5) was 19.4 % and that of equation (6) 19.3 %.

RICHTER (1973) has used equations of the form (6) to compute lake evaporation from Lake Stechlin and found for two separate periods the equations (7) and (8)

$$1963\text{–}1965 \quad E = (0.17 + 0.135\ V_2^{0.75})(e_o - e_2) \tag{7}$$

$$1969\text{–}1971 \quad E = (0.17 + 0.200\ V_2^{0.53})(e_o - e_2) \tag{8}$$

One has to be careful when comparing equation (6) with equations (7) and (8), as the lakes are morphologically different.

Figure 4 shows the Shuliakovski equation (2) together with the equations (5) and (6) in graphical form. At wind velocities of 2 to 3 m/s the equations give quite similar evaporation values. When the wind speed increases above 3 m/s the difference between the equations (5) and (6) and the Shuliakovski equation (2) becomes marked. For a wind speed of 6.4 m/s, which was the highest measured with reliable pan evaporation measurement, this leads to a difference of about 0.2 mm/mb. The largest vapour pressure difference $(e_o - e_2)$ observed was 17.8 mb. With those maximum values eq. (2) would give a daily evaporation 3.6 mm higher than equations (5) and (6). Even larger wind speeds, up to 10 m/s, were measured, but on those occasions the pan evaporation measurements had to be discarded.

The GGI-3000 pan evaporation measurements from 1977 were used for verification of the equations (2), (5) and (6). The evaporation values computed using these equations were compared to the measured evaporation values to obtain the mean errors. The errors amounted to, for the Shuliakovski equation (2) 25.8 %, for the linear equation (5) of the Shuliakovski form 24.2 % and for equation (6) 24.5 %. The verification period was of the same length as the measurement seasons of the calibration period, *i.e.* from 21 May to 20 October.

Figure 5 shows a part of the verification period from 25 June to 25 July. On many occasions the computed evaporations compare favourably with the measured ones, but *e.g.* on 4 and 5 July the deviations are remarkable. This can only be due to wave action disturbing the pan measurements.

The distribution curves of the daily evaporation values for the whole period 1973 to 1980, as computed using the three aerodynamical equations (2), (5) and (6) are shown in Figure 6. Again the period from 21 May to 20 October has been chosen. The Shuliakovski evaporation values, equation (2), are less frequent in the area of moderate evaporation when compared to the values obtained with equations (5) and (6). They occur more frequently in the area of high evaporation values. In the area of small evaporation values, all three equations gave the same results.

The monthly evaporation values as computed with the three aerodynamical

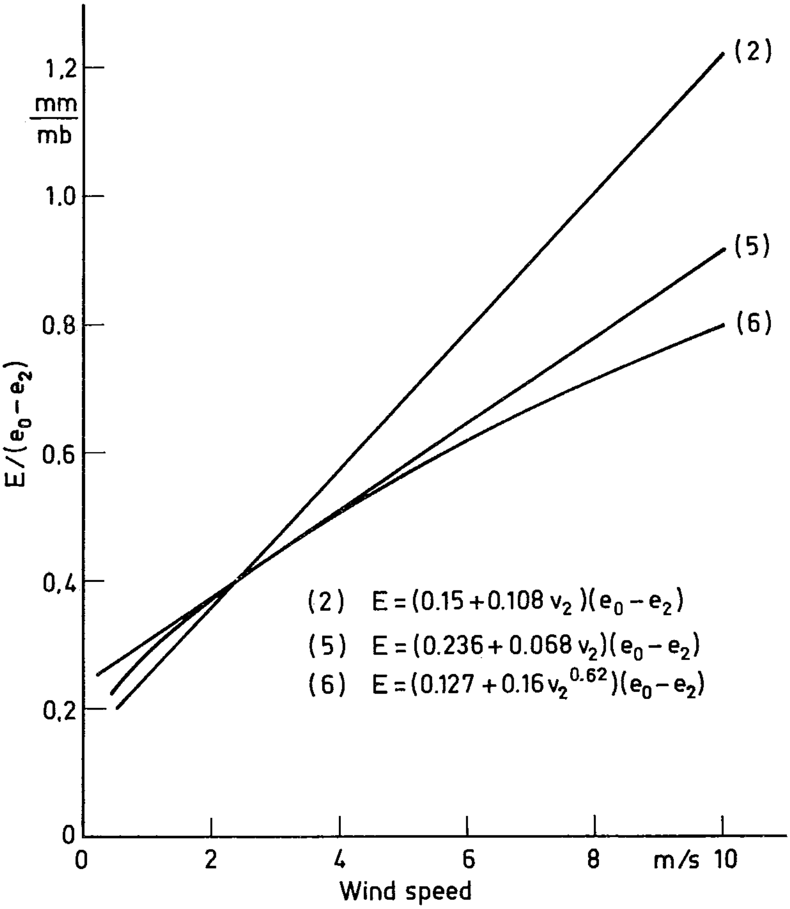

Fig. 4. Graphical representation of the aerodynamical evaporation equation.

equations (2), (5) and (6) for the years 1973 to 1980 are presented in Table 3. The monthly evaporation has been obtained by adding the daily evaporation values. The May and October values refer to those included in the season of study only, *i.e.* from 21 to 31 May and from 1 to 20 October only. The largest discrepancy is found in June of 1976, when equations (2) and (6) gave a difference of 23 mm. In May, September and October there is no difference of importance between the three equations. Using the values in Table 3 one obtains for the seasonal evaporations the mean differences:

(2) – (5) = 24.3 mm
(2) – (6) = 29.0 mm
(5) – (6) = 4.8 mm

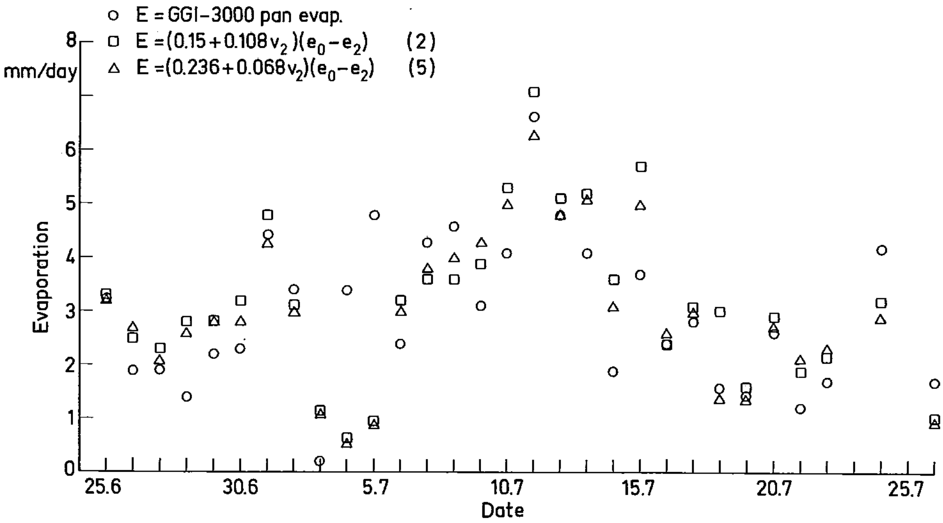

Fig. 5. The daily evaporation as measured with the floating GGI-3000 pan and calculated evaporation with equation (2) and (5).

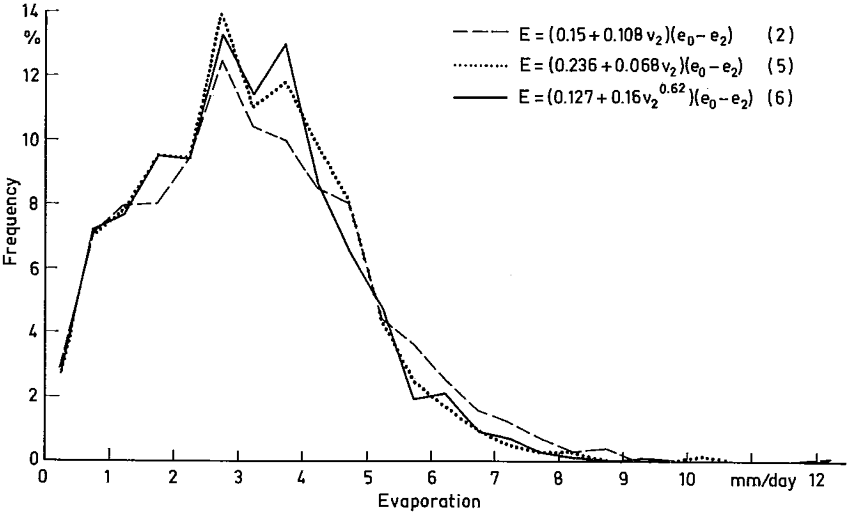

Fig. 6. The distribution of lake evaporation values calculated with equation (2), (5) and (6) for the years 1973 to 1980.

Table 3. The monthly evaporation from lake Tuusulanjärvi as computed with equations (2), (5) and (6) for the years 1973 to 1980. ( ) = part of the month only.

Month	Year											
	1973			1974			1975			1976		
	Equation			Equation			Equation			Equation		
	(2)	(5)	(6)	(2)	(5)	(6)	(2)	(5)	(6)	(2)	(5)	(6)
(May)	35	35	35	34	32	32	59	56	55	39	44	44
June	143	131	130	110	104	104	137	127	126	140	120	117
July	136	140	139	99	92	91	162	147	146	134	122	120
Aug.	116	107	106	91	85	85	132	127	127	120	116	115
Sept.	49	46	45	61	56	55	81	74	73	80	75	75
(Oct)	18	17	17	25	24	24	28	27	27	23	23	23
May–Oct	497	476	472	420	393	391	599	558	554	536	500	494

Month	Year											
	1977			1978			1979			1980		
	Equation			Equation			Equation			Equation		
	(2)	(5)	(6)	(2)	(5)	(6)	(2)	(5)	(6)	(2)	(5)	(6)
(May)	43	38	37	45	42	42	48	44	44	17	17	16
June	108	106	105	142	132	131	166	152	151	130	110	111
July	97	94	93	120	124	123	87	87	87	102	107	105
Aug.	84	86	85	91	89	87	107	103	102	95	98	97
Sept.	70	65	63	51	53	52	72	67	65	42	44	43
(Oct)	23	19	18	23	21	20	20	18	18	18	17	17
May–Oct	425	408	401	472	461	455	500	471	467	404	393	389

As a general conclusion of this study, one may state that equation (2) by Shuliakovski offers a good first approach for estimating lake evaporation in Finnish conditions. However, an extended study of the different lakes may lead to an improvement of the coefficients to be used in the aerodynamical method, especially in the cases with high winds.

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