

**RUBIDIUM MAGNETOMETER AND ITS SIMULTANEOUS USE  
FOR RECORDING PULSATIONS AND OBSERVATORY  
ONE-MINUTE VALUES**

by

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**A b s t r a c t**

The principle of operation of the rubidium magnetometer is described and its resonance frequency calculated basing on quantum mechanics. A three-component Rb magnetometer normally used in recording magnetic pulsations is also being used in the production of absolute one-minute values. The recorded values are shown here to be slightly dependent on other components, and the necessary corrections are given. The final absolute values are obtained by comparing calculated Rb values with conventional recordings made at quiet times. This method is found to produce digital one-minute or shorter values with higher absolute accuracy than methods such as curve digitizing.

*1. Principle of operation of the rubidium magnetometer sensor*

The operation of the rubidium magnetometer is based on the Zeeman effect, *i.e.* the splitting of the spectral lines of atoms in an external magnetic field. The energy difference between states that originally had equal energies depends on the magnitude of the external magnetic field, and this is the energy difference that is measured with a rubidium magnetometer sensor.

The spectral symbol for the ground state of rubidium is  $5^2 S_{1/2}$  (e.g. ENGELHARD, [9], p. 19), i.e.  $L = 0$  and  $S = J = \frac{1}{2}$ , where  $L$ ,  $S$  and  $J$  are the orbital, spin and total angular momentum quantum numbers of the electrons, or more precisely those of the single (valence) electron outside the closed shells. The transition from the ground state to the excited state  $2^2 P_{1/2}$  corresponds to an optical line whose wave length is 794.76 nm. This line is called the  $D_1$  line (e.g. ENGELHARD, [9], p. 19). The splitting of the energy levels caused by an external magnetic field can be calculated basing on perturbation theory (MERZBACHER, [15], p. 413): The Hamiltonian operator of the quantum mechanics includes two parts: an unperturbed part and a perturbation. Let  $H_p$  be the perturbation. In addition to a term taking the effect of the magnetic field into account,  $H_p$  includes a hyperfine structure term defining the interaction between the nuclear magnetic moment and the magnetic field created by the electrons at the nucleus (SOBEL'MAN, [20], p. 209). (According to KELLOGG and MILLMAN, [13], p. 343, there can be no quadrupole interaction between the nucleus and the electrons if  $J = \frac{1}{2}$ , as in the case of the rubidium states  $2^2 S_{1/2}$  and  $2^2 P_{1/2}$ .)

The total angular momentum  $\vec{F}$  of an atom is the sum of the total angular momentum of the electrons  $\vec{J}$  and nuclear spin  $\vec{I}$ :  $\vec{F} = \vec{I} + \vec{J}$ . The possible values of the corresponding quantum number  $F$  are  $I + J, I + J - 1, \dots, |I - J|$ . Hence when  $J = \frac{1}{2}$ ,  $F = I + \frac{1}{2}$  or  $F = I - \frac{1}{2}$  (if  $I \geq \frac{1}{2}$ ). The magnetic quantum number  $M_F$ , which is associated with  $F$ , has the values  $F, F-1, \dots, -F+1, -F$  (STUART, [22], p. 821).

The quantum states of an atom can be characterized by a group of quantum numbers. The state functions are denoted by  $|JIFM_F\rangle$ , where only quantum numbers are expressed (note that  $L$  and  $S$  are implicit). All  $(2I + 1)(2J + 1)$  states  $|JIFM_F\rangle$  ( $F = I + J, \dots, |I - J|$ ;  $M_F = F, \dots, -F$ ) are degenerate, i.e. they have equal energies, unless the perturbation  $H_p$  is taken into account. In calculating the effect of  $H_p$  therefore, degenerate perturbation theory must be used (MERZBACHER, [15], p. 425). In principle this means that, to obtain the first order corrections for the energy, the matrix of  $H_p$  with respect to the degenerate states must be diagonalized, i.e. the roots of the corresponding secular equation must be solved. In calculating the matrix elements,  $H_p$  can be expressed as follows:

$$H_p = \lambda \vec{I} \cdot \vec{J} + \frac{\mu_B g_J}{\hbar} \vec{J} \cdot \vec{B} + \frac{\mu_B g_I}{\hbar} \vec{I} \cdot \vec{B} \quad (1)$$

where  $\mu_B$  is the Bohr magneton,  $2\pi\hbar = h$  is Planck's constant,  $\vec{B}$  is the external magnetic field,  $g_J$  and  $g_I$  are the electronic and nuclear  $g$  factors respectively, and  $\lambda$  is a constant (WHITE *et al.*, [23], p. 23; MERZBACHER, [15], p. 434). (The  $g$  factors are defined as the negative proportionality factor between the magnetic moment expressed in units of  $\mu_B$  and the angular momentum expressed in units

of  $\hbar$ . Thus the  $g_J$  value of rubidium in its ground state is positive, while the  $g_I$  value of rubidium is negative (KELLOGG and MILLMAN, [13], p. 343). It can be seen that if  $J = \frac{1}{2}$ , the only off-diagonal matrix elements are

$$\left\langle \frac{1}{2} I I \pm \frac{1}{2} M_F | H_p | \frac{1}{2} I I \mp \frac{1}{2} M_F \right\rangle,$$

where  $M_F = I - \frac{1}{2}, I - \frac{3}{2}, \dots, -(I - \frac{1}{2})$ . So the diagonalization is not difficult (cf. MERZBACHER, [15], p. 435). The final result, giving the energy corrections of the various Zeeman sublevels, is the Breit-Rabi formula:

$$E_{I \pm \frac{1}{2}, M_F} = -\frac{\Delta W}{2(2I+1)} + \mu_B g_I M_F B \pm \frac{\Delta W}{2} \sqrt{1 + \frac{4M_F}{2I+1} x + x^2} \quad (2)$$

The quantity  $\Delta W$ , which depends on  $\lambda$ , is the difference between the energies of the two states  $I + \frac{1}{2}$  and  $I - \frac{1}{2}$  in a zero magnetic field, i.e. their hyperfine separation and

$$x = \frac{(g_J - g_I) \mu_B B}{\Delta W} \quad (3)$$

(KELLOGG and MILLMAN, [13], p. 343; BREIT and RABI, [6]). The possible values of  $M_F$  in equation (2) are  $I + \frac{1}{2}, I - \frac{1}{2}, \dots, -(I + \frac{1}{2})$  for  $I + \frac{1}{2}$ , and  $I - \frac{1}{2}, I - \frac{3}{2}, \dots, -(I - \frac{1}{2})$  for  $I - \frac{1}{2}$ . (KELLOGG and MILLMAN, [13], have written the energy in a different form for  $I + \frac{1}{2}$  when  $M_F = \pm(I + \frac{1}{2})$ , and in fact the inclusion of these in equation (2) requires that  $|x| < 1$ .) Owing to the non-vanishing off-diagonal matrix elements mentioned above, the states  $|\frac{1}{2} I I + \frac{1}{2} M_F\rangle$  and  $|\frac{1}{2} I I - \frac{1}{2} M_F\rangle$ , where  $M_F = I - \frac{1}{2}, \dots, -(I - \frac{1}{2})$ , are mixed by the perturbation. So the states corresponding to the energies  $E_{I \pm \frac{1}{2}, M_F}$  ( $M_F = I - \frac{1}{2}, \dots, -(I - \frac{1}{2})$ ) are not pure  $F = I + \frac{1}{2}$  states, and those corresponding to the energies  $E_{I - \frac{1}{2}, M_F}$  are not pure  $F = I - \frac{1}{2}$  states. But as the external magnetic field approaches zero, the former states become pure  $F = I + \frac{1}{2}$  states and the latter pure  $F = I - \frac{1}{2}$  states. For this reason the states associated with  $E_{I \pm \frac{1}{2}, M_F}$  can be called  $F = I + \frac{1}{2}$  states and those associated with  $E_{I - \frac{1}{2}, M_F}$  can be called  $F = I - \frac{1}{2}$  states. Note that both  $^2S_{1/2}$  and  $^2P_{1/2}$  have splittings of the kind shown in formula (2); in deriving the formula, the only assumption was that  $J = \frac{1}{2}$ . Figure 1 depicts the behaviour of the energies  $E_{I \pm \frac{1}{2}, M_F}$  (upper lines) and  $E_{I - \frac{1}{2}, M_F}$  (lower lines) for the ground state of  $\text{Rb}^{85}$ , as a function of the external magnetic field; the quantities in equation (2) have the following values:

$$\Delta W = 2.011351849 \times 10^{-24} \text{ J}$$

$$I = \frac{5}{2}$$

$$\mu_B = 0.92732 \times 10^{-23} \text{ Am}^2$$

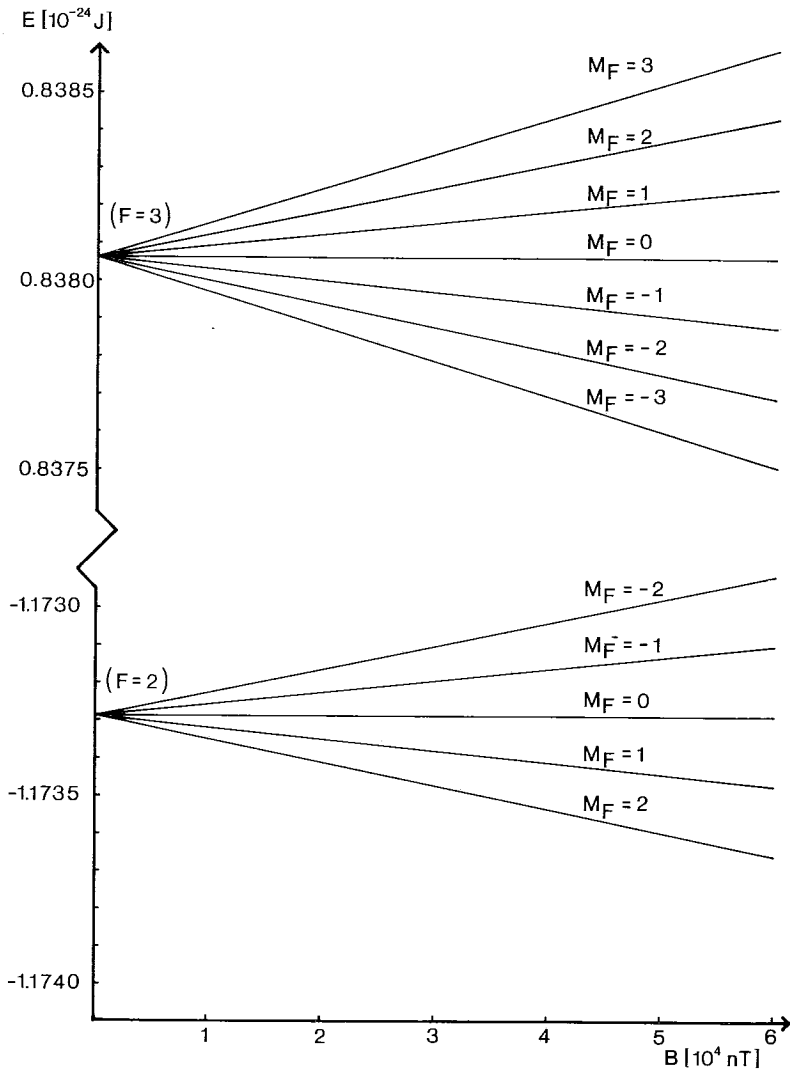


Fig. 1. Behaviour of the energy corrections for the ground state of rubidium-85, as a function of an external magnetic field. The upper and lower lines depict the states arising from the hyperfine levels  $F = 3$  and  $F = 2$ , respectively.

$$\frac{g_I}{g_J} = -0.2936400 \times 10^{-3} \text{ (WHITE } et al., [23], p. 29)$$

$$\frac{g_I}{g_J} = -1.4664908 \times 10^{-4} \text{ (WHITE } et al., [23], p. 27)$$

The hyperfine separation  $\Delta W$  was calculated from the equation  $\Delta W = h\Delta f$ , where

$h = 6.62559 \times 10^{-34}$  Js, and  $\Delta f = 3035732439$  Hz (PENSELIN *et al.*, [17]). The values of the magnetic field  $B$  in Fig. 1 are confined to those of the geomagnetic field, and here the energies seem to be almost linear functions of  $B$  (*cf.* KELOGG and MILLMAN, [13], Fig. 9).

The energy difference  $\Delta E$  between two Zeeman sublevels can easily be calculated from the Breit-Rabi formula (2). The frequency is obtained by dividing  $\Delta E$  by Planck's constant  $h$ , as follows:

$$f = \frac{\mu_B g_I}{h} B + \frac{\Delta W}{2h} \left( \sqrt{1 + \frac{4M_F}{2I+1} x + x^2} - \sqrt{1 + \frac{4(M_F-1)}{2I+1} x + x^2} \right) \quad (4)$$

Equation (4) corresponds to the transition  $M_F \leftrightarrow M_F - 1$  in the upper hyperfine  $F = I + \frac{1}{2}$  level. (Frequency is always non-negative. So, in principle,  $f =$  the absolute value of the right-hand side of (4).) If  $x$  is small enough, the square roots in equation (4) can be approximated by the first terms of a binomial series, so that

$$f = aB + bB^2, \quad (5)$$

where

$$a = \frac{\mu_B}{h(2I+1)} (g_J - 2I g_I) \quad (6)$$

and

$$b = -\frac{\mu_B^2}{h\Delta W} \frac{2M_F-1}{(2I+1)^2} (g_J - g_I)^2 \quad (7)$$

Equation (5) is valid if the magnitude of  $B$  is close to the geomagnetic field.

Either the isotope  $\text{Rb}^{85}$  ( $I = \frac{5}{2}$ ) or the isotope  $\text{Rb}^{87}$  ( $I = \frac{3}{2}$ ) can be used in rubidium magnetometers. According to ALLEN, [1], p. 198, the following values hold good for the ground state  $^2S_{1/2}$  of  $\text{Rb}^{85}$

$$a = 4.66737 \frac{\text{Hz}}{\text{nT}}$$

and

$$b = - \left\{ \begin{array}{c} 5 \\ 3 \\ 1 \\ -1 \\ -3 \\ -5 \end{array} \right\} \times 7.1887 \times 10^{-9} \frac{\text{Hz}}{(\text{nT})^2}$$

The corresponding values for  $\text{Rb}^{87}$ , according to ALLEN and BENDER, [2], p. 116, are:

$$a = 6.99577 \frac{\text{Hz}}{\text{nT}}$$

and

$$b = - \begin{Bmatrix} 3 \\ 1 \\ -1 \\ -3 \end{Bmatrix} \times 7.1892 \cdot 10^{-9} \frac{\text{Hz}}{(\text{nT})^2}$$

A fixed value  $B$  of the magnetic field gives rise to six frequencies in the ground state of  $\text{Rb}^{85}$ , and four in the ground state of  $\text{Rb}^{87}$ . The presence of a buffer gas changes the energy values slightly, but the effect on the frequencies is insignificant for geo-magnetic purposes.

If any of the above frequencies can be measured, the magnitude of the magnetic field, which is unknown, can be solved from equation (5). It should be stressed that only transitions in which  $M_F$  changes by 1 are considered here (*cf.* ALLEN and BENDER, [2], p. 115; SKILLMAN and BENDER, [19]), and that only the upper hyperfine  $F = I + \frac{1}{2}$  level is taken into account. The  $F = I - \frac{1}{2}$  level is less suitable for magnetic measurements (BLOOM, [5], p. 66; ALLEN, [2], p. 115), so it is omitted from this discussion.

In rubidium magnetometer sensors the frequency  $f$  (equation (5)) is measured using »optical pumping»: light from a rubidium lamp travels through an interference filter which admits only the optical  $D_1$  line. The light is then circularly (*e.g.* right-hand circularly) polarized, after which it passes through an absorption cell containing Rb vapour and an inert buffer gas (SKILLMAN and BENDER, [19]). As the light penetrates through the cell, the Rb atoms, which were originally in the ground state, are excited to the  $^2P_{1/2}$  state.

The orientation of the sensor should be such that the light travels roughly in the direction of the magnetic field or in the opposite direction (SKILLMAN and BENDER, [19]; ENGELHARD, [9], p. 9; KERZ, [14], p. 178). Assume now that the former is the case. The result of the right-hand circular polarization is that the quantum number  $M_F$  grows by one when the atoms are excited (ENGELHARD, [9], p. 7; SCHMIDT and AUSTER, [18], pp. 368–9), and the probabilities of transition from the different Zeeman sublevels of the ground state are not equal (*e.g.* BELL and BLOOM, [4], p. 1559). The atoms remain in the  $^2P_{1/2}$  state for about  $10^{-8}$  seconds (ENGELHARD, [9], p. 5; KERTZ, [14], p. 177). According to CAMPBELL, [7], the presence of the buffer gas prolongs the time till decay to the ground state. The levels of the ground state to which the atoms return from each sublevel of the excited state depend on the processes the atom undergoes in the excited state (FRANZ, [10], pp. 107–8; FRANZEN and EMSLIE, [11], p. 1455). According to BELL and BLOOM, [4] and FRANZ, [10], it can be assumed that all the sublevels of the ground

state have an equal probability of being populated by returning atoms if the pressure of the buffer gas is sufficient, say above a few hundred  $Pa$  (see BLOOM, [5], p. 61; ENGELHARD, [9], p. 15). In any case it can easily be supposed that the atoms tend in time to accumulate at the sublevels in the ground state from which the probabilities of excitation are small (DEHMELT, [8], p. 1488; FRANZEN, [12], p. 850). (Before the process is started, all the ground state sublevels can be regarded as equally populated (ENGELHARD, [9], p. 6; FRANZEN and EMSLIE, [11], p. 1454; DEHMELT, [8], p. 1489).) As  $\Delta M_F = +1$  in the absorption of light, the smaller  $M_F$  is, the higher the probability of transition becomes (DEHMELT, [8], p. 1488–9; BELL and BLOOM, [4], p. 1560; FRANZ, [10], pp. 106 and 108; FRANZEN and EMSLIE, [11], p. 1454). All these references consider the case  $I = \frac{3}{2}$  or  $I = 0$ , but it can be assumed that the same is also true for the probability when  $I = \frac{5}{2}$ . From the Breit-Rabi formula (2) it can be concluded that the energy increases with  $M_F$  ( $F = I + \frac{1}{2}$ , see Fig. 1). This mechanism, which makes the populations of the ground state levels differ from one another, is the process called optical pumping. In principle the end result of the pumping should be that all the atoms are at the ground-state level where  $M_F = F = I + \frac{1}{2}$ , because the selection rule  $\Delta M_F = +1$  prevents transitions from this level. But owing to relaxation processes, no such complete orientation is achieved (*cf.* BELL and BLOOM, [14], p. 1560; FRANZEN and EMSLIE, [11], pp. 1456–7; FRANZ, [10], p. 110; STUART, [22], p. 822).

As stated, optical pumping causes the less absorbant sublevels of the ground state to become populated at the expense of the more absorbant levels. This means that less light is absorbed in the cell, *i.e.* the intensity of the light travelling through the cell increases. (The light radiation emitted when the excited atoms return to the ground state has no effect on the light intensity observed through the cell, because the emitted light radiates in all directions (ENGELHARD, [9], p. 27).) If there is a coil around the rubidium vapour cell producing an oscillating magnetic field perpendicularly to the static magnetic field and the frequency of the oscillation corresponds to the difference in energy between two adjacent Zeeman sublevels of the ground state (equation (4) or (5)), induced transitions from the higher to the lower populated level will take place (SCHMIDT and AUSTER, [18], p. 369; KERTZ, [14], p. 178; ENGELHARD, [9], pp. 8–9; ALLEN and BENDER, [2], p. 106; BALLING, [3], p. 114). In the case discussed here, the transitions are induced emissions (SOBEL'MAN, [20], p. 294); if the level with higher energy was less populated, the transitions would be absorptions. (Spontaneous emissions between Zeeman sublevels are practically impossible (SCHMIDT and AUSTER, [18], p. 369; KERTZ, [14], p. 178; ENGELHARD, [9], p. 5, FRANZ, [10], p. 105).) The induced transitions show up as an increase of the absorption of the optical light in the cell, *i.e.* the intensity of the light passing through the cell decreases. So the correct

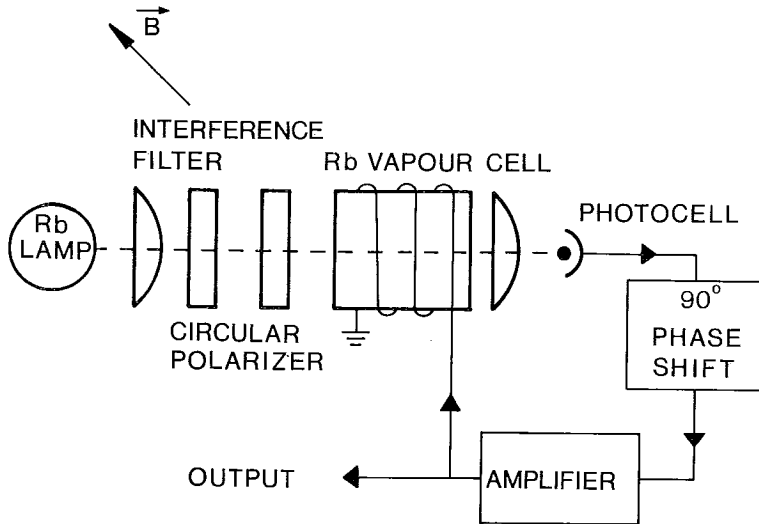


Fig. 2. Principle of the sensor normally used in rubidium magnetometers. The angle between the light beam and the magnetic field is roughly  $45^\circ$  (or  $135^\circ$ ). The axis of the coil is parallel to the light beam. The output of the photocell is fed back to the coil, thus producing a self-oscillating magnetometer (ENGELHARD, [9], p. 13).

frequency  $f$  from which the magnetic field to be measured should be calculated (equation (5)) can be found by monitoring the intensity of the light passing through the cell.

The rubidium magnetometer discussed above functions by the resonance method (ENGELHARD, [9], p. 8). Most Rb magnetometers employ another method (KERTZ, [14], p. 179), that shown in Fig. 2, in which the magnetic field and optical axis are not parallel. Instead the optimum angle between the light beam and the external magnetic field is somewhere near  $45^\circ$  (or  $135^\circ$ ), and the widest possible deviation from the optimum angle is about  $30^\circ$  (SCHMIDT and AUSTER, [18], p. 374). As pointed out by ENGELHARD, [9], p. 12, the sensor must always work in either an angle range of  $0^\circ \dots 90^\circ$  or  $90^\circ \dots 180^\circ$ . Another difference between the resonance method and that shown in Fig. 2 is the orientation of the coil producing the oscillating magnetic field: in Figure 2 the axis of the coil is parallel to the light beam, so that the angle between the oscillating and static magnetic fields is about  $45^\circ$ .

As can be concluded from the discussion on optical pumping, the absorption of the light depends on the components of the angular momenta of the atoms in the direction of the light beam; in the resonance method only the angular momentum components parallel to the external magnetic field are significant. In the



other method (Fig. 2), where the light beam and the magnetic field are not parallel, also the perpendicular components contribute to the absorption. Normally the angular momenta precess randomly around the direction of the magnetic field, so the perpendicular component of the total angular momentum of the Rb vapour is zero. But the oscillating magnetic field, which causes the transitions between Zeeman sublevels, forces the angular momenta to precess in phase if the frequency  $f$  corresponds to equation (5) (CAMPBELL, [7]; ENGELHARD, [9], pp. 10–11). The result is that the component of the total angular momentum of the Rb vapour perpendicular to the external magnetic field ceases to be zero and rotates at the frequency  $f$ . Consequently, the absorption of the part of the light that «sees» the angular momenta perpendicularly to the external magnetic field oscillates at the frequency  $f$ . This means that the intensity of the light passing through the cell is modulated by the frequency  $f$ , from which the magnetic field can be calculated. If the output of the photocell is then connected to the coil, the magnetometer becomes self-oscillating (STUART, [22], p. 826).

## 2. Description of the recording system for the geomagnetic field vector

The system has three rubidium sensors and is situated at the Nurmijärvi Geophysical Observatory in Finland. It is on loan from the British Institute of Geological Sciences (IGS). The station is part of the IGS network of pulsation recorders for the IMS (International Magnetospheric Study 1976–79).

The  $Z$  sensor, which measures the vertical intensity  $Z$  of the field, lies inside a horizontal Helmholtz coil that annuls the horizontal ( $H$ ) component of the field. The NE sensor is surrounded by two Helmholtz coils, one to compensate the vertical component and the other the NW component. Likewise the NW sensor is located inside two coils (STUART, [21], p. 1).

For pulsation recording, the signals from the sensors are taken to three independent frequency counters, and the frequencies are recorded digitally on cassette tape. The sampling interval is 2.5 seconds (MILLS *et al.*, [16], p. 2). The control electronics of the IGS equipment produce power for the three sensors and the Helmholtz coils. To record one-minute values, pulses from the sensors are collected in channels of a separate data logger. The numbers of pulses collected in one minute are counted and recorded digitally on cassette tape together with time data, etc. The data are later transferred from the cassette to computer magnetic tape.

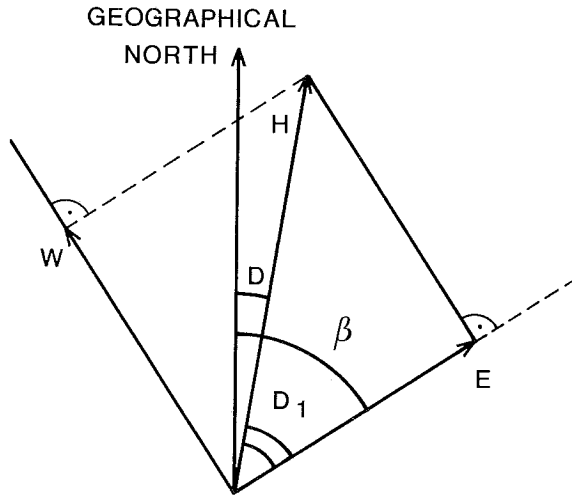


Fig. 3. NW component (denoted  $W$ ) and NE component (denoted  $E$ ) recorded by the Rb magnetometer.  $D$  = magnetic declination;  $D_1$  = angle between the horizontal intensity  $H$ , and  $E$ ;  $\beta$  = angle between geographical north and  $E$ .

### 3. Production of $Z$ , $H$ and $D$ values from the recorded $Z$ , NE and NW components

The rubidium magnetometer system gives the  $Z$  component of the field directly (after converting the frequency into magnetic units). As can be seen from Fig. 3, the following equations are satisfied

$$H = \sqrt{E^2 + W^2} \quad (8)$$

and

$$D = \beta - D_1 = \beta - \arctan \frac{W}{E} \quad (9)$$

where  $\beta$  is the angle between geographical north and NE. Note that NE is denoted simply by  $E$  and NW by  $W$ .

The measured values of the  $Z$ ,  $E$  and  $W$  components are correct only if the directions of the compensation coils are exactly right and their currents precisely dimensioned. If, for example, the Helmholtz coil connected to the  $Z$  sensor is not quite horizontal, but lies at an angle of  $0.1^\circ$  to the horizontal plane, the error of  $Z$  will be about 26 nT, when  $H = 15.000$  nT. The coils can be levelled and oriented by rotating them and observing the frequency reading on the display. It should be borne in mind here that, in practice, the Rb sensors rotate with the coils. But they will not function if their position relative to the magnetic field is arbitrary (*cf.*

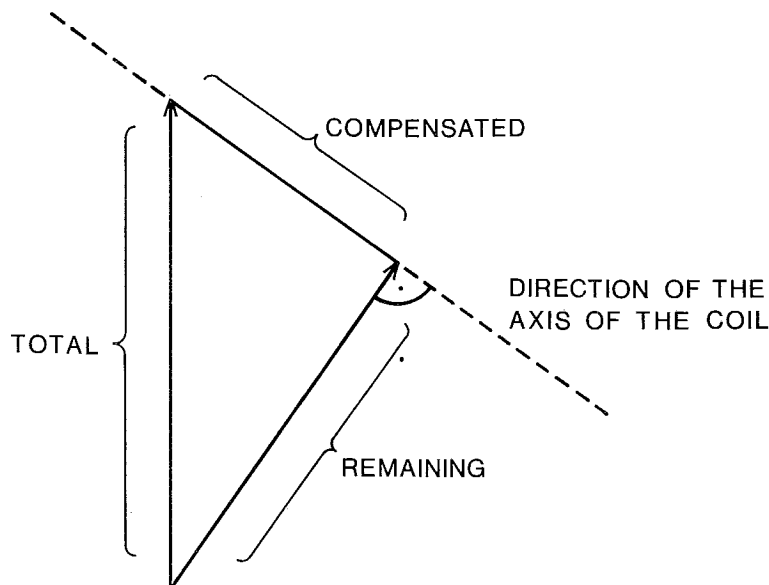


Fig. 4. Principle of compensating a component of the total field. When the frequency reading of the magnetometer is minimum, the remaining component of the field is perpendicular to the axis of the compensating coil.

section 1); this restricts the rotation. If necessary, the currents in the coils can be inverted.

Once the direction of a coil is correct, the right compensation current is obtained by reducing the frequency reading to minimum. The compensated component of the field vector and the remaining component are then perpendicular to each other, as can be seen from Fig. 4.

Even after accurate levelling and orientation of the Helmholtz coils and adjustment of the currents, small corrections still have to be made to the measured values since the magnetic field may differ from the adjustment value  $\vec{F}_0 = (Z_0, E_0, W_0)$ . The vector  $\vec{F}_0$  represents a typical field, because the adjustment must be performed at a quiet moment.

Let the magnetic field to be measured, be  $\vec{F}$ , with the components  $Z$ ,  $E$  and  $W$ , (Fig. 5). As the compensation produced by the Helmholtz coils is correct for  $\vec{F}_0$ , it can be concluded by geometry that

$$Z = \sqrt{Z'^2 - (E - E_0)^2 - (W - W_0)^2} \quad (10)$$

$$E = \sqrt{E'^2 - (W - W_0)^2 - (Z - Z_0)^2} \quad (11)$$

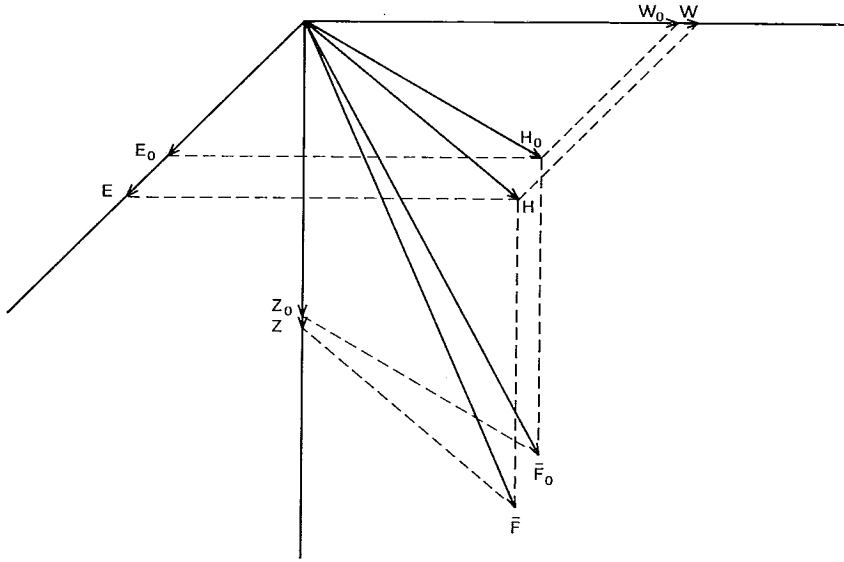


Fig. 5.  $\bar{F}_0$  = average magnetic field vector representing quiet conditions;  $\bar{F}$  = magnetic field vector at the moment of measurement. The NE, NW and vertical components of these vectors are shown.

$$W = \sqrt{W'^2 - (Z - Z_0)^2 - (E - E_0)^2} \quad (12)$$

where  $Z'$ ,  $E'$  and  $W'$  are the measured values. This is a rather complicated set of equations for the unknown quantities  $Z$ ,  $E$  and  $W$ . Since the uncorrected values  $Z'$ ,  $E'$  and  $W'$  are close to the correct values  $Z$ ,  $E$  and  $W$ , they can be substituted for  $Z$ ,  $E$  and  $W$  in the right-hand terms of equations (10) to (12). Hence

$$Z = \sqrt{Z'^2 - (E' - E_0)^2 - (W' - W_0)^2} \quad (13)$$

$$E = \sqrt{E'^2 - (W' - W_0)^2 - (Z' - Z_0)^2} \quad (14)$$

$$W = \sqrt{W'^2 - (Z' - Z_0)^2 - (E' - E_0)^2} \quad (15)$$

If the field is close to  $\bar{F}_0$ , then  $Z'$ ,  $E'$  and  $W'$  are practically the same as  $Z$ ,  $E$  and  $W$ . The correction given by formulas (13) to (15) becomes significant when one or more components of the field differ by more than, say, 100 nT from the normal quiet-time value, as can be seen from Table I, which shows the effect of changes in the NW component  $W$ .

Table I. Effect of changes in the NW component  $W$  on the measured values  $Z'$ ,  $E'$  and  $W'$  and the values  $Z$ ,  $E$  and  $W$ , corrected according to formulas (13), (14) and (15), respectively.  $Z$ ,  $E$  and  $W$  are the corresponding correct values. The calculations were made assuming that  $Z_0 = 48990$  nT and  $E_0 = W_0 = 10750$  nT. All the figures are in nT.

$W - W_0$	$Z$	$Z'$	$Z$ (13)	$E$	$E'$	$E$ (14)	$W = W'$	$W$ (15)
1000	48990	49000.2	48990.0	10750	10796.4	10750.0	11750	11749.9
500	48990	48992.6	48990.0	10750	10761.6	10750.0	11250	11250.0
100	48990	48990.1	48990.0	10750	10750.5	10750.0	10850	10850.0
0	48990	48990	48990	10750	10750	10750	10750	10750
-100	48990	48990.1	48990.0	10750	10750.5	10750.0	10650	10650.0
-500	48990	48992.6	48990.0	10750	10761.6	10750.0	10250	10250.0
-1000	48990	49000.2	48990.0	10750	10796.4	10750.0	9750	9749.9

As can be seen from Table I, the corrected values from equations (13) to (15) are very close to the correct values even with disturbances of 1,000 nT.

#### 4. Calculation of one-minute values from Rb magnetometer data

As stated earlier the rubidium magnetometer produces three values (numbers of pulses) every minute on magnetic tape. These are  $Z'$ ,  $E'$  and  $W'$ , which are proportional to the corresponding uncorrected components of the magnetic field. The corrected values  $Z$ ,  $E$  and  $W$  are computed from equations (14) to (15).

In principle, these values can be converted to magnetic units  $Z_m$ ,  $E_m$  and  $W_m$  with sufficient accuracy by using the linear proportionality factors  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$ , which are equal and constant in theory:

$$\alpha_1 = \frac{Z_m}{Z} \quad (16)$$

$$\alpha_2 = \frac{E_m}{E} \quad (17)$$

$$\alpha_3 = \frac{W_m}{W} \quad (18)$$

(, i.e. the non-linear term in equation (5) has been neglected).

In practice, however, the rubidium magnetometer system at Nurmijärvi is not very stable. So the proportionality factors  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are not exactly equal and they vary with time, so they have to be redetermined at fairly frequent intervals from equations (16) to (18), where  $Z_m$ ,  $E_m$  and  $W_m$  are known absolute values (calculated from absolute measurements or other recordings) and  $Z$ ,  $E$  and

$W$  are the corresponding corrected numbers of pulses.

When pulse numbers are used in formulas (13) to (15) in the manner stated above, the quiet-time values  $Z_0$ ,  $E_0$  and  $W_0$  also depend on the  $\alpha$ 's. Nevertheless, the effect of the errors of  $Z_0$ ,  $E_0$  and  $W_0$  is fairly small, as can be deduced from the figures in Table I. So there is no need to correct  $Z_0$ ,  $E_0$  and  $W_0$  except when the  $\alpha$ 's change by more than say 0.1 to 0.3 %, depending on the magnitude of the respective magnetic component. To be precise, formulas (13) to (15) should be used for pulse numbers only if the  $\alpha$  factors for each component are equal. The smallness of the corrections  $(E' - E_0)^2$ , etc., makes the effect of this error negligible in practice.

The final magnetic components are then obtained using the  $\alpha$  factors and, say, formulas (8) and (9).

The Rb magnetometer at Nurmijärvi is used for two other purposes besides pulsation recording: to check hand-scaled hourly means from conventional magnetograms, and to produce one-minute values.

In practice it has been found that the values of  $\alpha$  have to be determined one to three times a day. At Nurmijärvi we compute the  $\alpha$ 's for every hour by comparing the hand-scaled hourly mean values with the hourly means computed from the one-minute pulse numbers produced by the Rb magnetometer. The median (actually the average of the two middle values) of the corresponding eight  $\alpha$ 's is taken as the final  $\alpha$  for each 8-hour period, and for each element. This median is preferable to the overall average because it is not affected by the occasional very high or low value, *i.e.* mistakes in hand-scaling or disturbances in the Rb magnetometer.

The median  $\alpha$ 's are then used to produce the final one-minute values and, from them, the final hourly means for checking the hand-scaled values.

Finally, the one-minute values are computed in the form of  $X$ ,  $Y$ ,  $Z$  and  $F$  and recorded on magnetic tape in the IAGA format (IAGA News No. 15, 1976).

## 5. Conclusions

We could find no description of the basic principle of the rubidium magnetometer complete with the calculation of the energy levels of Rb in the literature. The first part of this paper therefore deals with the physical principles of the Rb magnetometer. The formulas given enable the reader to calculate the frequencies with sufficient accuracy for all practical geomagnetic uses of the Rb magnetometer.

The IGS rubidium magnetometer described in this paper was designed for digital recording of magnetic pulsations (2.5 s values) where no long-term stability is needed. It is demonstrated that this magnetometer is also useful for the produc-

tion of absolute one-minute and hourly mean values in observatory practice, provided these are compared often enough with the observatory standards. At the Nurmijärvi Observatory we make comparisons three times a day (this frequency is hardly necessary now that the sensor huts have been thermostated). The results obtained with the system show that hourly mean values produced using the Rb magnetometer have the same accuracy as hand-scaled magnetogram values at quiet times ( $\pm 1$  nT) and their accuracy is clearly better during disturbed periods. The Rb magnetometer is shown to be far superior to curve digitizing for the production of one-minute values. It also permits the production of faster samples, say 10 s or 20 s.

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