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ANNUAL VARIATION OF SOIL TEMPERATURE AT DEPTHS 20 TO 700 CM IN AN EXPERIMENTAL FIELD IN HYRYLÄ, SOUTH-FINLAND DURING 1969 TO 1973

by

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Abstract

Fourier analysis was applied to monthly mean soil temperature data, collected from a measuring field in South-Finland. Temperature measurements were carried out weekly from January 1969 to December 1973 at depths 20, 40, 80, 150, 250, 400 and 700 cm. Two harmonics were found to describe the observed temperature variations to 92 %; four harmonics did not give a significant improvement.

The diffusivity of the soil was calculated directly from the results of the Fourier analysis and from weekly observations applying a Laplace-transform method. Using only the first harmonic the diffusivity was found to be 16 % lower than the value obtained from weekly observations. The corresponding difference for two harmonics was 10 %.

1. *Introduction*

Temperature variations at different depths depend on several variables such as soil composition and water content, distance to the surface, depth and duration of snow cover etc. The main cause of variations at different depths close to the surface is the variation of heat transfer to and from the surface by radiation, convection, conduction etc. Therefore periodic variations with an annual cycle are to be expected. In this study the annual variation was calculated using Fourier analysis. Four harmonics were computed. The harmonics found were then used for the calculation of the diffusivity of the soil. The diffusivity was determined also directly from weekly observations of the same period 1969 to 1973.

In a detailed soil temperature study at Argonne, Illinois USA, CARSON [1] used measurements at depths from 1 to 884 cm and found that 93.0 to 99.8 % of the variance can be explained by the first harmonic. In the arid zone of India KRISHNAN and KUSHWAHA [4] used depths from 1 to 120 cm and found that the first harmonic explains 69 to 85 % of the total variance at different depths, while the second and third harmonic represent 9 to 22 % and 0.6 to 4 % of the total variance. A more complete list of papers concerning soil temperatures can be found in a report by GILMAN [2].

In this study measurements from a special experimental field in South-Finland, over a 5-year period, were subjected to analysis by the Fourier method.

Table 1. Mechanical composition of the soil at the experimental field.

depth cm	gravel	distribution, per cent sand	fine sand	silt	bulk density g/cm³
000—003					
003—040	1	49	48	2	1.28
040—060	2	73	25	—	1.44
060—080	—	16	69	15	1.47
080—095	—	4	71	25	1.39
095—105	6	75	19	—	1.37
105—125	—	1	74	25	1.40
125—195	1	78	21	—	1.36
195—200	—	26	74	—	1.30
200—222	6	78	16	—	1.30
222—224	—	7	73	20	
224—250	—	46	54	—	1.32
250—320	1	84	15	—	1.38
320—340	—	23	76	1	1.40
340—370	—	18	68	14	
370—415	5	76	19	—	
415—460	—	3	73	24	
460—625	8	70	22	—	
625—700	—	65	35	—	1.60
mean 000—700	2.7	56.8	36.4	4.1	

2. *Experimental field and soil temperature measurements*

The experimental field is situated in South-Finland at 60°23′ N latitude and 25°02′ E longitude. The area, about 3 km^2, lies on a glacifluvial delta formation. In the middle, on the groundwater divide, a special experimental field was established in a clearing in the woods. The size of the clearing is 200 × 430 m. The height above mean sea level is 59.75 m and the field has a slope of 1:2000.

The surrounding clay areas are at mean elevation of 42.0 m. The mechanical composition and dry bulk density of the soil are shown in table 1 [5].

The soil temperatures were measured using copper-constantan thermocouples at depths 20, 40, 80, 150, 400 and 700 cm below the ground surface. The point 700 cm was in a groundwater observation tube, and was the only measuring point below the water table. The precision of the measurements was ± 0.1 °C. The measurements were carried out weekly from January 1969 to December 1973. Mean values for each month were calculated from the weekly readings. 5-year averages for each month were then calculated from the monthly averages.

Fig. 1 shows the profiles of the soil temperature based on the 5-year (from 1969 to 1973) monthly averages. As can be seen, the amplitude is 17.5 °C at the depth of 20 cm and decreases to 0.8 °C at 700 cm. As expected, the decrease of the amplitude is close to exponential [6]. In Fig. 2 the maximum and minimum temperatures for the years from 1969 to 1973 at each measuring depth are shown. Here the decrease of the amplitude is also approximately exponential [6]. Departures from symmetry are caused by the latent heat of fusion of water causing an elevation of the lowest temperatures (below and around 0°C).

Fig. 3 shows the soil temperature vs. depth based on 5-year grand means. The temperature varies from 5.42 °C at the depth of 400 cm to 5.81 °C at 40 cm. During the same period from 1969 to 1973 the mean air temperature was 4.61 °C at the height of 200 cm. Evidently the main reason for the higher soil temperature compared to the mean air temperature is the insulating effect of the snow cover, since the mean vertical heat flow from the interior of the earth is only 35.0 mWm^{-2} in Finland [3]. The average 5-year soil temperatures are also shown in table 2 as well as the average amplitudes and their standard errors at different depths.

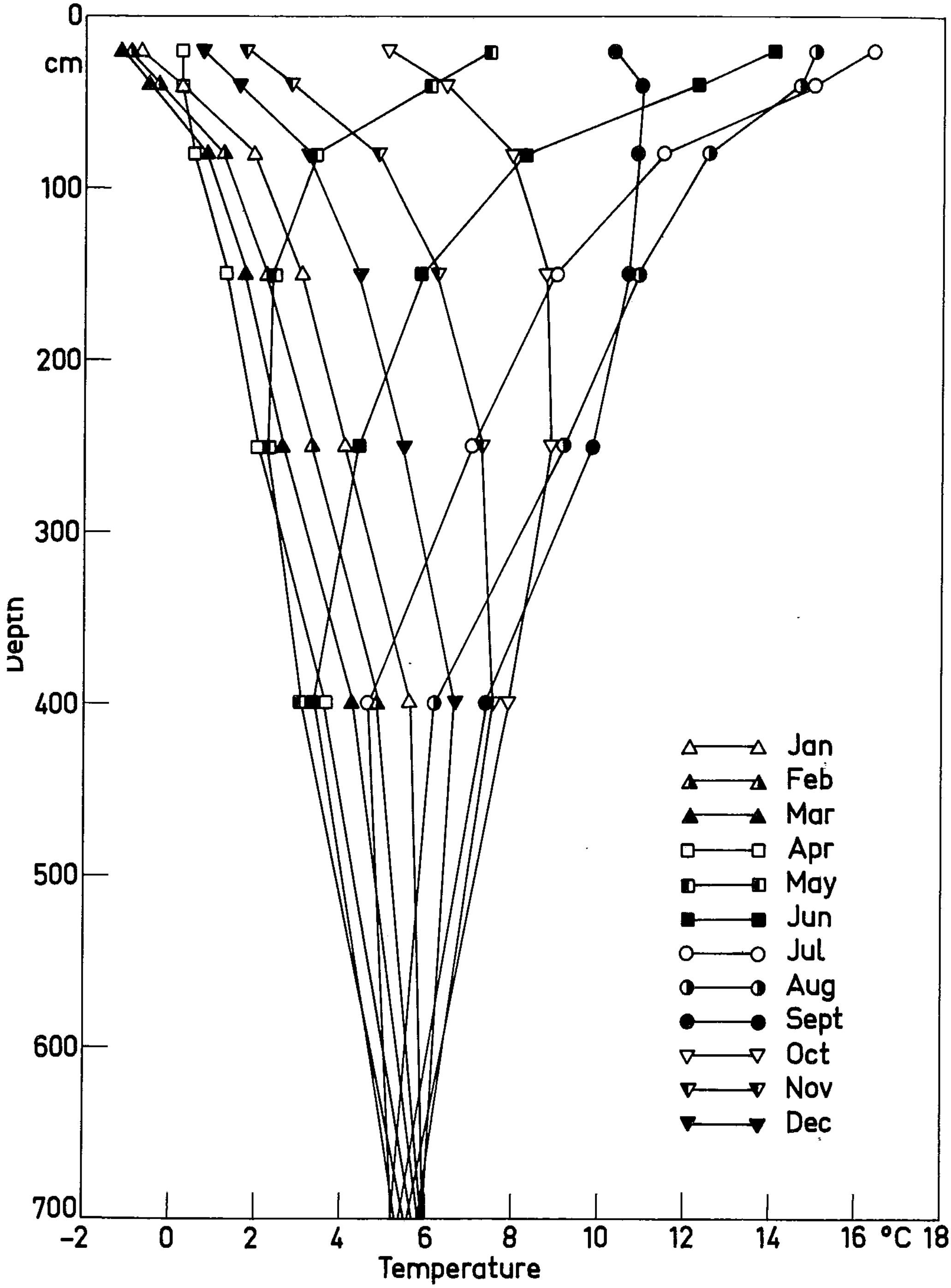

Fig. 1. Soil temperature profiles based on monthly averages of the 5-year period 1969 to 1973.

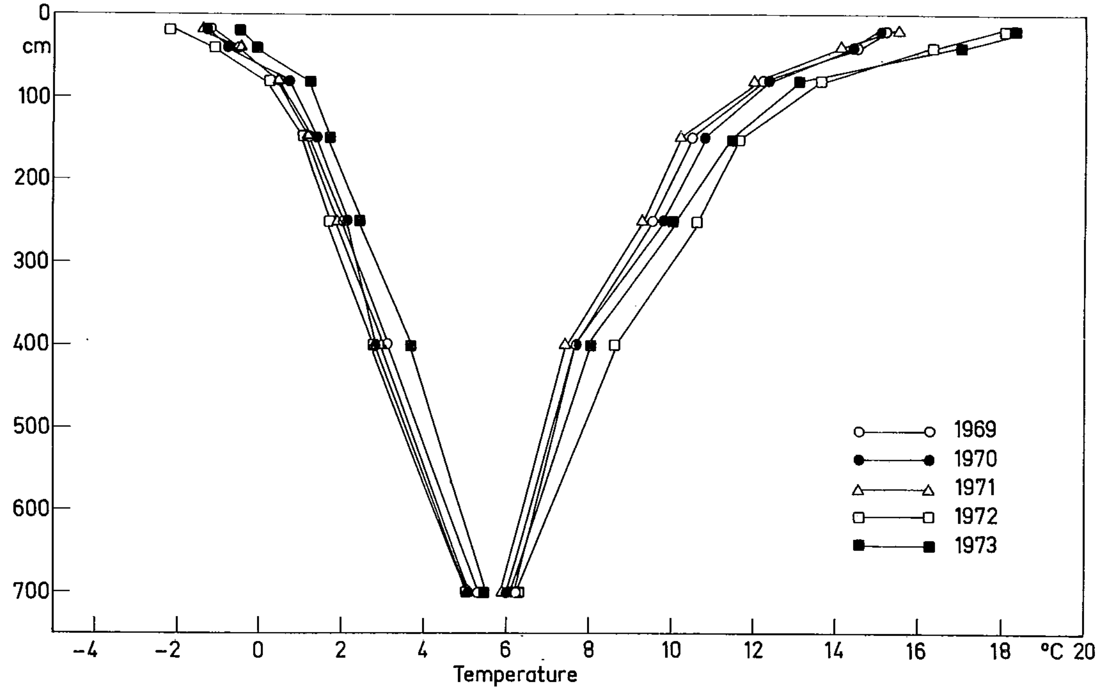

Fig. 2. Annual maximum and minimum soil temperatures for the years 1969 to 1973 at different depths.

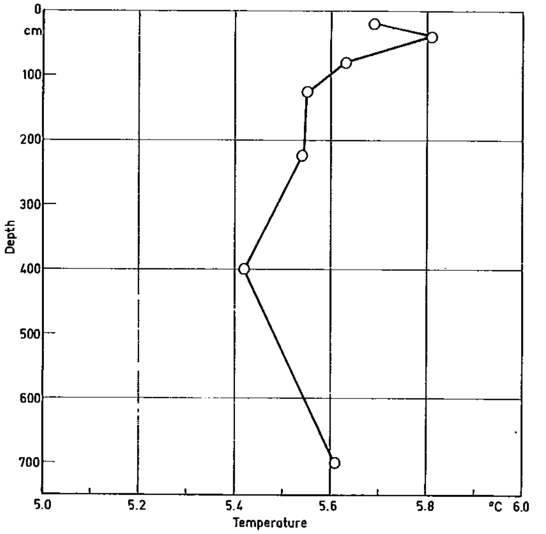

Fig. 3. Soil temperature vs. depth based on 5-year grand means 1969 to 1973.

Table 2. Mean soil temperature for the period 1969 to 1973 and the 5-year mean of the annual amplitude at different depths. S = standard error.

Depth cm	Mean temperature °C	S °C	Mean amplitude °C	S °C
20	5.69	0.27	17.78	1.71
40	5.81	0.30	15.74	1.39
80	5.63	0.29	12.00	0.74
150	5.55	0.28	9.60	0.58
250	5.54	0.28	7.82	0.61
400	5.42	0.28	4.82	0.59
700	5.61	0.11	0.84	0.09

3. *Harmonic analysis of soil temperatures*

To the time series of 12 monthly means calculated from the data for the years 1969 to 1973, a Fourier analysis was applied.

$$T = \bar{T} + \sum_{n=2}^{N/2} (S_n \sin \frac{360}{P} nt + C_n \cos \frac{360}{P} nt) \qquad (1)$$

where T is the temperature, $\bar{T}$ is the 5-year mean temperature, S_n and C_n are half range amplitudes, N is the number of data points (12), P is the period (12 months) and t is the time (0,5 to 11,5 months). The term in brackets is called the n:th harmonic. The amplitude A and the phase Φ of the n:th harmonic are

$$A_n = (S_n^2 + C_n^2)^{1/2} \qquad (2)$$

$$\Phi_n = \tan^{-1} (S_n/C_n) \qquad (3)$$

The total variance is the variance around the average $\bar{T}$, the variance of the n:th harmonic being $A_n^2/2$, except that of the last harmonic which is A_n^2 [4].

Fourier analysis was applied to the data sets of all measuring depths. Four harmonics were calculated (n = 1, 2, 3 and 4 describing 1/1, 1/2, 1/4 and 1/8 of the annual wave). As an example, the results from the depth of 80 cm are shown in Fig. 4. The observed 5-year monthly mean temperatures as well as the calculated ones using the two first harmonics are shown. As can be seen, two harmonics are adequate to describe the annual temperature variation rather well.

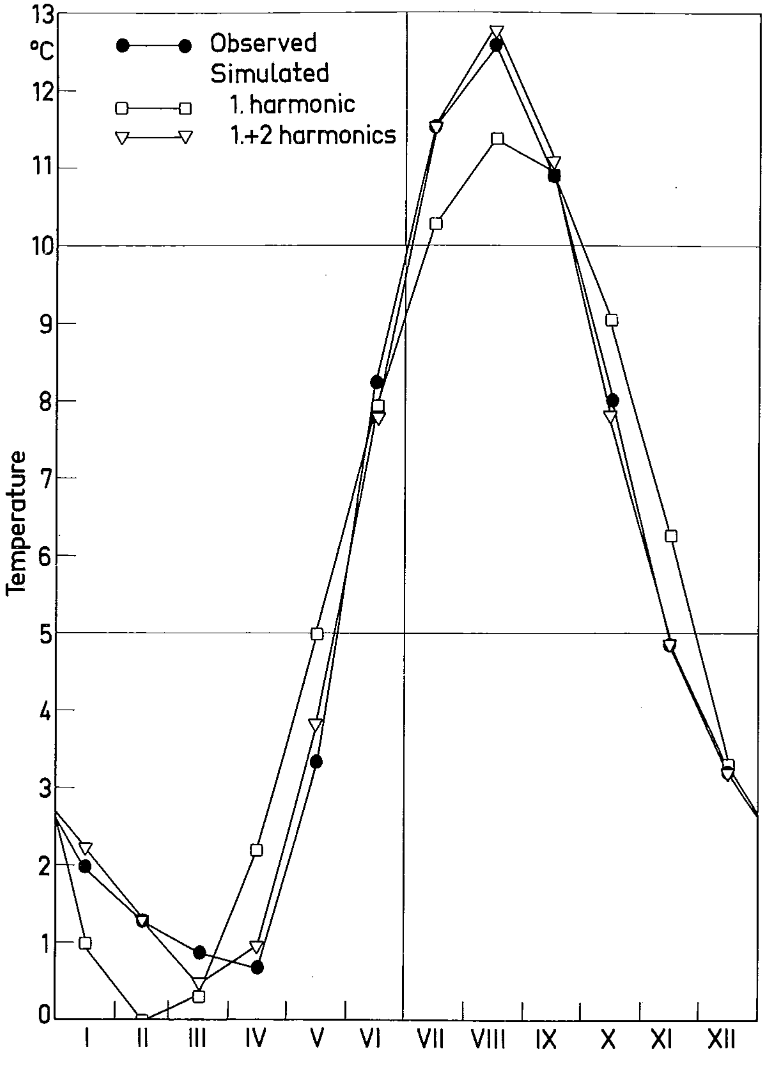

Fig. 4. Observed and calculated (one or two first harmonics) monthly mean temperatures at the depth of 80 cm, period 1969 to 1973.

In Fig. 5 $\log_e$ of the amplitude vs. depth and the phase vs. depth as calculated using the first harmonic are shown. Also standard error of each point is shown. In a homogeneous soil these points should fall on straight lines whose slopes are equal in magnitude but opposite in sign.

The correlation coefficient between the observed and simulated values are shown in table 3.

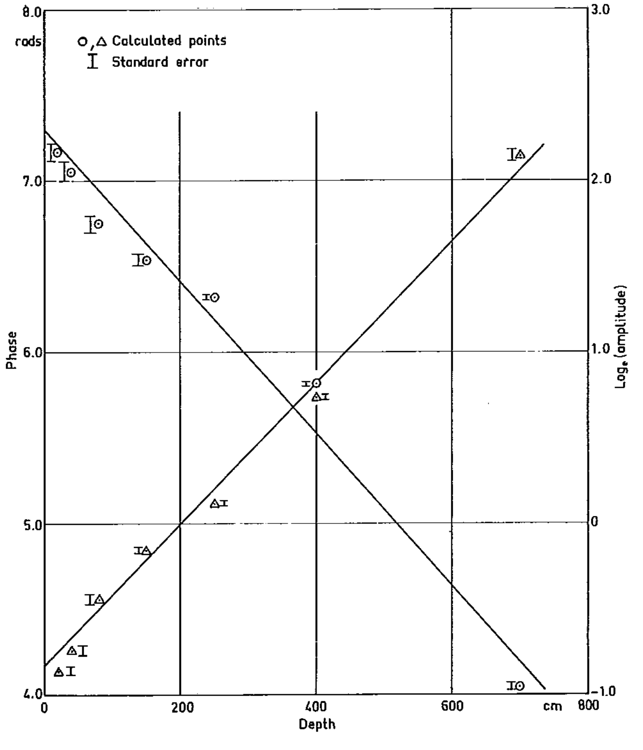

Fig. 5. Amplitude and phase of the first harmonic of the periodic temperature wave against the depth.

Table 3. The correlation coefficient between the observed and simulated monthly temperature values at different depths

Depth cm	Correlation coefficient			
	1. harmonic	1.+2. harmonic	1.+2.+3. harmonic	1.+2.+3.+4. harmonic
20	.9569	.9975	.9985	.9998
40	.9611	.9980	.9987	.9997
80	.9660	.9982	.9993	.9998
150	.9720	.9990	.9995	.9999
250	.9769	.9993	.9998	.9999
400	.9812	.9995	.99993	.99997
700	.9935	.9985	.9997	.9998

The correlation coefficient clearly increases with increasing depth. On the other hand, the improvement of the correlation coefficient is not significant after the second harmonic. The same is obvious from the table 4 showing the contribution of the variance of the different harmonics to the total variance.

Table 4. Contribution (in 1/100) of variance of different harmonics to the total variance. $\bar{X}$ = mean value for all depths, s = standard error.

Depth cm	Percentage of total variance					
	1. harmonic	2. harmonic	3. harmonic	4. harmonic	1.+2. harmonic	1.+2.+3.+4. harmonic
20	83.7	7.6	0.14	0.22	91.3	91.7
40	85.0	6.7	0.13	0.19	91.7	92.0
80	85.8	5.9	0.13	0.09	91.7	91.9
150	86.5	4.8	0.09	0.08	91.3	91.5
250	87.5	4.1	0.09	0.02	91.5	91.7
400	88.3	3.3	0.09	0.01	91.6	91.7
700	90.5	1.0	0.17	0.01	91.5	91.6
$\bar{X}$	86.8	4.8	0.12	0.09	91.5	91.7
s	2.2	2.2	0.03	0.09	0.2	0.2

4. *Diffusivity of the soil*

Assuming the soil to be homogeneous, the heat conduction equation (Fourier equation) is

$$a\frac{\delta^2 T}{\delta z^2} = \frac{\delta T}{\delta t} \tag{4}$$

where a is the diffusivity of the soil. Diffusivity is the ratio between the thermal conductivity and the heat capacity, and has the units m^2s^{-1}. Periodic solutions of this equation are

$$T_{(z,\,t)} = \bar{T}_z + T_0 e^{-z/D} \cos(\omega t - z/D) \tag{5}$$

where D = damping depth

$$D = (2a/\omega)^{1/2} \tag{6}$$

The heat conduction equation is linear so that periodic solutions of the form (5) can be superimposed to meet initial and boundary conditions. For any given Fourier component with angular frequency ω, the amplitude of the soil temperature variation is given as

$$A(\omega, z) = A_0(\omega)\, e^{-z/d} \tag{7}$$

and

$$\varnothing(\omega, z) = z/D \tag{8}$$

A plot of $\log_e$ of the amplitude against the depth yields a straight line, with a gradient $-1/D$, and a plot of the phase against the depth also yields a straight line, with a gradient $1/D$ [2]. In theory any harmonic may be used, but to minimize errors it is usual to take the first, and strongest, harmonic.

From (6),

$$a = D^2\omega/2 \tag{9}$$

where ω = 360/P, P is the period.

The diffusivity was calculated in eight cases. The points 20 and 40 cm deep were not used, because the soil frost went over 40 cm deep every year during 1969 to 1973 and also significant diurnal temperature variations can be expected to reach a depth of 40 cm. The diffusivities calculated are given in table 5.

The point 700 cm is in a groundwater observation tube. The groundwater table fluctuated between 628 and 705 cm during 1969 to 1973. Thus the point 700 cm was below the groundwater table most of the time. The diffusivities in the first two lines of table 5 are smaller than the later ones probably because of the warming effect of groundwater. Fig. 6 depicts $\log_e$ of the amplitude vs. depth and the phase vs. depth. A linear fit was calculated using the method of least squares for the depths 80, 150, 250 and 400 cm. One or two first harmonics were used. The results are shown in table 5. The slope of each line is also given in Fig 6.

The diffusivity was determined also directly from weekly observations for each year. The depths used were 80, 150, 250 and 400 cm. The average diffusivity for the five year period from this data set gave $a = 1.17 \times 10^{-6}$ $m^2 s^{-1}$. A Laplace-transform [7] from the same data set gave $a = 1.14 \times 10^{-6}$ $m^2 s^{-1}$. These values are comparable to the value of table 5: $a = 1.034 \times 10^{-6}$ $m^2 s^{-1}$.

Table 5. Diffusivity based on the results of Fourier analysis. A: Diffusivity as obtained from the change of phase as a function of depth. B: Diffusivity as obtained from the change of $\log_e$ (amplitude) as a function of depth.

Case	Diffusivity (x 10^{-6} m^2s^{-1})		
	A	B	Mean
5 points, 1. harmonic (80, 150, 250, 400 and 700 cm)	0.572	0.527	0.550
4 points, 1. harmonic (150, 250, 400 and 700 cm)	0.544	0.464	0.504
4 points, 1. harmonic (80, 150, 250 and 400 cm)	0.748	1.193	0.971
4 points, 1.+2. harmonic (80, 150, 250 and 400 cm)	0.882	1.185	1.034

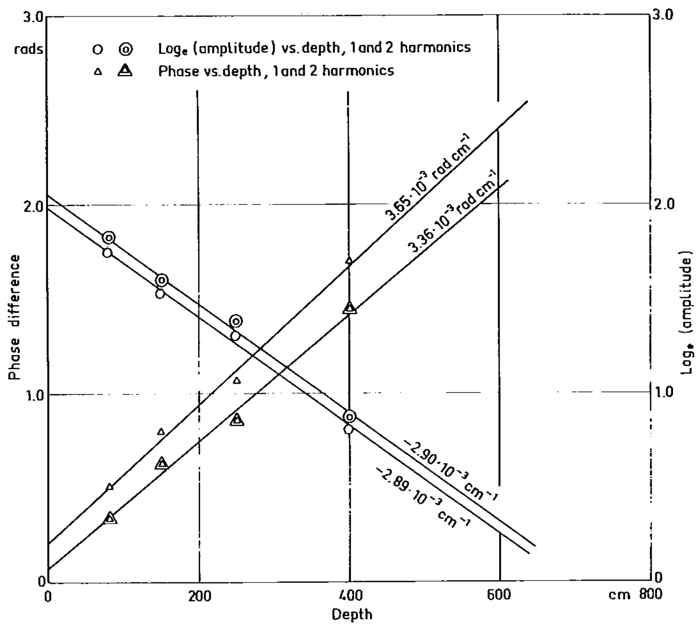

Fig. 6. Log_e of the amplitude and phase of the temperature, as represented by first or first and second harmonics as a function of depth.

5. *Conclusions*

Two harmonics represent 91.5 % of the observed annual variation of the temperature at depths 20 to 700 cm. The improvement of the correlation coefficients between observed and simulated values was insignificant when three or four harmonics were used: the calculated increases of the correlation coefficients between first two harmonics, two and three harmonics and three and four harmonics were 0.026, 0.001 and 0.0005.

The diffusivity of the soil was calculated from the monthly averages. The difference of the diffusivity calculated by Fourier techniques from monthly mean values and the diffusivity calculated from weekly observed values was 16 % when only the first harmonic was used, and 10 % when the first two harmonics were used.

Thus, for purposes where about 92 % of the variation is the accuracy required, two harmonics seem to be a simple way to simulate the temperature variations in the soil at different depths close to the surface. On the other hand, taking more harmonics seems not to improve the fit, showing that the remaining 8 % of the variance is caused by processes wich are aperiodic or have higher frequencies.

6. *Acknowledgements*

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