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# ON THE EFFICIENCY OF THE WIND TO GENERATE VERTICAL MIXING

by

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## 1. *Problem statement*

Vertical mixing in the sea is generally very weak due to the dominating stable stratification (see *e.g.* KULLENBERG [8]). For the surface layer the mechanical energy input from the wind, directly or via the waves, is often crucial for generating vertical mixing during the warm seasons. It has been established that only a small fraction of the energy input from the wind is consumed for vertical mixing (*e.g.* TURNER [20]; DENMAN and MIYAKE [1]; KULLENBERG [9]). This is also in agreement with laboratory experiments (*e.g.* TURNER [21]). It seems reasonable to expect that the strongest wind action occurs during storms and passages of meteorological fronts. The purpose of this note is to investigate the efficiency of wind-generated mixing during the passage of a storm using observations obtained during the Baltic Open Sea Experiment in September 1977, referred to as BOSEX 77. The vertical mixing during weak wind conditions was also investigated during BOSEX 77 by means of dye diffusion experiments. Thus conditions under fairly extreme situations can be compared.

## 2. *Theoretical background*

In the mixed layer below the wave zone horizontal homogeneity of the fluctuating fields may be assumed, implying that the mechanical energy equation can be written in the form

$$\frac{\delta}{\delta t}\left(\frac{\overline{E'^2}}{2}\right) + \frac{\delta}{\delta z}\overline{\left(w'\left(\frac{p'}{\varrho_o} + \frac{E'^2}{2}\right)\right)} + \overline{w'\vec{v}'} \cdot \frac{\delta \vec{V}}{\delta z} + g\,\frac{\overline{\varrho' w'}}{\varrho_o} + \varepsilon = 0 \qquad (1)$$

Here $\frac{1}{2}E'^2$ is the fluctuating kinetic energy per unit mass, $w'$, $p'$ and $\varrho'$ the fluctuating parts of the vertical velocity, pressure and density, respectively, $\varrho_o$ is the mean density, $\vec{V}$ is the horizontal velocity vector with the fluctuating part $\vec{v}'$, $g$ is acceleration of gravity and $\varepsilon$ is the rate of energy dissipation per unit mass. Absorption of solar radiation has been neglected.

The second term is the kinetic energy flux caused by vertical divergence, the third term gives the production of turbulent energy by interactions between the mean shear and the fluctuating velocity fields, and the fourth term gives the change of potential energy. Using the K-theory approximation this term may be written in the form

$$g \cdot \frac{\overline{\varrho' w'}}{\varrho_0} = -K\frac{g}{\varrho_0}\frac{d\varrho}{dz} = K N^2 \qquad (2a)$$

where $N$ is the Brunt-Väisälä frequency.

Similarly the production term may be written in the form

$$\overline{u'w'}\frac{dU}{dz} = K_m \cdot \left(\frac{dU}{dz}\right)^2 \qquad (2b)$$

where $K_m$ is the turbulent momentum transfer coefficient, and $u'$, $U$ are the fluctuating and mean velocities, respectively, in the $x$-direction assuming mean motion in this direction only.

The time required for redistribution of turbulent kinetic energy in the mixed layer is small compared to the time over which the energy input from the wind changes, implying that a steady state may be considered.

Niiler [13] suggested that away from boundaries a balance exist between the divergence, the buoyancy and the dissipation term. However, at the base of the mixed layer some production of turbulent energy will take place due to the shear there. A balance which is often assumed is between the production, buoyancy and dissipation terms, for instance analysed by Townsend [19] in the atmospheric boundary layer.

Deepening of the mixed layer in the stably stratified case occurs through entrainment of fluid from the lower layer into the mixed layer due to well developed turbulence in the upper layer (see *e.g.* Phillips [15], Turner [21]). The entrainment process appears to be dominating during high (storm) wind speeds. By means of a simple energy argument (*e.g.* Turner [21]) one finds that

$$u_e = c \cdot u_* \cdot Ri_*^{-1} \tag{3}$$

where $u_e$ and $u_*$ are the entrainment and friction velocities, respectively, $Ri_*$ *is overall Richardson number and* $c$ a constant. KATO and PILLIPS [4] found $c \simeq 2.5$ from laboratory experiments. The rate of deepening of the mixed layer varies with time, in the early stage being proportional to $t^{1/2}$, after about one inertial period decreasing to $t^{1/3}$. These regimes have been found theoretically and to some extent confirmed experimentally (KRAUS and TURNER [5], POLLARD, RHINES and THOMPSON [16], de SZOEKE and RHINES [18], KUNDU [11]). Most models, however, have as initial conditions a fluid at rest with a constant slope density profile and a well-defined onset time of the wind stress or stirring, which also is treated as constant. In the sea these conditions are never met, and mostly the initial conditions are not even known.

It is clearly of interest to analyse individual storm cases in order to obtain more field information on the entrainment process and the efficiency of strong winds to generate vertical mixing in stably stratified conditions. Here we study the change of potential energy relative to the mechanical energy available in the wind field at about 10 m above the sea surface. The rate of work per unit area by the wind stress at the 10 m level is written in the form

$$E_{10} = \tau_o W_{10} = c_d \varrho_a W_{10}^3 \tag{4}$$

The energy input per unit time and area from the wind can be expressed as

$$E = k \tau_o W_{10} = k_1 u_* \tau_o \tag{5}$$

Here $W_{10}$ is the wind speed, $\tau_o$ the wind stress, $c_d$ the drag coefficient, $\varrho_a$ the density of the air, $k$ the windfactor, and $k_1$ a numerical constant.

The potential energy per unit area of the water column is defined as

$$E_{pot} = g \int_{-D}^{o} (\varrho - \bar{\varrho})\,(z + D)dz \tag{6}$$

where $\bar{\varrho}$ is the mean reference density, and $D$ the water depth. The value of $E_{pot}$ is negative in stable conditions and zero for the totally mixed water column. The change of potential energy and the work by the wind over a period of time $\tau$ are found by integration of (6) and (4), respectively, and the ratio $R_1$ becomes

$$R_I = \frac{\frac{g}{T}\int_{-D}^{0} (\varrho(t_1) - \bar{\varrho}(t_1))\,(z+D)\,dz - \frac{g}{T}\int_{-D}^{0} [\varrho(t_2) - \bar{\varrho}(t_2)]\,(z+D)\,dz}{\frac{1}{T}\varrho_a \int_{t_1}^{t_2} c_d W_{10}^3\,dt} \tag{7}$$

$$= \frac{\frac{g}{T}\int_{t_1}^{t_2}\int_{-D}^{0} \frac{\delta}{\delta t'} [(\varrho(t') - \overline{\varrho(t')}]\,(z+D)\,dz\,dt'}{\frac{1}{T}\varrho_a \int_{t_1}^{t_2} c_d W_{10}^3\,dt}$$

The aim is to determine this ratio from the observations.

The local flux Richardson number $Rf$ is defined as the ratio between the potential energy change generated by the mechanical mixing and the total turbulent energy available, *i.e.*

$$Rf = \frac{\Delta E_{pot}}{E_T} = \frac{KN^2}{K_m\left(\frac{dU}{dz}\right)^2} = \frac{K}{K_m}\cdot Ri \tag{8}$$

where $E_T$ is the total turbulent energy avaiable and $Ri$ is the gradient Richardson number. This formulation assumes that the only production of turbulence occurs through the interactions between the mean shear and the fluctuating velocities. In the present case we may assume that the energy input from the wind $E$ is the source of turbulent energy so that $E = E_T$. Taking the mean of $\Delta E_{pot}$ and $E_t$ over some time period and assuming that the wind factor $k$ is constant over this period we find

$$Rf_c \simeq R_1/k$$

where $Rf_c$ is the critical flux Richardson number, *i.e.* the maximum value of $Rf$.

## 3. *Data and results*

During BOSEX 77 three Danish current meter stations were operated over the period 7 to 19 September (Table 1) which will be used for the present study. The time period analysed here is from 2 a.m. on 12 September to 2 a.m. on 14 September during which a storm passed the area (Table 2). The current spee at 20, 30 and 40 m nominal depths at station 611 are shown in Figs. 1—3. The recording interval was 10 minutes.

Table 1. Aanderaa current meter measurements during BOSEX 77.

| Station no. | Latitude | Longitude | Mean Depths of Current meters (m) | Depth (m) | Period of operation |
|---|---|---|---|---|---|
| 610 | 55°50′ N | 18°15′ E | 22, 32, 43, 58, 69 | 78 | 770906/10.10 am-770920/ 2.00 p.m. |
| 611 | 55°51′ N | 18°44′ E | 17, 27, 37, 85, 96 | 109 | 770906/ 1.10 p.m.-770920/ 2.00 p.m. |
| 612 | 55°59′ N | 18°31′ E | 12, 22, 33, 80, 92 | 106 | 770906/ 3.40 p.m.-770919/12.00 |

Table 2. Wind conditions during BOSEX 77
* The windspeed was given in Beaufort scale
** From Krauss 1978.

| Time | R/V "ALKOR" * direction | m/s | R/V "POSEIDON" ** m/s |
|---|---|---|---|
| 770912/ 4.00 am | SSW | 6.5 | |
| 8.00 am | SW | 12.5 | |
| 9.00 am | | | 11 |
| 12.00 | W | 19 | 26 |
| 4.00 p.m. | NNW | 15.5 | 30 |
| 6.00 p.m. | | | 28 |
| 8.00 p.m. | N | 19 | 24 |
| 770913/ 0.00 | NW | 12.5 | 20 |
| 4.00 am | NW | 12.5 | |
| 8.00 am | NW | 14 | |
| 12.00 | NW | 14 | 16 |
| 4.00 p.m. | NW | 14 | |
| 8.00 p.m. | NW | 12.5 | |
| 770914/ 0.00 | NW | 12.5 | 15 |

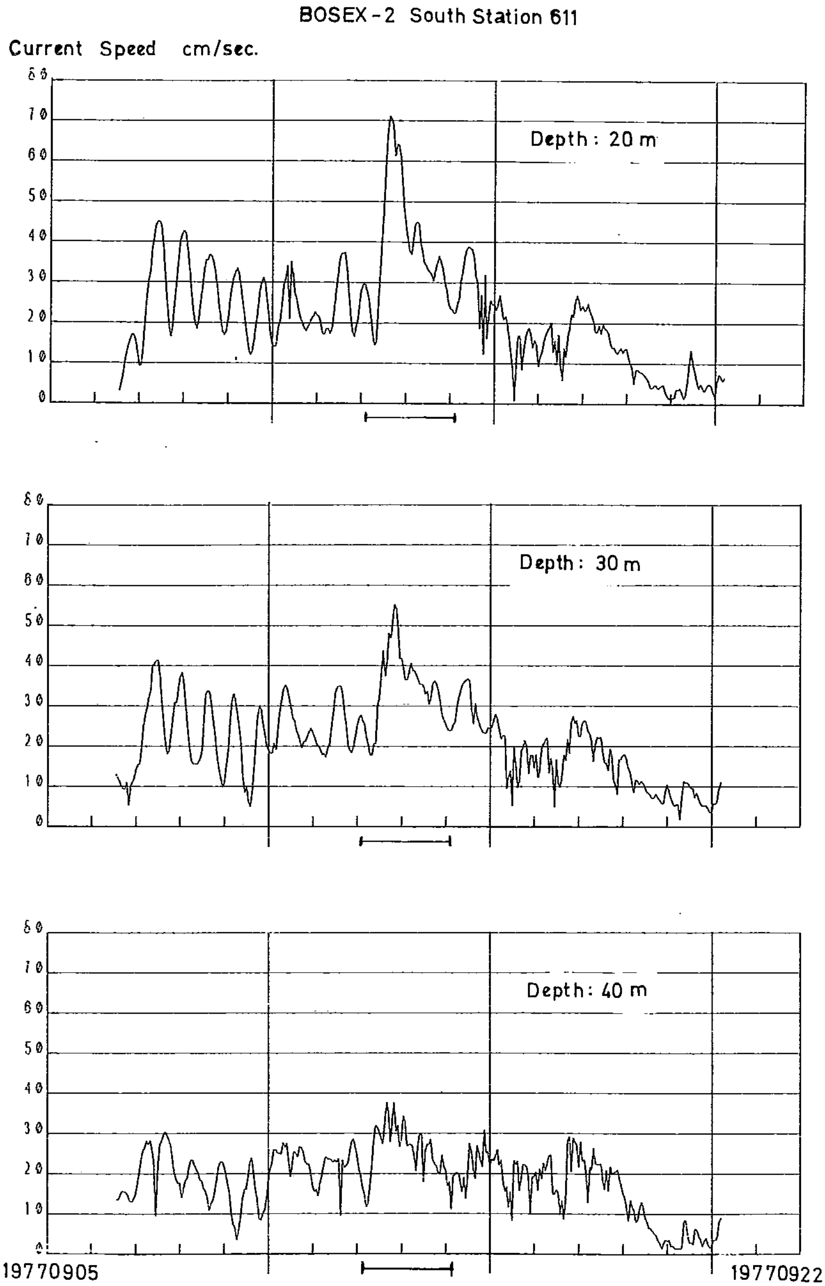


Fig. 1—3. Current meter records from station BOSEX-2, 611, nominal dephts 20 m, 30 m and 40 m.

The temperatur record (Fig. 4) shows a relatively rapid decrease of temperature in the upper layer and a rise in the intermediate layer, demonstrating the combined effect of atmospheric cooling, entrainment and diffusion in the water. Real time temperature was used in the calculations. However, horizontal advection changed the salinity field with the inertial period (Fig. 5) especially in the lower layer. For this reason the mean salinity during the period was used in the calculations. The change in potential energy was calculated over different periods $T$, ranging from 8 to 48 hours. The wind observations from R/V Alkor were used (Table 2) and, separately, the wind speeds observed by R/V Poseidon. These were considerably higher than those from R/V Alkor, implying an increase of $E_{10}$ by a factor of 2.5 The values of the ratio $R_1$ based on the Alkor wind data are shown in Fig. 6,7 for different

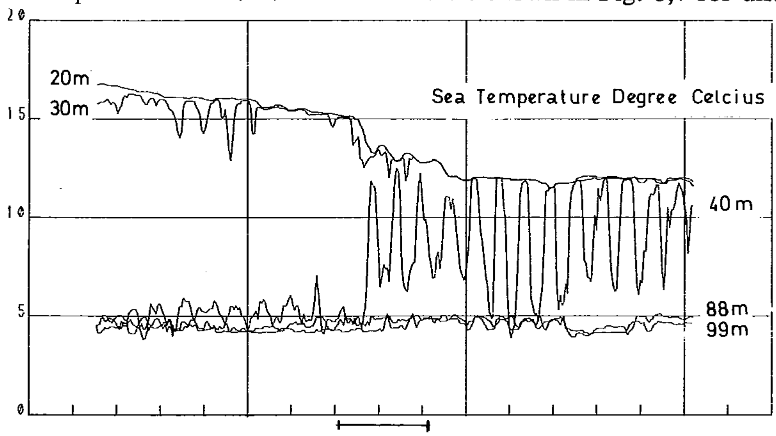


Fig. 4. Temperature record at station BOSEX-2, 611, nominal depths.

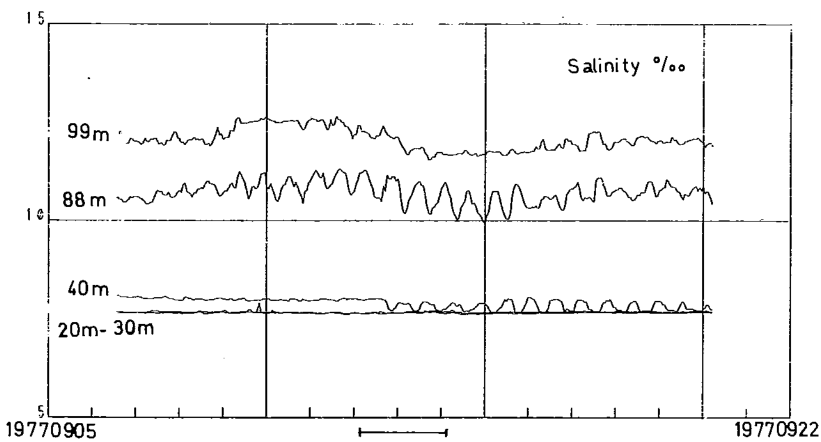


Fig. 5. Salinity records at station BOSEX-2 611, nominal depths.

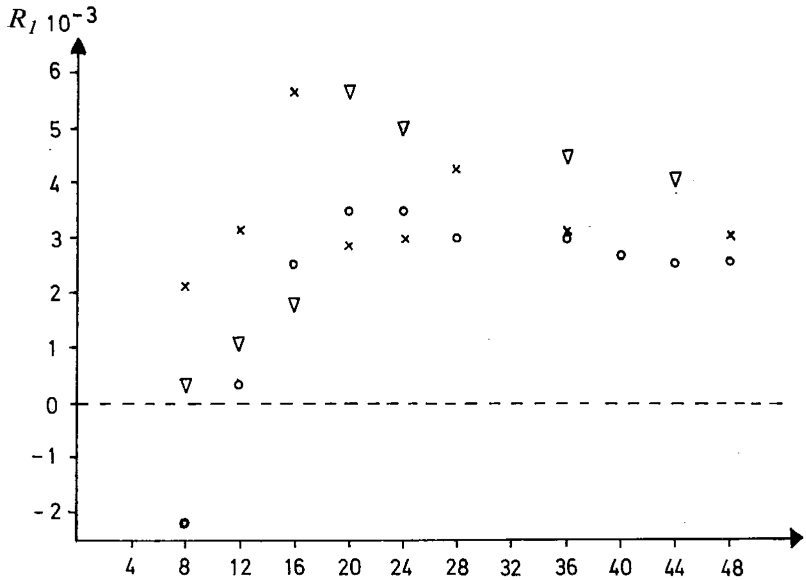


Fig. 6. Efficiency factor $R_1$, calculated with increasing integration time. o = 610 $x$ = 611 ∇ = 612

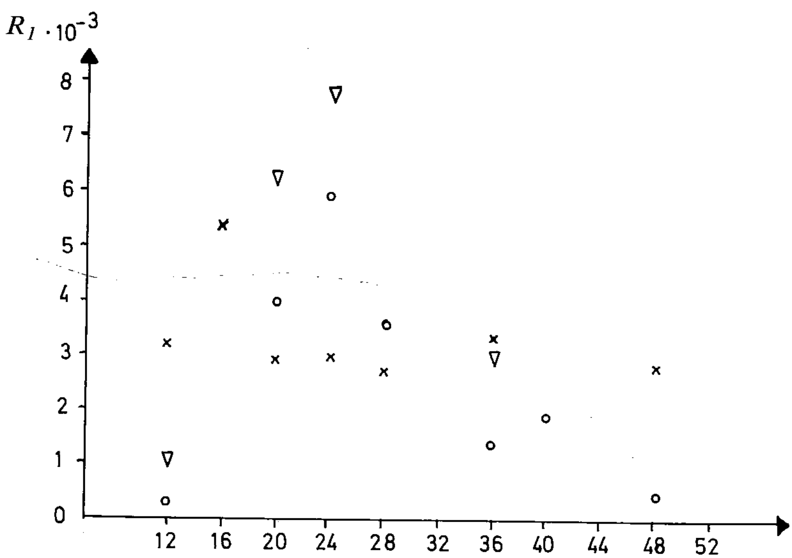


Fig. 7. Efficiency factor $R_1$, calculated with constant, 12 hours, integration time. Same notations as in Fig. 6.

integration periods, and using $\varrho_a = 1.25$ kg $\cdot$ m$^{-3}$ and $c_d = (0.75 + 0.067 W_{10})$ GARRATT [2]. Using wind data from Poseidon the values of $R_1$ are reduced by a factor of 2.5.

In order to be able to compare the effective mixing during the storm period with the mixing across the pycnocline during weaker winds, it is pertinent to estimate an effective vertical mixing coefficient. This is done by means of (2a) and the results of the efficiency calculations above. The potential energy change per unit mass and lenght water column is

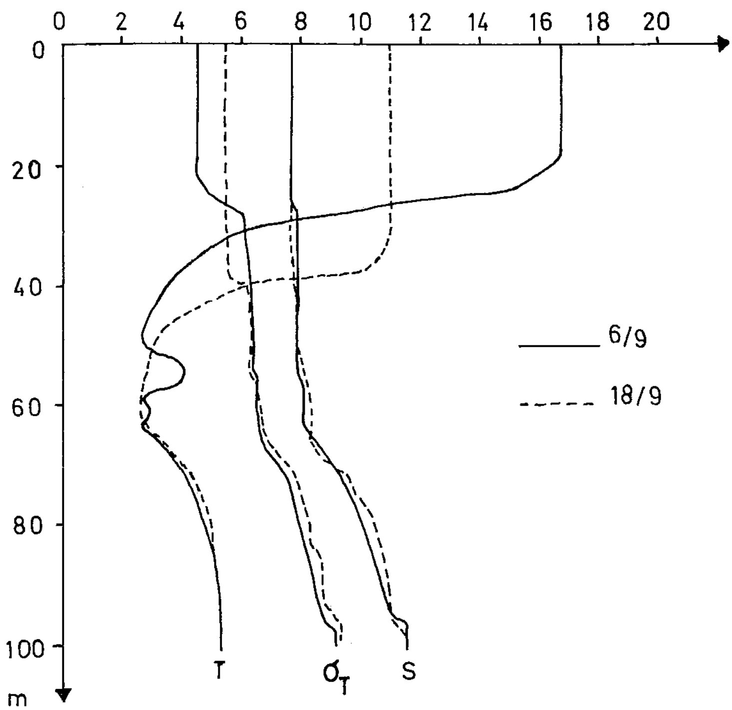


Fig. 8. Conductivity — temperature — depth profiles observed before (full drawn, 6 September) and after (dashed, 18 September) the storm at station 612.

$$\Delta P = \frac{\overline{R}_1 \cdot \overline{E}_{10}}{\varrho \cdot H} = K\overline{N^2} \tag{9}$$

The value of $N^2$ is determined from conductivity — temperature — depth (CTD profiles (Fig. 8)), obtained before the storm period. With $\overline{R}_1 = 3 \cdot 10^{-3}$, $\overline{H} = 30$ m, $\overline{W}_{10} = 20$ m $\cdot$ s$^{-1}$, $\overline{N^2} = 3 \cdot 10^{-3}$ s$^{-2}$ we find

$K = 7.10^{-4}$ m$^2 \cdot$ s$^{-1}$.

This result may be used to indirectly determine the entrainment velocity $u_e$ during this period by means of the expression (KULLENBERG, [10]).

$$K \frac{d\varrho}{dz} = u_e \cdot \Delta\varrho \tag{10}$$

The density gradient and the density difference across the interface are determined from the CTD profiles before the storm. Using the values $\Delta\varrho = 1.5\ kg \cdot m^{-3}$ *and* $\Delta z = 6$ m we find $u_e = 1.2 \cdot 10^{-4}\ m \cdot s^{-1}$.

4. *Vertical mixing during low winds*

During periods of low wind conditions before and after the storm period, the vertical mixing was investigated in the thermocline region by means of dye experiements. Briefly (see *e.g.*; KULLENBERG, [8, 10] for details) about 100 liters of rhodamine B dye solution is injected in a subsurface layer of about 2 m thickness, with the density of the solution adjusted as nearly as possible to that at the level of injection. The dye is traced in situ, over periods of half a day to several days, by towing a fluorometer after the ship cruising in a regular fashion over the area, navigating relative to a drogue or using Decca navigation. The fluorometer signal is recorded on board together with the temperature profile from a thermistor on the fluorometer. The observed dye concentration distributions are used to determine vertical and horizontal mixing parameters.

In stratified conditions with vertical shear flow the dye is normally fund to be distributed in layers, with a thickness varying in the range 20—200 cm, depending upon the conditions. In the Baltic such a layered structure has been observed earlier in the Arkona and Bornholm basins (KULLENBERG [8, 10]). Examples from the BOSEX observations are shown in Fig. 9. The layers are usually very well-defined with sharp boundaries, almost pulse-like.

Individual layers can often be identifield by the thickness, shape and position in the temperature profile. The dye concentration decreases at various rates but the thickness is virtually constant over limited periods of time. The layered structure is clearly related to the temperature and salinity profiles as well as to the current profiles.

By means of the observed dye concentration decrease in such a layer an effective vertical mixing coefficient K can be calculated from the formula

$$K = \frac{h^2}{\pi^2 \cdot (t_2 - t_1)} \cdot \ln \frac{C_1}{C_2} \tag{11}$$

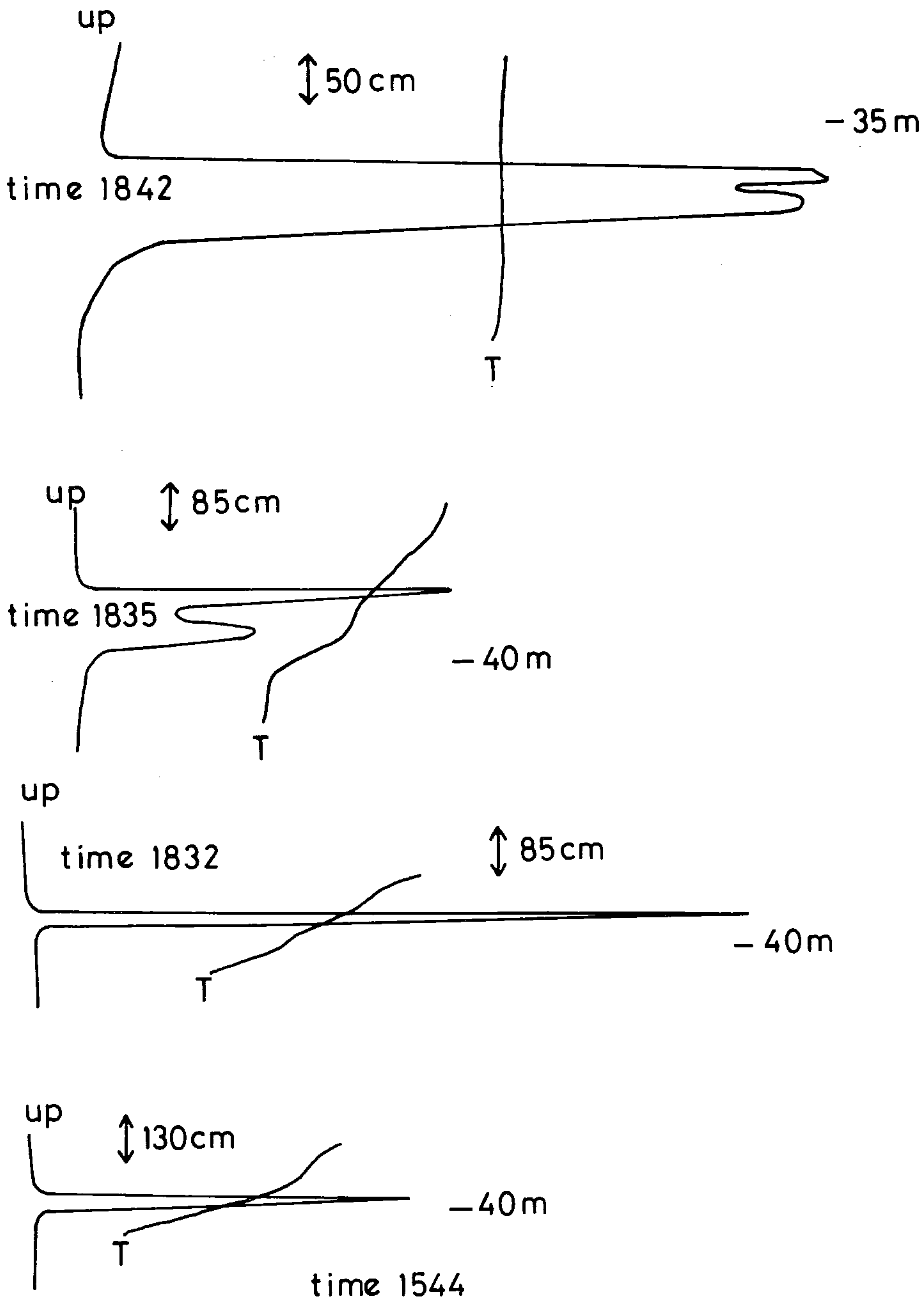


Fig. 9. Examples of dye profiles with temperature trace, the fluorometer going downwards. Time of observation on 18 September, thickness and depht of layer indicated for each profile. Time after injection 5.2—5.3 hours and 2.3 hours (lowest profile).

where $h$ is the layer thickness, $C_1$ and $C_2$ the maximum concentration at time $t_1$ and $t_2$, respectively (*e.g.* KULLENBERG [8]) Values of K have been determined for various parts of the pycnocline layers (Table 3).

In conjunction with the dye tracing, profiles of salinity and temperature were taken at intervals by CTD, from which the density stratification has been determined. This varied (given as $N^2$) in the range (2.8—5.0) · $10^{-3}$ $s^{-2}$ across the pycnocline layer. During the first dye experiment the wind varied in the range 3—8 m · $s^{-1}$ and during the second in the range 4—8 m · $s^{-1}$.

Table 3. Vertical mixing coefficients determined from dye experiments during BOSEX 77: time interval $\Delta$ t= $t_2$—$t_1$, mean thickness of dye layer h, concentration ratio ratio $C_1/C_2$, depth $D$ of layer and mixing coefficient $K$.

| Date | $\Delta t$ hours | $h$ m | $C_1/C_2$ — | $D$ m | $K \cdot 10^4$ $m^2 \cdot s^{-1}$ |
|---|---|---|---|---|---|
| September 1977 | | | | | |
| 8 | 0.8 | 0.55 | 2.0 | 30 | 0.2 |
| N56°05′ E18°23′ | 2.7 | 1.40 | 8.4 | 38 | 0.4 |
| duration 10 hours | 2.9 | 1.55 | 2.5 | 38 | 0.2 |
| 18 | 3.1 | 1.60 | 14 | 28 | 0.6 |
| N56°06′ E18°23′ | 3.3 | 1.05 | 6.4 | 40 | 0.2 |
| duration | 1.0 | 0.90 | 1.9 | 42 | 0.15 |
| 12 hours | 2.9 | 1.00 | 5 | 34 | 0.15 |
| | 3.6 | 0.25 | 32 | 40 | 0.02*) |

*) in sharp temperature gradients

## 5. *Discussion of results*

The absolute values of $R_1$ (Fig. 6,7) should be taken with much precaution. Besides the uncertainty regarding the wind speed, discussed above, errors in defining the layers may cause an error in the calculated energy change, especially for small integration periods $T$. However, the variation of $R_1$ with time (periods of integration $T$) is similar at the 3 stations and should be real. The temperature records show a maximum rate of decrease of temperature in the upper layer for the period 8—12 hours after the start of the storm. Considering that the inertial periods is about 14 hours this might indicate the time when the rate of entrainment decreases as predicted by various theoretical models, *e.g.* DE SZOEKE and RHINES [18].

The method of using progressively longer periods of integration (or averaging) will tend to decrease the fluctuations of $R_1$. Alternatively a fixed period may be used (Fig. 7), where 12 hours integration time was used throughout. The maximum again comes out for all three stations at approximately the same time as before.

The average values are 2.5, 3.3 and 4.5, all times $10^{-3}$, for stations 610, 611, 612, respectively, whereas the other method gave the mean values, 2.2, 3.4 and $3.2 \cdot 10^{-3}$, respectively. It is noticed that the long time values, 36 to 48 hours, which are the most reliable, show a relatively small scatter (Figs. 6,7).

KUNDU [11] found a variation with time of $R_1$, the maximum value being $2.0 \cdot 10^{-3}$, for larger times decreasing to a constant value of $1.25 \cdot 10^{-3}$. This yields a ratio of maximum to mean of 1.6. In the present case this ratio is 1.4 — 1.8 for the first method of integration and 1.7—2.4 for the second.

Other reported values of $R_1$ are in the range $(0.8—2.9) \cdot 10^{-3}$, based on laboratory experiments as well as field studies (KRAUS and TURNER [5], KATO and PHILLIPS [4], DENMAN and MIYAKE [1], KULLENBERG [9]).

The present $R_1$ values show a similar time behaviour as found by KUNDU [11]. The response of the sea seems in the present case to be somewhat slower than in model case studied by Kundu. This could, however, be due to the difficulty in defining the onset time of the storm, and could also be related to the difference in initial conditions between model and reality.

Finally the value of the critical flux Richardson number based on the relation $Rf_c = R_1/k$ is found to be in the range 0.09—0.18, using $k = 2.5 \cdot 10^{-2}$ and the range of mean values of $R_1$. This range for $Rf_c$ agrees quite well with what has been reported in the literature (*e.g.* TURNER [21], PEDERSEN [14]).

The vertical mixing coefficients determined for the different conditions vary by more than two orders of magnitude. This is clearly related to the variability of the winds as well as the details of the stratification in the water column. Compared to values of the mixing coefficient observed during comparable conditions in other areas as well as in the Baltic Sea, the results appear very plausible (*e.g.* KUKKENBERG [8], MATTHAUS [12]). Similar values have also been determined in the nothern part of the Baltic by HELA [3] and SIMOJOKI [17], who both used the penetration of the seasonal heat wave to calculate an effective vertical mixing coefficient. SIMOJOKI [17] found a value of about $0.1 \cdot 10^{-4}\ m^2 \cdot s^{-1}$ for the thermocline layer in the northern Baltic.

The entrainment velocity determined indirectly for the storm case may be compared with other results by means of the overall Richardson number $Ri_*$

$$Ri_* = \frac{g \cdot H \cdot \Delta\varrho}{\varrho\, u_*^2}$$

With $\overline{W}_{10} = 20$ m · s$^{-1}$ we find $u_*^2 = 10.5 \cdot 10^{-4}$ m$^2$s$^{-2}$, and $Ri_* = 430$ with $H = 30$ m and $\Delta\varrho = 1.5$ kg · m$^{-3}$. According to the results of KATO and PHILLIPS [4] this should give an entrainment velocity of $1.9 \cdot 10^{-4}$ m · s$^{-1}$, which is a factors of 1.6 larger than the value estimated from the field results. This results may suggest that our calculations are fairly consistent and that the values of $R_1$ may not really be too large. The ratio of $u_e/u_* = 3.8 \cdot 10^{-3}$ also compares well with other results for similar magnitudes of $Ri_*$ (see *e.g.* KULLENBERG [10]). Although all the absolute values should be considered with precaution, the apparent consistency of the results seems encouraging.

The study clearly demonstrates the importance of the wind forcing for the vertical mixing. In particular it is indicanted that the realtive efficiency is fairly large during a storm for a time period up to about the length of the inertial period. It should be noted that the wind duration as well as the wind force is important for the effects in the water column.

*Acknowledgements:* The second author would like to express his appreciation to the Nordic University Group for Physical Oceanography and to his Colleagues from Finland in particular for the cooperation in BOSEX 77. Financial support from the Danish Environmetal Protection Agency and the Danish Natural Science Council is also acknowledged.

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