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MAGNITUDE-SCALING OF EARTHQUAKES IN FENNOSCANDIA

by

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Abstract

Richter's original magnitude concept M_L is constructed for southern California events recorded with Wood-Anderson short-period torsion seismometers. The present paper proposes a technique for application of a concept as close to M_L as possible also to other areas and other types of instruments. Transformations are performed numerically for Fennoscandia and for the instruments in operation in Finland and Sweden. Derived expressions are $M_L = \log a + \log V(T) + 1.61 \log \Delta - 3.22$ for readings from Grenet-Coulomb short-period vertical-component instruments and $M_L = \log a + \log V(T) + 1.61 \log \Delta - 2.76$ for readings from Benioff short-period vertical-component instruments. a is the ground amplitude (μm), Δ is the epicentral distance (km) and $V(T)$ is the Wood-Anderson seismometer magnification at period T.

1. *Introduction*

Various methods already exist in Fennoscandia for calculation of magnitudes of regional earthquakes. Formulae in BÅTH [3] were used to assign macroseismic magnitudes to Fennoscandian earthquakes 1891–1950 (BÅTH [4]). KORHONEN [11] for Finnish earthquakes correlated the lengths of signals recorded at Sodankylä (SOD) with macroseismic magnitudes calculated from the formulae of BÅTH [3] and so derived a signal duration magnitude scale for this station. From a spectral approach BÅTH *et al.* [7] developed an amplitude-based magnitude scale for application to the Swedish seismograph station network.

By relating recorded amplitudes of crustal waves from near-located seismic events to magnitudes of a conventional scale determined from teleseismically recorded waves, amplitude-distance relations can be established and magnitudes can be calculated solely from crustal wave data (see *e.g.* TANER [20]). Unfortunately, this excellent and simple technique to refer to a conventional magnitude concept is not applicable to areas with small-size and rare-occurrence near events. Fennoscandia is one such area. Instead, RICHTER's [14] original magnitude concept for regional events, M_L, has to be used, and this involves some complications.

The M_L-scale was constructed with reference to a particular seismic region and a particular type of seismometer. The generalization of M_L to other regions or other types of instruments involves many steps, and the similarity to the original concept is impossible to preserve exactly. The present study endeavours to take these steps without withdrawing from the original concept more than necessary. Many regional magnitude scales based on instrumental amplitude readings do not aim at the greatest possible coincidence with M_L. These scales generally show high internal consistency, *i.e.* the relative sizes of the various events in the actual seismic region are accurately calculated, but at the sacrifice of feasible transformations to conventional scales. Used methods for regional magnitude calculation are reviewed in BÅTH [6], LEE and WETMILLER [12] and ADAMS [1].

The present work describes the generalization problems with particular application to Fennoscandian crust and existing instrument conditions. From observational data from Finnish and Swedish station records amplitude-distance relations are derived and a magnitude scale is established. The influence of individual stations is investigated. For Swedish earthquakes 1963–1976 a comparison with the scale of BÅTH *et al.* [7] is done.

The technique to be presented in the following chapters can easily be applied to other seismic regions or types of instruments.

2. *Generalization of the M_L-scale to other areas and types of seismometers*

»The magnitude of any shock is taken as the logarithm of the trace amplitude, expressed in microns, with which the standard short-period torsion seismometer (T_0 = 0.8 s, V = 2800, h = 0.8) would register that shock at an epicentral distance of 100 kilometers.» Starting with this definition of M_L, Richter gives amplitude values, $\log A_0$, corresponding to $M_L = 0$ at every 5 km within the epicentral distance range 25–600 km. Throughout the present paper log stands for logarithm to the base 10. The variation of $\log A_0$ with distance reflects the influence of two factors:

1. The attenuation of crustal waves in southern California. Events utilized for the determination of $\log A_0$ values are assumed to be located exclusively within the crust.

2. The instrumental amplitude response characteristics. Recorded periods of crustal waves usually increase with increasing distance, and since the definition is based on

trace amplitudes no adjustment is made for the variation of magnification with frequency. For practical applications, an amplitude-distance relation corresponding to the $\log A_0$ distribution has to be established from the conditions of the actual crust.

This work is based on records of short-period vertical-component instruments operated in the seismic networks in Finland and Sweden. One should be careful to select those trace amplitudes which yield the maximum amplitudes when transformed to the Wood-Anderson reference seismometer. Richter's M_L-concept and scales related to it include the term log (amplitude). The term log (amplitude/period) is used in many magnitude scales for near events. For short-period instruments like those used in Finland and Sweden, log (amplitude) is to be preferred according to ADAMS [1].

Richter's definition is based on maximum amplitude independent not only of recorded period but also of recorded wave type. The present study, however, is restricted to Sg-waves. For the material utilized, these waves show almost without exception the largest amplitudes and also yield the largest amplitudes when transformed to trace amplitudes of the reference seismometer. The derived concept is, therefore, in practice not diverging from the original M_L in this respect. The main reason for the restriction is the appearance of short-period surface waves, Rg, generated by near-surface events. Rg-waves are rapidly attenuated with increasing distance from the source. At short distances, say up to 200 km, Rg-waves usually have larger amplitudes than Sg-waves and other recorded waves from near-surface events. Such events are not included in the present study. Except for chemical explosions and rockbursts, Rg-waves are only rarely recorded at Finnish and Swedish stations. If readings of Rg should enter the material used for the construction of amplitude-distance relations, then magnitude calculations for a station near the epicentre would yield too large a value for near-surface events (Rg recorded and measured) and too small a value for deeper events (Rg not recorded, Sg measured). Pg-waves have smaller amplitudes than Sg-waves, except possibly at very short distances, say a few km, at which the scale is not applicable anyway.

Richter's $\log A_0$ values and calculated magnitudes were obtained from readings of horizontal-component seismometers. Most Finnish and Swedish stations are equipped only with short-period vertical-component seismometers. The scale established in the present paper is therefore based on readings from vertical-component instruments exclusively. These are more objective than the horizontal-component in the sense that the amplitudes of waves recorded by the latter vary with the station to source azimuth, while a directional dependence does not exist for the former, that is if we disregard possible focal mechanism influence and crust inhomogeneities. However, tentative studies and experience show that average amplitudes (stations evenly distributed around the epicentre) of crustal waves do not differ much between the vertical- and the horizontal-components (see also BÅTH *et al.* [7]). The change of components does therefore not introduce a severe divergence from the original M_L concept.

Last it should be noted, that since the M_L value is due to the expected amplitude at a certain non-zero distance (100 km), the same released seismic energy may give different M_L-values for different crustal regions, even if each medium were perfectly isotropic and each source energy radiation pattern were perfectly equal in all directions. Therefore, the magnitude, M_L, is by definition forced to be an index of energy in a relative sense only.

3. *Observational data*

This study uses readings from five Finnish Benioff, three Swedish Benioff and seven Swedish Grenet (Grenet-Coulomb) seismometers, all of which are short-period vertical-component instruments. Locations and instrument constants of stations have in some cases been slightly changed during the actual time interval. Table 1 gives data valid in 1974 for stations operated at that time. For stations closed before 1974, data valid at the last time of operation are given. Examples of amplitude response curves for Grenet and Benioff instruments are given together with the Wood-Anderson amplitude response curve in Fig. 1. Station locations are plotted in Fig. 2. All stations use photographic registration with a drum speed of 1 mm/s.

56 Baltic Shield earthquakes in the time interval 1958–1974 have been used. Source parameter data were obtained from SEISMIC EVENTS IN NORTHERN EUROPE [17], SEISMOLOGICAL BULLETIN, HELSINKI [18] and SEISMOLOGICAL BULLETIN, UPPSALA [19]. A renewed analysis of the records led to minor revisions of the bulletin data. Among other improvements, regional travel time tables, which did not exist when the older bulletins were prepared, were now used. Source parameters are given in Table 2. Epicentre locations are plotted together with station locations in Fig. 2.

The following requirements are set for the selection of data:

1. At least one station of each of the Finnish and Swedish networks have recorded the event.

2. At least three stations at epicentral distances $\geqslant$ 100 km within an instrument group (Grenet or Benioff) have recorded the event with trace amplitudes of Sg (that would yield maximum amplitudes by the standard seismometer) of the size $\geqslant$ 0.20 mm, but not so large such that there is a risk for missing high peaks.

Table 1. Seismograph station data.

Station	Country	Station coordinates		Seismograph	Seismometer period	Galvanometer period	Maximum magnification	Time of operation
Uppsala (UPP)	S	59.858°N	17.627°E	Benioff**)	1.0 s	0.7 s	40000	1955–
Kiruna (KIR)	S	67.840	20.417	Grenet	1.3	0.7	13310	1951–
Skalstugan (SKA)	S	63.580	12.280	Grenet	1.4	0.8	12040	1957–
Umeå (UME)	S	63.815	20.237	Grenet	1.4	0.7	12990	1960–61
Umeå (UME) *)				Benioff	1.0	0.7	75000	1962–
Uddeholm (UDD)	S	60.090	13.607	Grenet	1.4	0.7	12990	1966–67, Oct 20
Uddeholm (UDD)				Benioff	1.0	0.7	75000	1967, Oct 20–
Delary (DEL)	S	56.472	13.868	Grenet	1.4	0.7	12990	1968–
Göteborg (GOT)	S	57.698	11.978	Grenet	1.4	0.5	10530	1958–68
Karlskrona (KLS)	S	56.165	15.592	Grenet	1.5	0.7	11590	1961–68
Nurmijärvi (NUR) *)	F	60.509	24.655	Benioff	1.0	0.75	37000	1962–
Kajaani (KJN)	F	64.085	27.712	Benioff portable	1.0	0.75	74000	1964–70, Jul 9
Kajaani (KJF)	F	64.199	27.715	Benioff portable	1.0	0.75	37000	1970, Jul 9–
Sodankylä (SOD)	F	67.371	26.629	Benioff	0.9	0.2	77000	1956–
Kevo (KEV) *)	F	69.755	27.007	Benioff	1.0	0.75	37000	1960–

*) Belong to the WWSSN

**) Calibrated by comparison with Grenet and therefore belongs to the Grenet group (see chapter 4).

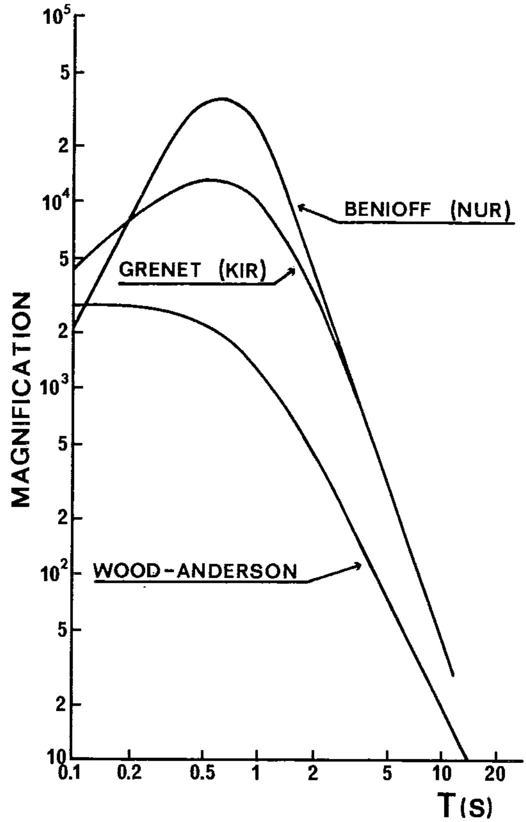

Fig. 1. Examples of amplitude response curves: Benioff at NUR and Grenet at KIR. Wood-Anderson short-period torsion seismometer amplitude response curve.

Richter also regards 0.20 mm as the reliable minimum value. The upper limit is determined by the exposure quality of the record. The amplitude in the Sg-wave train that would yield the largest trace amplitude on a Wood-Anderson standard seismometer record was picked, with its corresponding period, at each available record. Accuracies of measurements are: trace amplitudes ± 0.025 mm and periods ± 0.025 s. Calculated epicentral distances have an accuracy of ± 10 km.

The epicentral distance limit (100 km) is due to the great increase of amplitudes at small distances. Therefore, if data from small distances were included in a mathematically simple amplitude-distance relation covering a large distance range, then this relation would not be a good approximation towards either end of the distance range. To use different analytical expressions for different distance ranges, such as BAKER [2] does, for example, would be insignificant for this meagre material. The

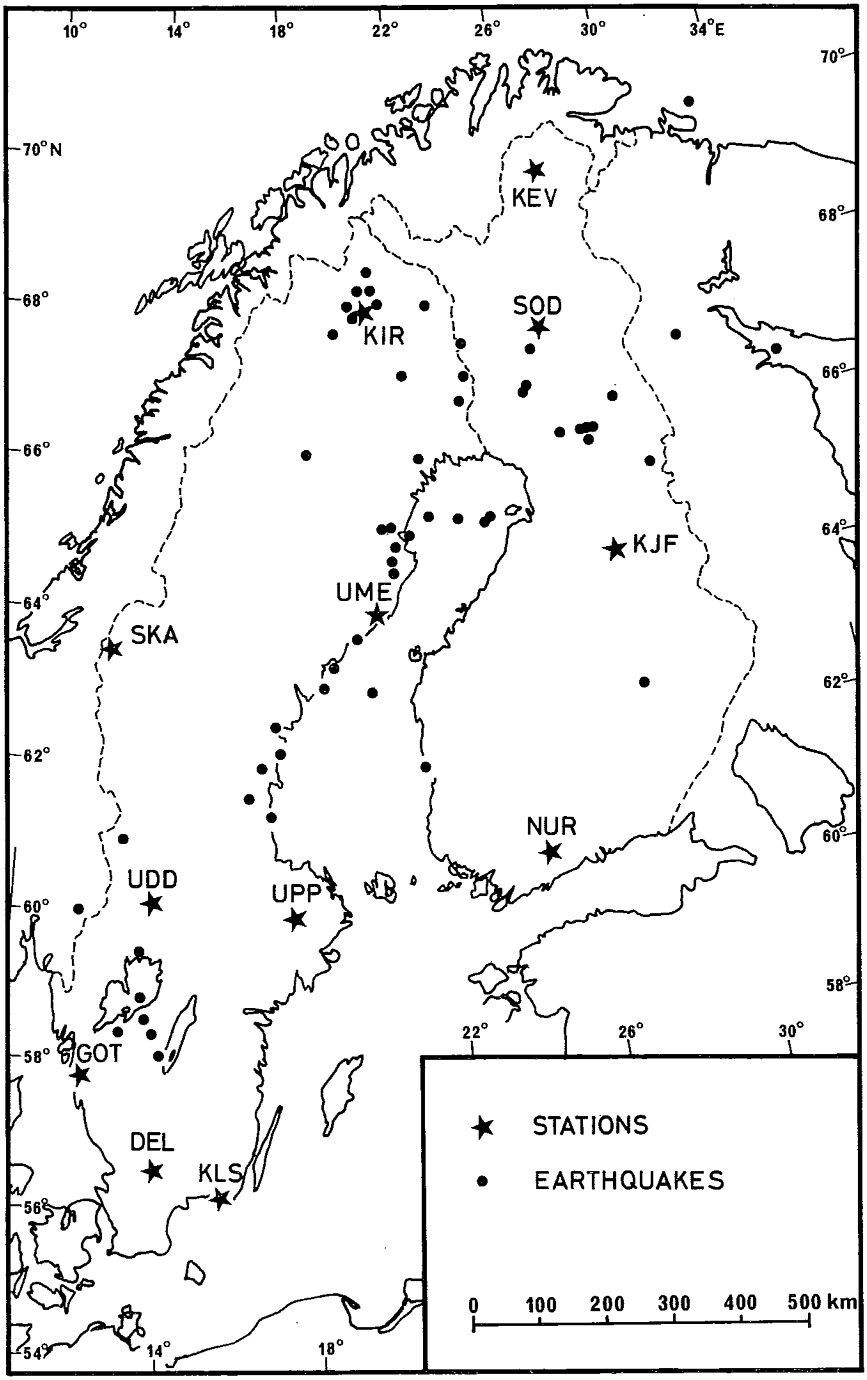

Fig. 2. Locations of epicentres and stations.

Table 2. Earthquake data.

Event number	Date	Origin time	Epicentral coordinates	
1	58 01 19	19 45	67.0°N	21.6°E
2	60 02 02	12 32	67.0	30.9
3	60 02 20	00 52	66.3	28.4
4	60 07 21	07 31	70.0	33.0
5	61 02 17	03 21	65.1	24.1
6	62 09 28	17 22	64.5	20.7
7	63 08 01	16 02	62.5	28.0
8	63 10 29	18 28	65.8	21.9
9	65 01 23	11 09	64.9	23.9
10	65 02 23	13 55	64.7	20.9
11	65 03 20	02 44	67.2	26.0
12	67 01 04	04 44	67.9	21.0
13	67 02 04	15 34	59.5	13.3
14	67 04 10	05 14	65.1	22.9
15	67 04 13	08 46	68.1	20.8
16	67 04 13	09 03	63.2	18.9
17	67 05 20	23 18	66.5	33.9
18	67 07 22	19 22	66.0	26.5
19	67 08 24	23 11	64.8	21.3
20	67 11 07	06 32	66.0	27.5
21	67 12 04	04 59	66.6	23.4
22	68 03 12	07 32	58.6	13.5
23	68 06 01	04 50	68.3	20.7
24	68 06 13	04 50	64.4	20.8
25	68 09 04	17 09	66.9	23.7
26	68 12 25	15 28	68.2	20.3
27	69 05 23	18 40	66.0	27.5
28	69 07 04	22 28	67.6	19.4
29	70 03 28	07 28	67.3	23.6
30	70 05 12	14 14	61.0	12.8
31	70 06 14	17 33	65.1	22.1
32	70 08 12	19 28	61.5	16.4
33	71 04 17	08 05	67.8	22.6
34	71 04 20	23 33	64.3	20.8
35	71 07 28	23 24	62.1	17.3
36	71 08 13	13 50	62.8	20.0
37	71 09 07	02 41	61.2	17.0
38	71 10 10	05 29	61.8	21.3
39	72 03 06	16 03	64.9	20.5
40	72 08 20	02 52	61.9	16.8
41	72 12 16	10 09	63.5	19.7
42	73 02 13	00 05	66.0	18.3
43	73 04 11	05 01	58.8	13.4
44	73 04 17	06 17	67.9	20.0
45	73 07 22	04 02	58.3	13.8
46	73 10 01	16 44	60.0	11.9
47	73 11 26	21 45	62.9	18.5
48	73 12 10	20 03	66.6	25.6
49	73 12 10	20 07	66.7	25.7
50	74 02 05	22 33	58.1	14.0

Table 2 (cont.)

Event number	Date	Origin time	Epicentral coordinates	
51	74 03 04	13 43	65.5	29.3
52	74 05 21	16 51	58.3	12.8
53	74 06 04	23 13	62.3	17.2
54	74 06 21	06 31	66.0	27.2
55	74 11 06	05 22	65.8	27.4
56	74 12 01	19 35	67.8	20.1

distance range restriction is an obstacle for the immediate application of the magnitude scale to distances < 100 km. Richter supplies a simple technique for magnitude determinations from readings at very small distances.

4. *Amplitude-distance relations*

Let us assume that for a limited seismic region

- all earthquakes within the region are located at approximately the same depth;
- the crust of the region is fairly homogeneous;
- the ground conditions beneath the stations within the region are similar;
- the azimuthal distribution of source energy radiation is fairly uniform;
- the relative distribution of energy with frequency is roughly similar for events of various sizes.

Then, the variation of wave amplitude with epicentral distance can, within the region, be expected to be similar for all the earthquakes. This means, that if for each shock the recorded ground amplitudes or log (ground amplitudes) are plotted versus distance or log (distance) and a curve is fitted to the points, then approximately parallel curves would result.

Richter found this »parallelism hypothesis» to be applicable for his data of southern California earthquakes during January 1932 recorded by the Wood-Anderson standard seismometers of the Pasadena network. Since he used only one kind of instrument, he could use trace amplitudes instead of ground amplitudes. Thus, he obtained zero-magnitude values, $\log A_0$, as discrete values on a curve parallel to the amplitude-distance curves and passing through the zero-magnitude definition point, *i.e.* a trace amplitude of 1 μm at a distance of 100 km. Magnitudes were then calculated as average values of M_L from

$$M_L = \log A - \log A_0 \tag{1}$$

for available recorded maximum amplitudes A.

A-values are either read directly from Wood-Anderson instrument records or are transformed for readings from other types of instruments – see BÅTH [6], LEE and WETMILLER [12] and ADAMS [1]. At several stations or networks in various parts of the world the numerical $\log A_0$ values in RICHTER's [14] Table I or RICHTER's [15] Table 22-1 are (or have been) used for the magnitude calculations. This introduces an error, for stations not located in southern California, which is not easy to estimate. TOCHER [22] showed that areas as near each other as southern and central California have clearly different amplitude-distance relations. For many other areas, for instance Fennoscandia as shown by the present study, the differences in this respect from southern California are significant. $\log A_0$ values must therefore be based on the attenuation of the actual crust.

The Baltic Shield is a homogeneous region, and at least the second assumption in the beginning of this chapter should be no obstacle for the validity of the parallelism hypothesis. If the amplitude-distance data from the present study are plotted separately for the two instrument types, then roughly parallel curves are obtained. For each event where the data yield both a Grenet and a Benioff amplitude-distance curve, the latter is below the former. Due to the difference, the observational material for the present study is divided into two groups: Grenet readings and Benioff readings. UPP, although equipped with Benioff, belongs to the Grenet group since the response characteristics of the seismometer have been determined by a comparison with a Grenet seismometer, which was in operation simultaneously.

That Benioff readings yield small magnitudes is known from several previous studies. Body wave magnitudes calculated from Benioff readings are in average about 0.75 units smaller than magnitudes determined from Wood-Anderson readings at Berkeley and Pasadena (ROMNEY [16]). BÅTH [5] reported that body wave magnitudes calculated from UPP and KIR short-period vertical-component seismometers – the latter a Grenet, the former calibrated with a Grenet – are up to 0.7 units larger than magnitudes calculated from the WWSSN operating Benioff instruments. BUNE *et al.* [8] observed a difference of this size for body wave magnitudes calculated from Kirnos instruments in the USSR and from instruments of the WWSSN. ROMNEY [16], BÅTH [5] and BUNE *et al.* [8] all refer to large teleseismic events. The present study shows a similar relation between Grenet and Benioff magnitudes for small near events in Fennoscandia.

The physical explanation of the differences comes from the different instrumental band widths. The Benioff short-period instrument has a narrower band than the Grenet and Wood-Anderson instruments.

CHRISTOSKOV [9] found a dependence of the absolute magnitude for PH- and SH-waves at small distances upon the amplitude-distance relation – the smaller the magnitude the more rapidly do amplitudes decrease with increasing distance. He postulated that a similar dependence may also be expected for pure crustal waves, such as Sg. This presumptive factor of violence on the parallelism hypothesis (the last assumption in the beginning of this chapter) is here ignored since the magnitude range of Fennoscandian earthquakes is relatively narrow. The curves of the shocks are, as mentioned, found to be roughly parallel.

An amplitude-distance relation can be expressed

$$\log a = \sigma(\Delta) + m_j \tag{2}$$

where a is the ground amplitude, Δ the epicentral distance, σ a distance function and m_j constant for event j indicating its relative size. Three $\sigma(\Delta)$ functions are applied for each instrument group

$$\sigma_1(\Delta) = k_1 \cdot \Delta \tag{3.1}$$

$$\sigma_2(\Delta) = k_2 \cdot \Delta^2 + k_3 \cdot \Delta \tag{3.2}$$

$$\sigma_L(\Delta) = k_4 \cdot \log \Delta \tag{3.3}$$

Various $\sigma(\Delta)$ functions have been used in various studies, and (3.3) is by far the most frequently occurring (see BÅTH [6], LEE and WETMILLER [12] and ADAMS [1]). Observations are treated in least-squares solutions (WAHLSTRÖM [23]) yielding coefficients $k_1, \ldots, k_4$ (Table 3a).

Obtained ground amplitudes are transformed to ground amplitudes that would have been recorded at a distance of 100 km, *i.e.* from (2)

$$\log a_{100} = \log a - \sigma(\Delta) + \sigma(100) \tag{4}$$

where a_{100} is a at Δ = 100 km. a_{100} values are then converted to presumptive standard seismometer trace amplitudes at this distance, A_{100},

$$\log A_{100} = \log a + \log V(T) - \sigma(\Delta) + \sigma(100) \tag{5}$$

where T is the period measured at distance Δ and $V(T)$ is the standard seismometer magnification at period T. Putting $M_L = \log A_{100}$, as prescribed in the definition, a scale based on (5) was developed for Fennoscandian events by BÅTH *et al.* [7].

Table 3a. Coefficients of $\sigma(\Delta)$ functions.

Instrument type	k_1	k_2	k_3	k_4	number of events	number of readings
Grenet	$-1.01 \cdot 10^{-3}$	$1.03 \cdot 10^{-6}$	$-2.56 \cdot 10^{-3}$	-1.49	27	101
Benioff	$-1.44 \cdot 10^{-3}$	$0.84 \cdot 10^{-6}$	$-2.40 \cdot 10^{-3}$	-1.47	46	181

As will be shown in the next chapter, application of (5) implies a deviation from the M_L concept giving too small magnitudes.

5. M_L-*scale*

According to (1), M_L should be calculated as the difference between the logarithms of the trace amplitude and the zero-magnitude amplitude at the same distance, both amplitudes related to the standard seismometer. The zero-magnitude values can not be obtained directly from the $\sigma(\Delta)$ functions. These express the ground amplitude-distance dependence. The period is in general increasing with increasing distance, and the Wood-Anderson seismometer exhibits a continuous decrease in magnification with increasing T (see DI FILIPPO and MARCELLI [10] or the ISC manual, WILLMORE and KÁRNÍK [24]). The combined effect is that $\log A_0$ values decrease more rapidly than $\sigma(\Delta)$ with increasing distance. Consequently, at distances greater than the definition distance of 100 km, magnitudes in general are too small when calculated from (5). To express the above in another way: when ground amplitude calculated at a distance of $\Delta > 100$ km is transferred to ground amplitude at a distance of $\Delta = 100$ km and then to standard seismometer trace amplitude at $\Delta = 100$ km, T in (5) should correspond to $\Delta = 100$ km. What we have is T measured at a distance of $\Delta > 100$ km, which is in general greater than T at $\Delta = 100$ km. Therefore, too small a presumptive Wood-Anderson trace amplitude at $\Delta = 100$ km, *i.e.* too small M_L, results.

The scale of BÅTH *et al.* [7] implicitly contains the assumption that the variation of T with Δ is negligible. To be as true as possible to the original M_L concept this assumption is not made in the present study. Two ways are then available:

- to investigate and include the $T-\Delta$ dependence;
- to develop functions, $\sigma_{ss}(\Delta)$, that describe the variation of presumptive standard seismometer trace amplitude, A, with distance, and then replace $\sigma(\Delta)$ terms in (5) with $\sigma_{ss}(\Delta)$ terms.

Each approach is successful only if the actually recorded periods would have been the same if recorded by the standard seismometer. This assumption is made and is in accordance with recommendations from the ISC (WILLMORE and KÁRNÍK [24]). It is a reasonable assumption for the short-period Grenet and Benioff instruments. Readings from local networks equipped with instruments with peak response at considerably higher frequencies must undergo a normalization (THATCHER [21]).

In this study the second approach is chosen. Analogous with $\sigma(\Delta)$ functions three $\sigma_{ss}(\Delta)$ functions are applied for each instrument group

$$\sigma_{ss1}(\Delta) = k_{ss1} \cdot \Delta \tag{6.1}$$

$$\sigma_{ss2}(\Delta) = k_{ss2} \cdot \Delta^2 + k_{ss3} \cdot \Delta \tag{6.2}$$

$$\sigma_{ssL}(\Delta) = k_{ss4} \cdot \log \Delta \tag{6.3}$$

$k_{ss1},..., k_{ss4}$ (Table 3b) are derived by replacing the ground amplitudes used in the derivation of $k_1,..., k_4$ with standard seismometer trace amplitudes at the very same distances and then perform the least-squares solutions.

Table 3b. Coefficients of $\sigma_{ss}(\Delta)$ functions.

Instrument type	k_{ss1}	k_{ss2}	k_{ss3}	k_{ss4}	number of events	number of readings
Grenet	$-1.14 \cdot 10^{-3}$	$1.00 \cdot 10^{-6}$	$-2.63 \cdot 10^{-3}$	-1.64	27	101
Benioff	$-1.54 \cdot 10^{-3}$	$0.98 \cdot 10^{-6}$	$-2.65 \cdot 10^{-3}$	-1.58	46	181

Analogous to equations (4) and (5)

$$M_L = \log A_{100} = \log A - \sigma_{ss}(\Delta) + \sigma_{ss}(100) =$$
$$= \log a + \log V(T) - \sigma_{ss}(\Delta) + \sigma_{ss}(100) \tag{7}$$

By this equation the ground amplitude, a, observed at Δ is converted to the trace amplitude, A, which would be recorded at the same distance by the Wood-Anderson seismometer. A is then transferred to the trace amplitude, A_{100}, at Δ = 100 km. M_L is properly obtained by using the measured period, T.

Subtracting (5) from (7) gives

$$M_L(\text{diff}) = -\sigma_{ss}(\Delta) + \sigma(\Delta) + \sigma_{ss}(100) - \sigma(100) \tag{8}$$

For the Fennoscandian data $k_{ss1} < k_1$ and $k_{ss4} < k_4$ for both instrument groups and $k_{ss2} < k_2$ and $k_{ss3} < k_3$ for the Grenet group. In these cases M_L(diff) increases with increasing distance and is positive for distances greater than 100 km. For function (6.2) and the Benioff group, where $k_{ss2} > k_2$ and $k_{ss3} < k_3$, the numerical values yield increasing M_L(diff) with increasing distance up to approximately 940 km and then decreasing M_L(diff). M_L(diff) is positive for distances between 100 km and approximately 1780 km. Therefore, within the whole relevant distance range all six $\sigma_{ss}(\Delta)$ functions yield larger M_L values than corresponding $\sigma(\Delta)$ functions, as expected. M_L values obtained from the various $\sigma_{ss}(\Delta)$ functions are given in WAHLSTRÖM [23].

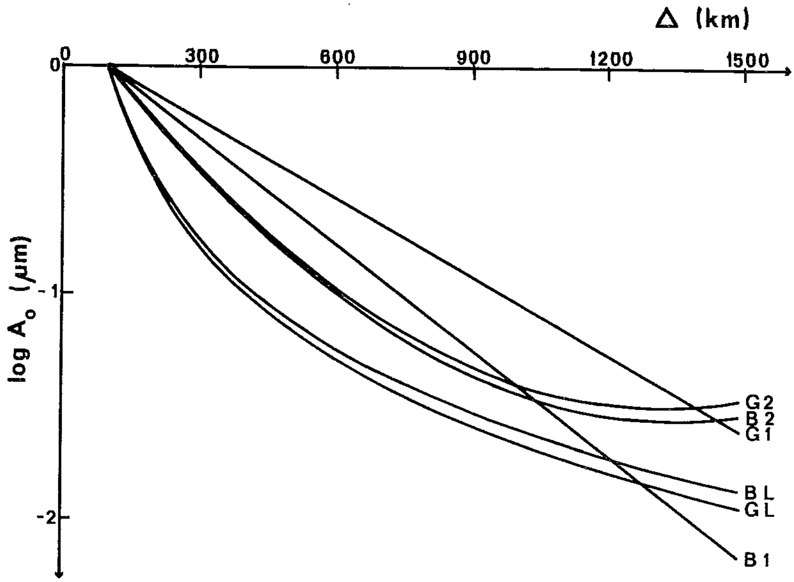

Fig. 3. Zero-magnitude functions. G1: 1st order approximation for Grenet; G2: 2nd order approximation for Grenet; GL: log-function approximation for Grenet. Analogous for Benioff: B1, B2 and BL.

The zero-magnitude functions are obtained by putting $M_L = 0$ in (7)

$$\log A_0 = \sigma_{ss}(\Delta) - \sigma_{ss}(100) \tag{9}$$

Comparing the various $\log A_0$ functions, *e.g.* in a plot such as Fig. 3, it is found that magnitudes calculated from $\sigma_{ssL}(\Delta)$ are the largest within each instrument group, except for Benioff at distances above approximately 1200 km. Mean values of differences in magnitude are for the Grenet group events

$$M_L \text{ for } \sigma_{ss2}(\Delta) = 0.31 + M_L \text{ for } \sigma_{ss1}(\Delta) \tag{10.1}$$

$$M_L \text{ for } \sigma_{ssL}(\Delta) = 0.61 + M_L \text{ for } \sigma_{ss1}(\Delta) \tag{10.2}$$

and for the Benioff group events

$$M_L \text{ for } \sigma_{ss2}(\Delta) = 0.15 + M_L \text{ for } \sigma_{ss1}(\Delta) \tag{11.1}$$

$$M_L \text{ for } \sigma_{ssL}(\Delta) = 0.40 + M_L \text{ for } \sigma_{ss1}(\Delta) \tag{11.2}$$

To bring better agreement between magnitudes obtained from the various $\sigma_{ss}(\Delta)$ functions within an instrument group, the normalization can be done at a more

»representative» distance than 100 km. Say Δ_R is a »representative» distance of a sample. Then normalizing at $\Delta = \Delta_R$ means that $\log A_0$ has the same value at Δ_R for all the three $\sigma_{ss}(\Delta)$ functions within a group. This value can for example be taken as the mean of the $\log A_0$ values at Δ_R for the three functions. However, normalization at a distance not equal to 100 km violates the definition of M_L and implies that a standard seismometer trace amplitude of 1 μm recorded at a distance of 100 km does not in general give $M_L = 0$. Therefore, no such normalization is undertaken in the present study.

The linear $\log a$-$\log \Delta$ relation obtained by NUTTLI [13] from empirical data was found to be a fair approximation, in limited distance intervals, of what is expected theoretically of the combined effect of spherical spreading and anelastic attenuation. As mentioned, this design of empirical relation is utilized in most amplitude-based magnitude scales. It is also accepted in this study.

The data contain 17 events for which magnitudes are calculated from both Grenet and Benioff readings. Mean values of differences in magnitude calculated for these events are

$$\text{for } \sigma_{ss1}(\Delta): M_L \text{ for Grenet} = 0.28 + M_L \text{ for Benioff} \tag{12.1}$$

$$\text{for } \sigma_{ss2}(\Delta): M_L \text{ for Grenet} = 0.44 + M_L \text{ for Benioff} \tag{12.2}$$

$$\text{for } \sigma_{ssL}(\Delta): M_L \text{ for Grenet} = 0.51 + M_L \text{ for Benioff} \tag{12.3}$$

k_{ss} coefficients are intended to reflect the behaviour of standard seismometers in the Fennoscandian crust. The values should be independent of the type of instrument from which they are derived (in the absence of standard seismometers). Since priority can be given to neither of Grenet or Benioff concerning the obtained k_{ss} values, the relation

$$\overline{k_{ss}} = \frac{(k_{ss})\text{ Grenet} + (k_{ss})\text{ Benioff}}{2} \tag{13}$$

is used. Then,

$$\overline{k_{ss4}} = -1.61 \tag{14}$$

for the applied function, $\overline{\sigma_{ssL}}(\Delta)$. Recalculating magnitudes using this value

$$M_L \text{ for Grenet} = 0.46 + M_L \text{ for Benioff} \tag{15}$$

is acquired for $\overline{\sigma_{ssL}}(\Delta)$ as a substitute for (12.3) for the 17 events included in both instrument groups. The value 0.46 is ascribed to the instrumental band width difference.

To be consistent with Berkeley and Pasadena magnitudes and with magnitudes for teleseismic events calculated at UPP and KIR and reported in the Uppsala monthly bulletins, preference is given to Grenet magnitudes. Equation (7) may now be reformulated for the instruments discussed

$$\text{Grenet: } M_L = \log a + \log V(T) - \overline{\sigma_{ssL}}(\Delta) + \overline{\sigma_{ssL}}(100) \qquad (16.1)$$

$$\text{Benioff: } M_L = \log a + \log V(T) - \overline{\sigma_{ssL}}(\Delta) + \overline{\sigma_{ssL}}(100) + \text{constant} \qquad (16.2)$$

From equations (14), (15), (16.1) and (16.2), the data of the present study yield

$$\text{Grenet: } M_L = \log a + \log V(T) + 1.61 \log \Delta - 3.22 \qquad (17.1)$$

$$\text{Benioff: } M_L = \log a + \log V(T) + 1.61 \log \Delta - 2.76 \qquad (17.2)$$

In practice, the magnitude of an event is calculated as the mean of M_L values obtained from available Grenet and Benioff readings. The magnitude standard deviation is calculated from

$$\text{S.D.} = \sqrt{\frac{\sum_S (M_{L,S} - M_L)^2}{N-1}} \qquad (18)$$

where M_L is the event magnitude, $M_{L,S}$ stands for magnitude for station S and N is the number of recording stations. Magnitudes, together with standard deviations, of the 56 events are given in Table 4.

Table 4. Magnitudes calculated using formulae (17.1) and (17.2).

Event number	M_L with standard deviation		Number of stations used
1	2.82	± 0.03	3
2	4.45	± 0.17	4
3	3.86	± 0.11	3
4	3.32	± 0.12	3
5	3.57	± 0.18	4
6	3.82	± 0.08	5
7	3.50	± 0.19	5
8	3.16	± 0.06	3
9	3.31	± 0.08	8
10	2.59	± 0.49	3
11	3.30	± 0.21	7
12	3.09	± 0.10	7
13	3.61	± 0.21	7
14	3.12	± 0.15	8
15	3.55	± 0.12	5
16	3.49	± 0.22	9
17	5.08	± 0.07	6

Table 4 (cont.)

Event number	M_L with standard deviation		Number of stations used
18	2.70	± 0.17	3
19	2.44	± 0.28	3
20	2.79	± 0.18	3
21	2.40	± 0.18	4
22	3.06	± 0.25	7
23	2.74	± 0.11	4
24	2.81	± 0.06	3
25	3.33	± 0.19	10
26	2.61	± 0.26	3
27	3.25	± 0.15	9
28	3.03	± 0.22	5
29	2.74	± 0.38	4
30	3.13	± 0.18	9
31	2.99	± 0.22	5
32	2.99	± 0.18	5
33	3.16	± 0.24	6
34	3.07	± 0.24	7
35	3.03	± 0.15	5
36	2.29	± 0.32	3
37	2.69	± 0.23	3
38	2.71	± 0.17	3
39	2.37	± 0.55	3
40	2.97	± 0.26	3
41	2.79	± 0.18	8
42	3.23	± 0.20	5
43	3.85	± 0.20	8
44	3.32	± 0.34	6
45	3.07	± 0.28	4
46	3.01	± 0.16	3
47	3.17	± 0.14	8
48	3.38	± 0.16	3
49	2.80	± 0.07	4
50	2.85	± 0.24	4
51	2.36	± 0.37	4
52	3.35	± 0.24	7
53	3.63	± 0.15	6
54	3.50	± 0.29	6
55	2.33	± 0.43	3
56	3.13	± 0.23	3

If a network consists of not one or two but three or more different types of seismometers, then all steps in this chapter can still be performed in an analogous way, only the reference instrument (in this case Grenet) is selected.

6. *Station corrections*

One of the assumptions for the parallelism hypothesis is the similarity of the station conditions. The biasing of M_L determined at individual stations is investigated by calculating C_S from the usually applied relation

$$C_S = -\frac{\sum_j (M_{L,S,j} - M_{L,j})}{N} \tag{19}$$

where C_S is the station correction for station S, $M_{L,j}$ the magnitude for event j, $M_{L,S,j}$ the magnitude for station S for event j and N the number of events for which M_L has been calculated at station S. C_S reflects the structural properties of the crust beneath S and the instrument characteristics. At sites where both Grenet and Benioff seismometers have been in operation, one C_S is therefore assigned to each instrument.

Results of C_S calculations are in WAHLSTRÖM [23]. Except for DEL, with seven available readings only, and UME Grenet, where only one reading is available and therefore no standard deviation is attainable, all obtained values are within 1 standard deviation from $C_S = 0$. Therefore, it does not seem to be motivated to add station correction terms to (17.1) or (17.2). A future study comprehending more data might yield revisions on this point. It can be mentioned that the largest station correction found by Richter using the relation (19) is 0.40. A more detailed study of corrections due to factors like the location of an event, station to source azimuth, epicentral distance, etc. is here omitted. To make such an investigation meaningful, it is necessary to have a more extensive set of data.

7. *Application to Swedish earthquakes*

Magnitudes for Swedish earthquakes 1963–1976 are calculated, with standard deviations, from the formulae (17.1) and (17.2) and from the scale of BÅTH *et al.* [7] (WAHLSTRÖM [23]). Due to different limitations for the application of the various scales, corresponding magnitudes are for some events not based on the same set of amplitude readings. For 123 of the altogether 158 events recorded and located during these 14 years, magnitudes obtained from the two scales are calculated from the same data set. The mean difference of magnitude for these events is

$$M_L(\text{present study}) - M_L(\text{BÅTH } et\ al.\ [7]) = 0.07 \tag{20}$$

For 97 of the 123 events, on which this relation is based, standard deviations of magnitudes are obtained. For the remaining events only one reading is available. The pooled variances (VAR) of magnitudes of the 97 events are calculated from

$$\mathrm{VAR} = \frac{\sum_j \sum_i (M_{L,i,j} - M_{L,j})^2}{\sum_j (n_j - 1)} \tag{21}$$

where i is station index, j is event index and n_j is number of stations on which $M_{L,j}$ is based. Then,

from the scale of BÅTH *et al.* [7] $\sqrt{\mathrm{VAR}} = 0.31$ (22.1)

from the scale of the present study $\sqrt{\mathrm{VAR}} = 0.26$ (22.2)

The most marked contribution to the greater dispersion and also to the smaller M_L values from the scale of BÅTH *et al.* [7] is, that for this scale $M_{L,i,j}$ obtained from readings of large periods are often considerably smaller than the mean value, $M_{L,j}$, of corresponding events. For commonly appearing periods (0.30 – 0.50 s), the differences in obtained magnitudes are small between the two scales.

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