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EXTREME DISTRIBUTION FUNCTIONS FOR DAILY AND MONTHLY PRECIPITATION IN FINLAND

by

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Abstract

A general form for the cumulative distribution function of the largest (smallest) values proposed by Jenkinson has been used in this study. The form includes the Fisher-Tippett Type I, Type II and Type III distributions, which are the three possible solutions for the Stability Postulate. The estimates for the parameters x_0, α and k have been obtained by the method of maximum likelihood. The distributions of Type I have been fitted from both annual and 5-year maxima, and Type II (or III) only from annual maxima. The distributions have been determined for 25 stations around Finland. No corrections have been made to the precipitation data used.

The results show that the distributions for daily precipitation fitted from annual maxima are of Type II in western Finland and of Type III in eastern Finland, Type I being represented near a line from south to north. The distributions for monthly precipitation fitted from annual maxima are of Type II on the west coast and of Type III in other areas. The reason for this unlimited or limited feature is partly the higher intensity and greater water contents of small and large scale disturbances in western Finland, and partly the effect of Köli mountain. The continentality can thus also be found among extreme values of precipitation. The results also show that the modal values of the distributions decrease towards the north and, thus all the distribution is shifted towards smaller values.

From the practical point of view the Type I distribution function fitted from 5-year maxima is considered the best among the largest extremes, and the areal analyses for return periods of 50 and 200 years have been made on the basis of this distribution. The Type I distribution function fitted from annual maxima in western Finland underestimates and in eastern Finland overestimates possible extremes, while the Type II (or III) distribution function fitted from annual maxima estimates them too high (or too limited).

1. *Introduction*

The problem of how to estimate the greatest (or smallest) value that a meteorological quantity can attain has been studied extensively for planning and decision making purposes. Because of relatively short observation periods much emphasis has been laid upon theoretical aspects in order to get maximum information about extreme events in terms of probability.

In this study, theory, especially the solutions obtained by JENKINSON [6, 7], has been applied to annual maximum values for daily and monthly precipitation in Finland. The study will be practically oriented and only the basic principles of the theory will be described. All the calculations have been programmed and run on the Institute's computer.

Notations

$x_i (i = 1,...,n)$	= a series of n independent daily or monthly (calendar) annual precipitation maxima
x	= a general value of the element
$f(x)$	= probability density function (PDF)
y	= reduced variate
$P(X \leqslant x)$	= probability that value X (annual maximum) is less than x
$F(x)$	= cumulative distribution function of x (CDF)
L	= logarithm of likelihood function
D_n	= Kolmogorov statistic
K_n	= Kimball's statistic
T	= return period
x_0	= the modal value of the x_i terms
α	= the slope of x, y curve
k	= the curvature parameter

In order to estimate the future values of a variable (x) from a sample of data, one has to know the distribution function (CDF) of the variable. FRÉCHET [3] was the first to obtain an asymptotic CDF of the largest value, one of the three possible CDFs. He also introduced the Stability Postulate, according to which the CDF of the largest value must satisfy the functional equation

$$F^n(x) = F(a_n x + b_n) \tag{1}$$

the parameters a_n and b_n being functions of the sample size n.

FISHER and TIPPETT [1] found two others in addition to Fréchet's asymptotic CDF. The three solutions are now known as Fisher-Tippett Type I, perhaps better known as the Gumbel distribution, Fisher-Tippett Type II (found by Fréchet) and Fisher-Tippett Type III, also known as the Weibull distribution. Type I CDF results from any initial PDF of the exponential type which converges to an exponential function as x increases. Type II CDF results from an initial PDF of the Cauchy type, and Type III CDF results from a limited initial PDF.

JENKINSON [6] obtained a general solution to the functional equation, $F(x)$ being the »Double Exponential Distribution» $exp(-e^{-y(x)})$, in the form

$$x = x_0 + \alpha(1 - e^{-ky})/k \qquad (2)$$

$$\Rightarrow y = -\frac{1}{k}\log\{1 - k(x - x_0)/\alpha\}$$

All the Fisher-Tippett types are included and are characterized by the sign of k.

$k > 0$: Fisher-Tippett Type III
$k = 0$: Fisher-Tippett Type I
$k < 0$: Fisher-Tippett Type II

x_0 is the value of x at $y = 0$, and α is the slope of the x, y curve at $(x_0, 0)$; k is a curvature parameter. When $k = 0$ expression (2) simplifies to the straight line

$$x = x_0 + \alpha y \qquad (3)$$

At least three methods have been developed for estimating parameters x_0 and α in this case. Perhaps the most common is a »least squares» method, by which GUMBEL [4] obtains estimates which are functions of sample size n. This graphical method, which is not a real least squares method, is nowadays of less importance, because modern computers have enough processing power to solve the estimation problem in a more advanced way. KIMBALL [8] was the first to propose the method of maximum likelihood, but it was considered long too complicated. LIEBLEIN [10] directs his solution for the estimates of x_0 and α towards minimizing the variance of the resulting estimate of each of n random values. He has prepared a table of values of coefficients to be used for samples of size 2–6. FRANSEN [2] has calculated a table for sample sizes up to 31. This method can be considered a real least squares method, but the solution for CDF of Type II has not yet been presented.

PANCHANG [11] has also presented a solution for calculating maximum likelihood estimates for Type I CDF.

In this study the method of maximum likelihood has been used for estimating the parameters of the general solution, *i.e.* expression (2) proposed by JENKINSON [6]. JENKINSON [7] has presented the maximum likelihood solution for the estimates of x_0, α and k and the solution is described fully in his text. Only the basic features will be summarized here.

The method oı maximum likelihood provides the answer to the question: »What values of x_0, α and k would make the probability of the observed x_i values occurring as annual peaks within a certain time interval, a maximum?» The probability of x_1 occurring as an annual peak is

$$f(x_1) = \frac{dF(x_1)}{dx} = \frac{F(x_1)}{\alpha} e^{-y(1-k)} \tag{4}$$

which is the probability density or the value of the probability density function $f(x)$ (PDF) at the point x_1. In the same way, for a sample of size n we get all the other probability densities $f(x_2),..., f(x_n)$. The likelihood $g(x_1,..., x_n)$ is equal to the product of the PDFs

$$g(x_1, x_2,..., x_n) = \prod_{i=1}^{n} f(x_i) \tag{5}$$

L, the logarithm of the likelihood, is equal to $\sum_{i=1}^{n} f(x_i)$. The maximum likelihood estimates for k, α, x_0 are those which maximize L and are obtained by a quick limiting process. At these values of k, α, x_0

$$\frac{\partial L}{\partial k} = 0; \quad \frac{\partial L}{\partial \alpha} = 0; \quad \frac{\partial L}{\partial x_0} = 0$$

High negative values for parameter k have been found to be quite common in rainfall data. In these cases the CDF predicts rainfall so heavy that there is not the remotest possibility of it occurring naturally.

JENKINSON [7] has suggested that, because there are not many independent days of rainfall in the year, only the upper 20 % of a set of annual maxima closely follow the extreme value distribution, while the lowest 30–40 % may not satisfy the underlying presumptions at all. On this basis he proposes that estimates should also be made from the 5-year maxima. This is done by assuming that the actual order in which the annual maxima occur in time is random. All possible randomizations of the data will give a set of 5-year maxima, and they are weighted according to their frequencies in these sets.

In the case of the 5-year maxima the author succeeded only in programming the solution for Type I CDF. Equivalent 1-year maxima can then be obtained from the expressions

$$\begin{aligned} \alpha(\text{1-year}) &= \alpha(\text{5-year}) \\ x_0(\text{1-year}) &= x_0(\text{5-year}) - \alpha \log 5 \end{aligned} \tag{6}$$

After obtaining estimates for the parameters of the CDF, one can determine the return period, often the primary objective of extreme statistics. The return period is the average interval of time within which the magnitude of the event will be equalled or exceeded once, and is designated by T.

If event X equal to or greater than x occurs once in T years, the probability $P(X \geqslant x)$ is equal to

$$P(X \geqslant x) = \frac{1}{T} \tag{7}$$

Hence,

$$T(x) = \frac{1}{P(X \geqslant x)} = \frac{1}{1 - P(X < x)} \tag{8}$$

The plotting positions for graphical analysis in this study are determined using the form

$$F(x_m) = P(X \leqslant x_m) = \frac{m - 0.31}{n + 0.38} \tag{9}$$

where m is the serial number of an annual maximum in the ordered set of n maxima. The respective y value can be calculated by applying the result of (9) to (8) and using the relationship

$$T(x) = \frac{1}{1 - F(x)} = \frac{1}{1 - \exp(-e^{-y})} \tag{10}$$

from which

$$y = -\log \log [T(x)/(T(x) - 1)] \tag{11}$$

Scales of $F(x)$, $T(x)$ are placed alongside the values of the reduced variate y.

Two criteria have been used to measure goodness of fit. These are the two-sided Kolmogorov statistic D_n, and a distribution-free statistic K_n based on coverages (KIMBALL [9]). Specifically,

$$D_n = \sup |F_n(x_i) - F(x_i)| \qquad (i = 1,..., n) \tag{12}$$

where $F(x)$ is the CDF in question and $F_n(x)$ is the empirical CDF. The $n+1$ coverages associated with the n -order statistic $x_{1,n} \leqslant ... \leqslant x_{n,n}$ are defined by

$$C_j = F(x_{j,n}) - F(x_{j-1,n}) \qquad 1 \leqslant j \leqslant n+1 \tag{13}$$

where $F(x_0, 0) = 0$ and $F(x_{n+1,n}) = 1$. Kimball's statistic is then given by

$$K_n = \sum_{i=1}^{n+1} [C_i - E(C_i)]^2 / E(C_i) \tag{14}$$

As a consequence, large values of D_n and K_n suggest poor fits.

2. *Distribution of daily and monthly precipitation in Finland*

In order to better understand precipitation maxima in Finland, some common features of the distributions of daily and monthly precipitation are introduced. For this purpose Finland has been divided into two parts

southern Finland $\varphi \leqslant 65°$
northern Finland $\varphi > 65°$

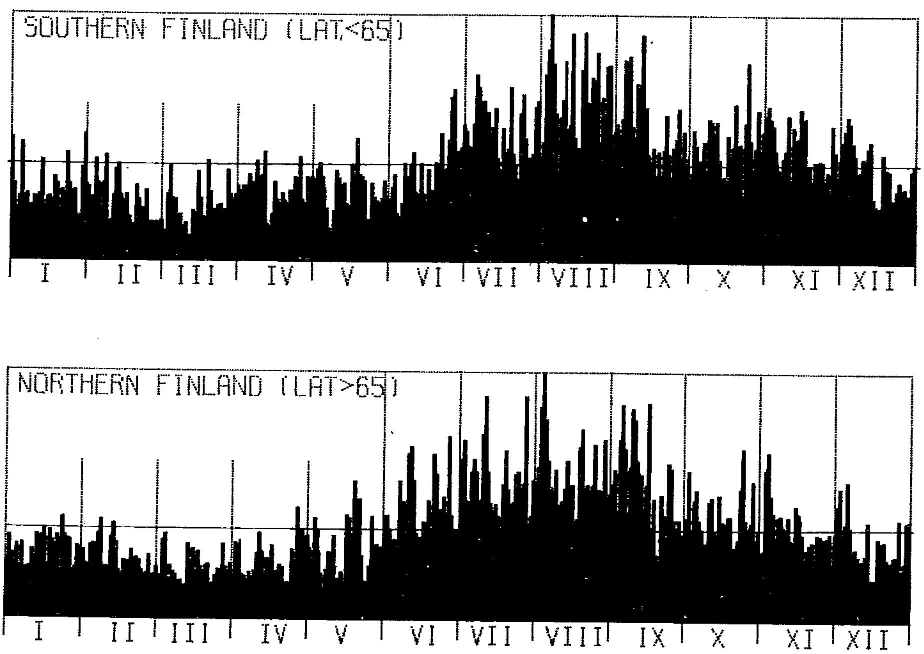


Fig. 1. Annual variation of daily precipitation in southern and northern Finland.

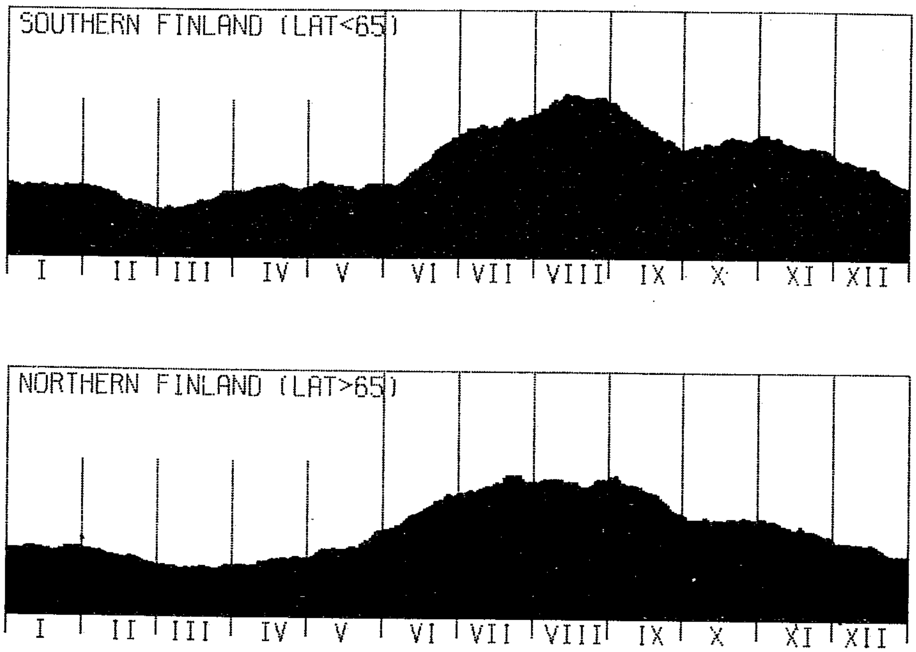


Fig. 2. Moving average of 31 days of daily precipitation in southern and northern Finland.

so that some areal differences can be seen. It should, however, be remembered that the differences within the areas themselves may be even greater.

The annual variation in the areal mean for daily precipitation can be seen in Fig. 1. Each column H_i (i = 1,..., 365) has been calculated from the data of the climatological data register using the formula

$$H_i = \frac{\sum_{k=1}^{n} \sum_{j=1}^{t_k} r_{ijk}}{\sum_{k=1}^{n} t_k} \tag{15}$$

where i = day (1,..., 365)

n = number of stations (54 for southern Finland, 24 for northern Finland)

t_k = length of period at station k in years (average 16 years, 1959–1974)

r_{ijk} = precipitation on day i, in year j, at station k.

The columns H_i in the two figures are comparable, ΣH_i being 538 mm for southern Finland and 495 mm for northern Finland. Thus the annual areal mean for daily

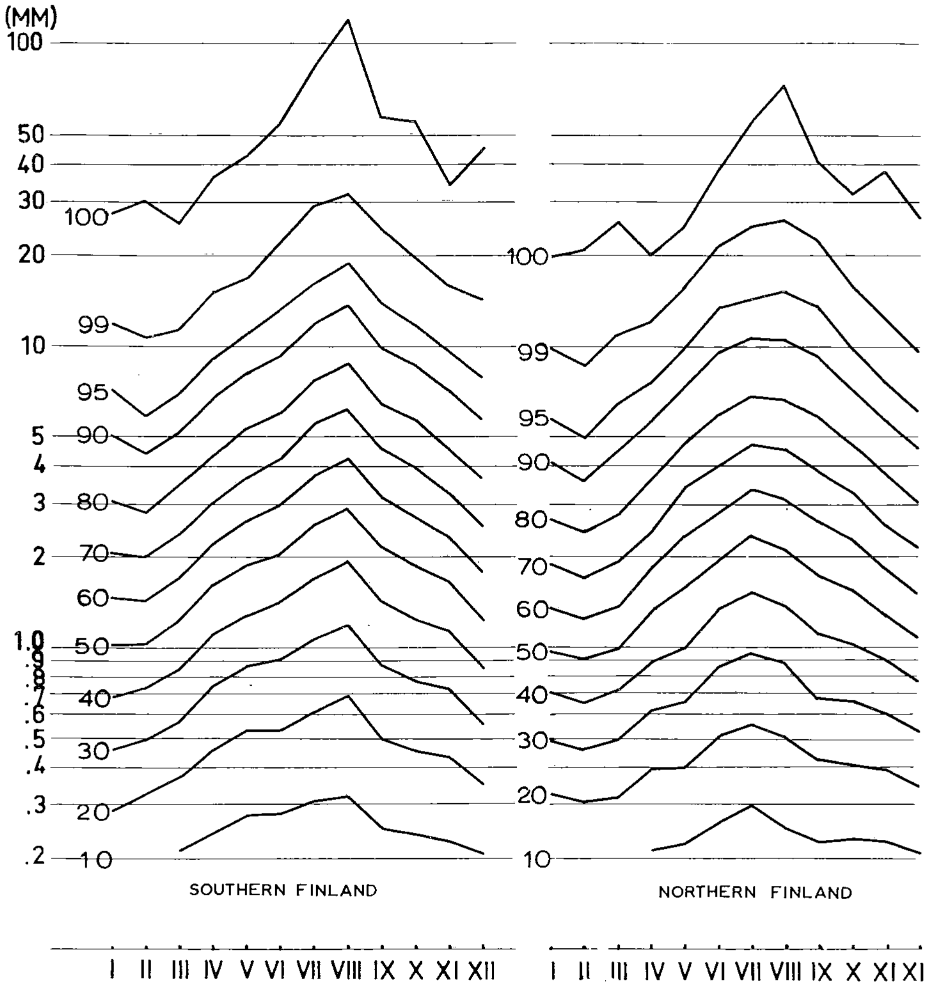


Fig. 3. The empirical monthly CDFs for daily precipitation in southern and northern Finland.

precipitation, the straight line, has values of 1.47 mm and 1.36 mm, respectively. The amounts of precipitation are rather high during the summer and the annual maximum values, which should be independent of each other, can be taken from a calendar year. Large cyclones or cyclone families are weighted and can be seen as broader »towers».

By taking the moving average of 31 days (Fig. 2) a secondary maximum can be seen in autumn in both southern and northern Finland.

The empirical monthly CDFs for daily precipitation are represented in Fig. 3

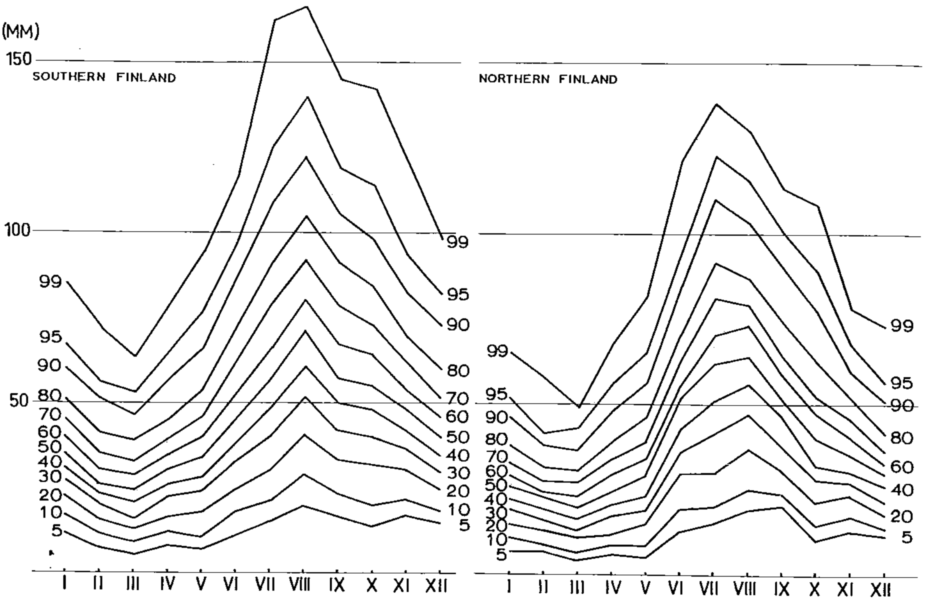


Fig. 4. The empirical CDFs for monthly precipitation in southern and northern Finland.

Table 1. The empirical annual CDFs for daily precipitation in southern (A) and northern (B) Finland represented by percentage points.

%	10	20	30	40	50	60	70	80	90	95	99
A	0.23	0.42	0.70	1.09	1.61	2.32	3.34	4.93	7.95	11.25	20.30 (mm)
B	0.22	0.32	0.62	0.92	1.35	1.95	2.80	4.17	6.75	9.65	17.80 (mm)

by percentage points 10, 20,..., 90, 95, 99 and 100. The precipitation scale is logarithmic. Precipitation amounts of $\leqslant 0.1$ mm have been omitted because of the errors involved. Precipitation is lowest in February, increasing in southern Finland up to August and in northern Finland up to July-August. The growth is approximately three times the value on the same percentage line in February. Values at the same percentage points are given in Table 1 for the whole year.

In figure 4 the empirical monthly CDFs for precipitation are represented by percentage points 5, 10, 20,..., 90, 95 and 99. It can be seen that the CDFs are roughly normal between points 5 and 95, as shown by HUOVILA [5].

The most likely months for maximum monthly precipitation are July, August

Table 2. Percentage occurrence of monthly precipitation maxima for each month.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	
A	–	–	–	–	1.1	2.6	25.5	57.7	9.6	2.7	0.8	–	(%)
B	–	–	–	–	0.8	14.0	35.9	35.1	11.6	2.6	–	–	(%)

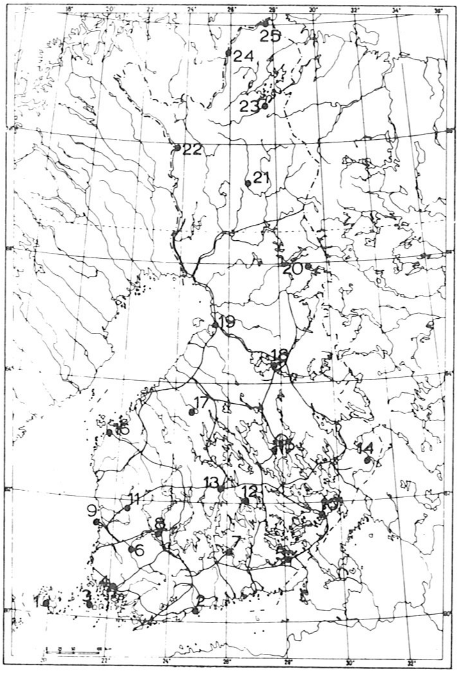
and September in southern Finland and June, July, August and September in northern Finland, as can be seen from Table 2.

3. *The extreme CDFs for precipitation*

The parameters of the extreme CDF have been estimated for the stations listed in Table 3.

Table 3. List of stations. Where there are no comments, daily and monthly precipitation maxima from the whole period have been used.
Comments: 1) the period for daily precipitation maxima, 2) the period for monthly precipitation maxima, 3) only daily precipitation maxima have been used.

No.	Station	Location		
1	Maarianhamina	(60°07′,	19°54′)	1886–1972 1) 1901–1974 2)
2	Helsinki, Kaisaniemi	(60°10′,	24°57′)	1881–1972 1) 1913–1974 2)
3	Houtskär	(60°13′,	21°16′)	1909–1972 3)
4	Turku	(60°31′,	22°16′)	1901–1974 2)
5	Lappeenranta	(61°03′,	28°09′)	1901–1974
6	Huittinen	(61°11′,	22°43′)	1894–1972 3)
7	Heinola	(61°13′,	26°02′)	1909–1974
8	Tampere	(61°28′,	23°44′)	1901–1974
9	Mäntyluoto	(61°36′,	21°29′)	1901–1974
10	Punkaharju	(61°48′,	29°20′)	1905–1974
11	Niinisalo	(61°51′,	22°28′)	1942–1974
12	Kangasniemi	(62°06′,	26°44′)	1910–1972
13	Jyväskylä	(62°14′,	25°44′)	1901–1974
14	Ilomantsi	(62°41′,	30°57′)	1933–1974
15	Kuopio	(62°54′,	27°41′)	1901–1974
16	Vaasa	(63°05′,	21°39′)	1901–1974
17	Lestijärvi	(63°26′,	24°30′)	1909–1972 3)
18	Kajaani	(64°17′,	27°41′)	1901–1974
19	Oulu	(65°01′,	25°29′)	1901–1974
20	Kuusamo	(65°58′,	29°11′)	1908–1974
21	Sodankylä	(67°22′,	26°39′)	1908–1974
22	Muonio	(67°58′,	23°41′)	1947–1974
23	Ivalo	(68°36′,	27°25′)	1946–1974
24	Utsjoki, Outakoski	(69°35′,	25°58′)	1910–1972 3)
25	Utsjoki, Nuorgam	(70°05′,	27°57′)	1929–1974



The length of the period varies from 24 to 92 years. The method of maximum likelihood is normally considered essential, especially if the data do not comprise a large sample or if the data are rather irregular. This is why the relatively short periods have also been included and why they are considered to be comparable with the others.

3.1 The extreme CDFs for daily precipitation

The extreme Fisher-Tippett Type I and Type II (or Type III) CDFs for the annual maximum values of daily precipitation are presented in **Table 4.**

Only 5 of these distributions are close to the Type I CDF, the values of k lying between $-0.05 - +0.04$. Type III, with $k > 0.04$, is represented at 7 stations, and at the other 13 stations the extreme CDF is of Type II, with $k < -0.05$.

The two criteria for goodness of fit based on D_n and K_n suggest that Type II (or III) is slightly superior to Type I. This can perhaps be seen better from **Fig. 5**

Table 4. The parameters for the extreme Fisher-Tippett Type I and Type II (or III) CDFs fitted from annual maxima of daily precipitation with the respective D_n and K_n values. Type I was the only solution for the Kuusamo station.

		Type I		Type II/III			D_n		K_n	
	n	x_0	α	x_0	α	k	I	II/III	I	II/III
Maarianhamina	87	24.66	8.91	23.49	7.76	−0.263	0.080	0.051	0.797	0.659
Helsinki	92	27.21	6.67	27.12	6.61	−0.024	0.051	0.051	0.973	0.980
Houtskär	63	24.02	7.89	23.51	7.53	−0.120	0.071	0.084	1.269	1.222
Turku	74	25.56	9.03	24.91	8.52	−0.135	0.073	0.057	1.003	0.976
Lappeenranta	73	24.15	7.65	24.52	8.33	0.111	0.088	0.113	0.994	1.089
Huittinen	79	24.48	7.76	24.04	7.41	−0.108	0.047	0.029	0.794	0.778
Heinola	66	25.24	8.46	25.29	8.47	0.012	0.075	0.077	1.050	1.053
Tampere	74	23.36	6.67	22.74	6.12	−0.182	0.076	0.063	0.975	0.906
Mäntyluoto	63	23.02	7.19	22.52	6.79	−0.134	0.073	0.053	0.834	0.795
Punkaharju	54	25.19	6.81	25.60	7.05	0.110	0.057	0.054	0.824	0.862
Niinisalo	33	23.39	6.23	22.77	5.69	−0.167	0.128	0.131	1.222	1.273
Kangasniemi	63	28.12	9.80	26.79	8.48	−0.259	0.062	0.061	0.944	0.930
Jyväskylä	73	26.89	10.30	26.31	9.85	−0.105	0.056	0.064	1.058	1.018
Ilomantsi	24	25.50	7.31	26.94	7.76	0.362	0.094	0.064	0.710	0.552
Kuopio	55	23.15	6.06	22.69	5.24	−0.066	0.133		0.629	
Vaasa	74	25.05	6.91	24.51	6.46	−0.143	0.058	0.059	1.052	1.088
Lestijärvi	64	21.88	5.79	21.73	5.71	−0.048	0.104	0.094	1.162	1.121
Kajaani	73	23.67	7.57	23.27	7.30	−0.093	0.059	0.061	0.676	0.692
Oulu	74	21.68	6.93	21.53	6.87	−0.037	0.058	0.055	0.959	0.944
Kuusamo	66	21.60	5.61				0.064		0.835	
Sodankylä	67	21.69	5.97	21.98	6.06	0.090	0.070	0.052	1.111	1.057
Muonio	28	22.05	7.20	21.51	6.74	−0.145	0.097	0.101	1.172	1.065
Ivalo	29	19.35	6.25	19.56	6.36	0.060	0.060	0.053	0.587	0.558
Utsjoki, Outakoski	62	19.13	5.79	18.80	5.50	−0.110	0.115	0.109	1.294	1.298
Utsjoki, Nuorgam	35	18.42	6.66	19.21	6.84	0.222	0.071	0.059	0.748	0.733

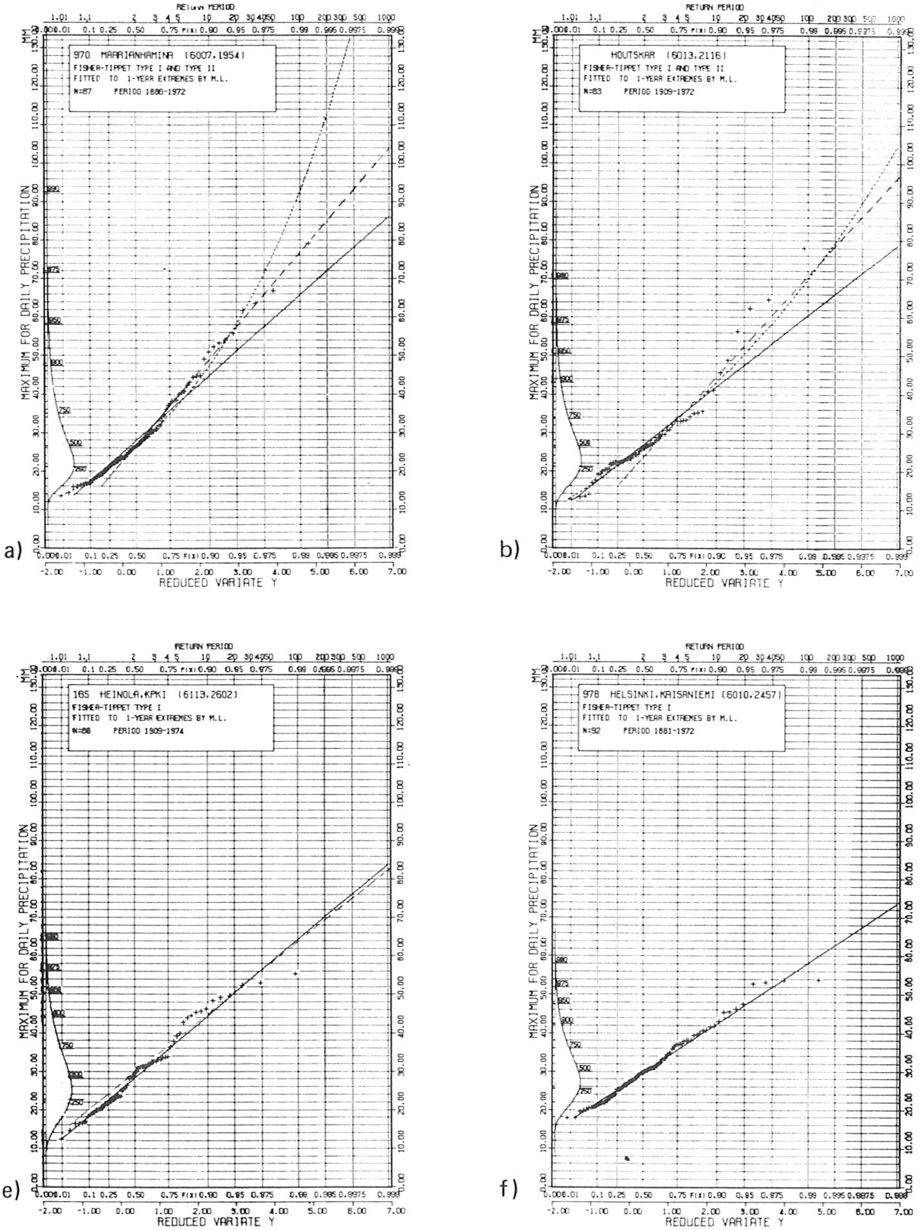
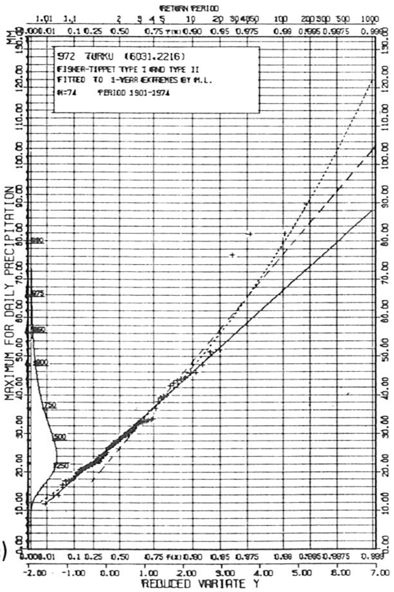


Fig. 5. Extreme CDFs for daily precipitation at 8 stations; a) Maarianhamina, b) Houtskär, c) Turku, d) Huittinen, e) Heinola, f) Helsinki, g) Punkaharju, h) Ilomantsi. Fisher-Tippett Type I fitted from annual maxima (solid straight line); Fisher-Tippett Type I fitted from



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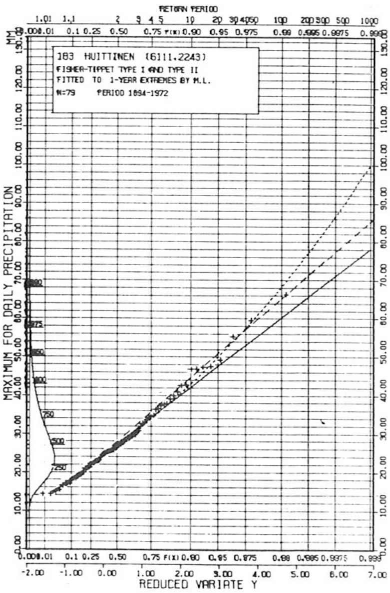

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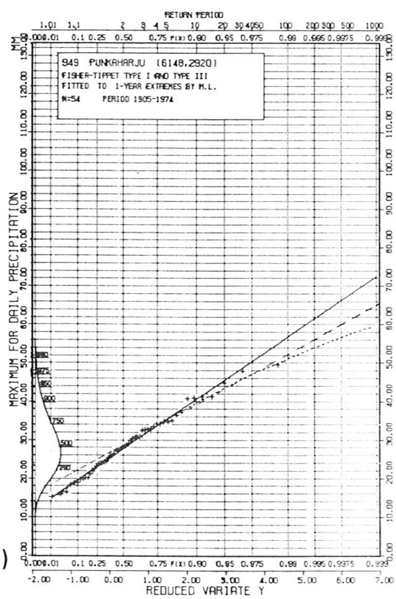

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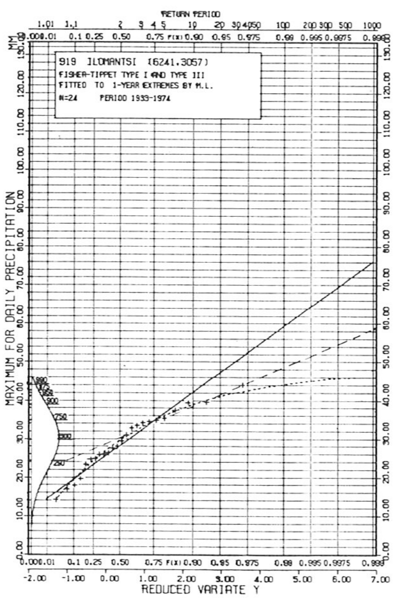


h)

5-year maxima (broken straight line); Fisher-Tippett Type II (or III) fitted from annual maxima, (broken curve); the »observed distribution» + marks. The PDF for Type II (or III) can be seen on the ordinate (x-axis).

where some of these distributions are represented graphically. Type I is drawn with a solid straight line and Type II (or III) with a broken curve. It seems obvious that a curve like Type II will give a satisfactory fit to the annual maxima at the Maarianhamina, Houtskär and Turku stations. Huittinen is already quite close to the Type I distribution while for Heinola the only solution has been a Type I distribution. The same also holds true for Helsinki.

A Type III gives a better fit for Punkaharju, although it is quite close to a Type I CDF. For Ilomantsi, still further to the east, the CDF is quite clearly of Type III.

Because a Type II (or III) CDF adjusts itself better to the »observed distribution», an areal analysis of its parameters x_0, α and k is made in **Fig. 6**.

The areal distribution for the slope parameter α is not presented here, but it is greatest in the southern and central parts of Finland, the maximum being near Jyväskylä. It can be seen from the figures that the modal value x_0 is greatest in the southern, southeastern and central parts of Finland, the maximum being 27.1 mm. From this line x_0 decreases steadily towards the north and a little bit faster in coastal regions of the Gulf of Bothnia. The minimum value for x_0 is 18.8 mm. The greatest modal value is thus 44 % greater than the smallest.

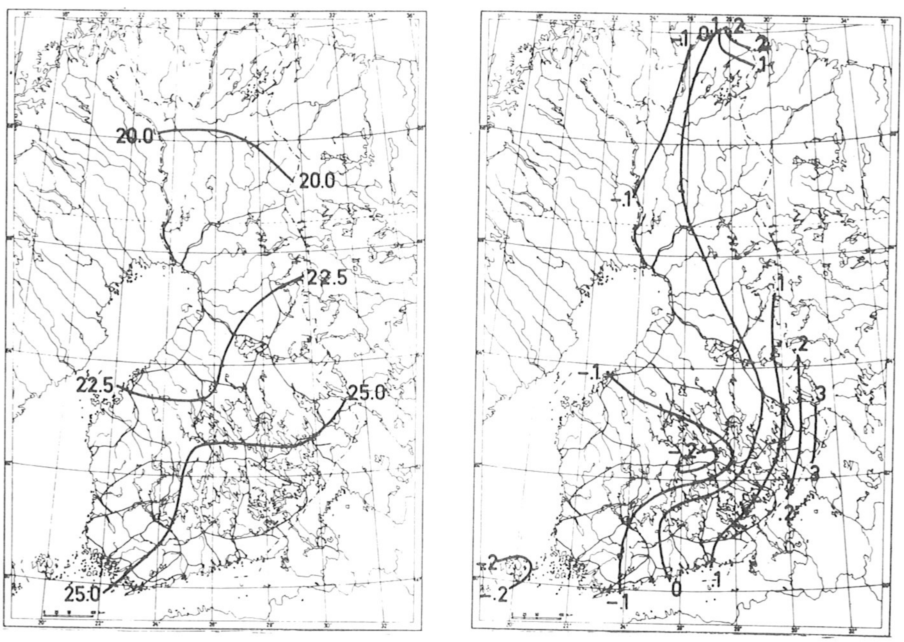


Fig. 6. An areal analysis of the parameters x_0 and k of Type II (or III) CDFs fitted from annual daily precipitation maxima.

The curvature parameter k, on the other hand, has a special zero line from south to north. West of this line k acquires negative values and to the **east positive values.** Thus the extreme CDFs in these areas are unlimited and limited, respectively.

Although the number of maxima (n) varies from station to station and the density of the station net is perhaps low, it can be stated that: 1) the local parameter x_0, and with it the whole extreme CDF, shifts from the southern 27.1 mm to the northern 18.8 mm; 2) the deviations of the largest maxima from the modal value are greater in western Finland, including the western parts of northern Finland, than in the respective areas of eastern Finland; 3) the largest maxima in Finland occur on the west coast in the region between Vaasa and Turku regions, especially in Ahvenanmaa and the Turku archipelago, and in the area around Jyväskylä in central Finland.

The extreme CDFs are also determined for monthly maximum values of daily precipitation; (this means 12 CDFs for each station). Although these CDFs are not part of this study, the distribution of parameters (x_0, α) for CDFs of Type I is presented in Fig. 7. There seems to be an interesting linear dependence between the parameters. According to this, the greater the modal value, the greater the deviations from this value are expected to be at fixed return periods. The dependence among parameters for CDFs fitted from annual maxima does not seem to be quite so strong.

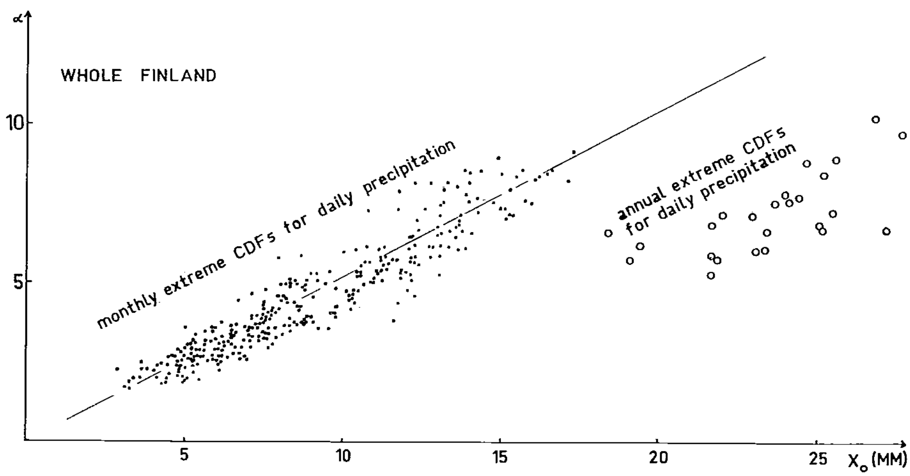


Fig. 7. Distribution of parameters (x_0, α) of extreme Type I CDFs fitted from 1-year monthly maxima of daily precipitation.

Table 5. The parameters for the extreme Fisher-Tippett Type I CDFs fitted from 5-year daily precipitation maxima.

	n	x_0	α
Maarianhamina	87	22.39	11.81
Helsinki	92	27.84	6.69
Houtskär	63	19.38	11.17
Turku	74	21.65	11.94
Lappeenranta	73	19.88	10.15
Huittinen	79	23.44	9.07
Heinola	66	26.45	8.10
Tampere	74	22.64	7.84
Mäntyluoto	63	21.06	9.09
Punkaharju	54	26.97	5.68
Niinisalo	33	17.54	9.42
Kangasniemi	63	20.68	15.31
Jyväskylä	73	25.23	11.56
Ilomantsi	24	29.52	4.14
Kuopio	55	15.94	10.26
Vaasa	74	22.12	8.99
Lestijärvi	64	20.92	6.81
Kajaani	73	20.66	9.37
Oulu	74	20.16	7.58
Kuusamo	60	23.35	4.43
Sodankylä	67	22.22	5.17
Muonio	28	22.64	7.89
Ivalo	29	20.78	5.29
Utsjoki, Outakoski	62	20.00	5.78
Utsjoki, Nuorgam	35	20.73	4.71

The extreme CDFs of Type I are also determined for 5-year maxima. The respective x_0 and α values are presented in Table 5 for each station. In Fig. 5 these CDFs can be seen as broken straight lines.

The goodness of fit seems to be quite satisfactory among the largest maxima, and here this CDF can be considered as a good compromise between Type I and Type II (or III) CDFs obtained from annual maxima.

An areal analysis for return periods of 50 and 200 years is presented in Fig. 8, based on extreme Type I CDFs for 5-year maxima.

The analyses should be considered as first approximations, and the isolines on the borders and coasts should be seen with a critical eye. A jump from T = 50 to T = 200 seems to mean a jump of 10 mm for the isoline values.

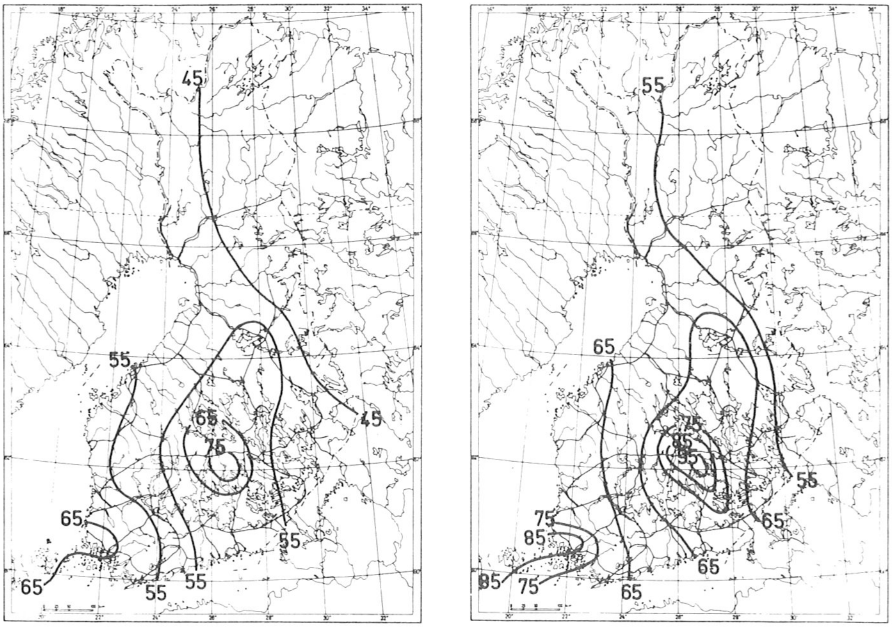


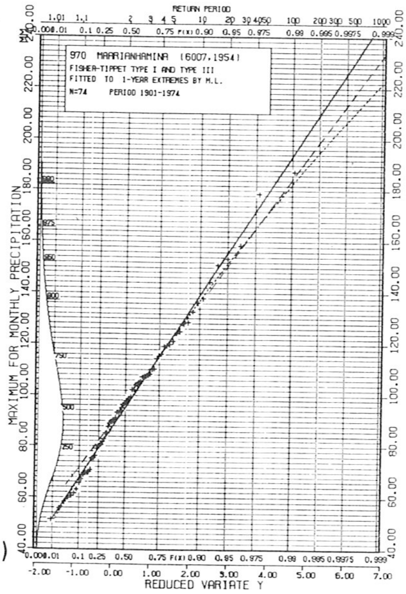
Fig. 8. An areal analysis of daily precipitation maxima (mm) with return periods of 50 years and 200 years.

3.2. The extreme CDFs for monthly precipitation

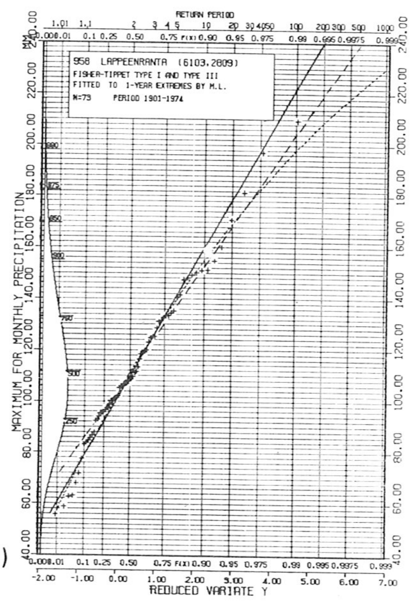
The extreme Fisher-Tippett Type I and Type II (or III) CDFs for annual maximum values of calendar month precipitation are presented in Table 6.

Here again, the goodness of fit criteria suggest that Type III (or II) is slightly superior to Type I. The same can also be seen from Fig. 9, where some of these distributions are presented graphically. Only one of the distributions is of Type I ($-0.05 < k < 0.04$). Type II is presented only at 2 stations, and all the other 17 CDFs have Type III characteristics.

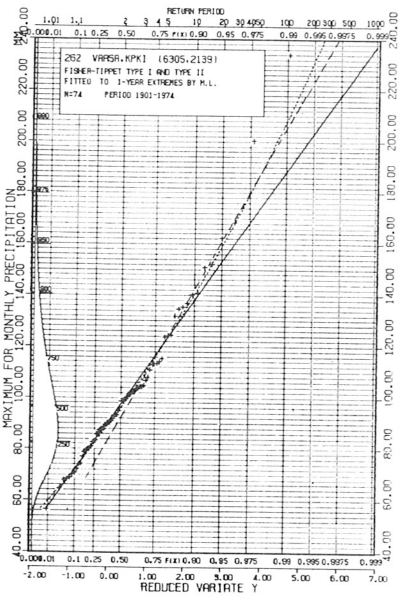
An annual analysis of x_0 and k (Fig. 10) shows that x_0, as expected, decreases from south to north; along the west coast, however, it decreases slightly faster. A zero line for parameter k lies on the west coast, and from this line k increases to the south, north and east. Along the eastern and northern borders the CDFs are strongly limited.



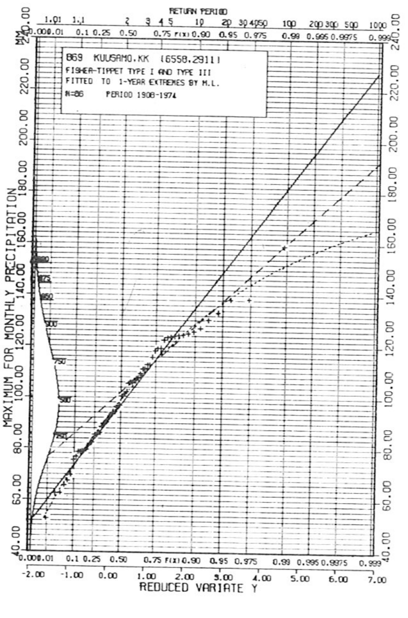
a)



b)

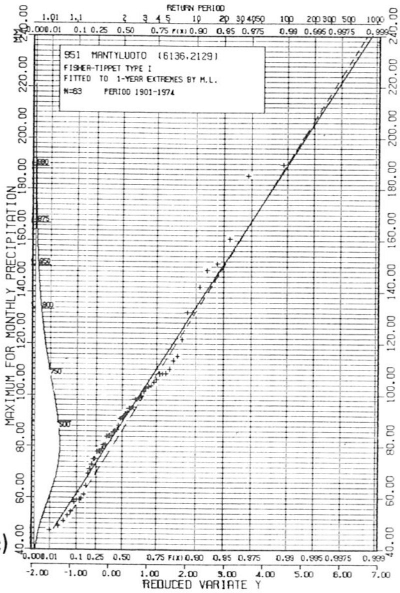


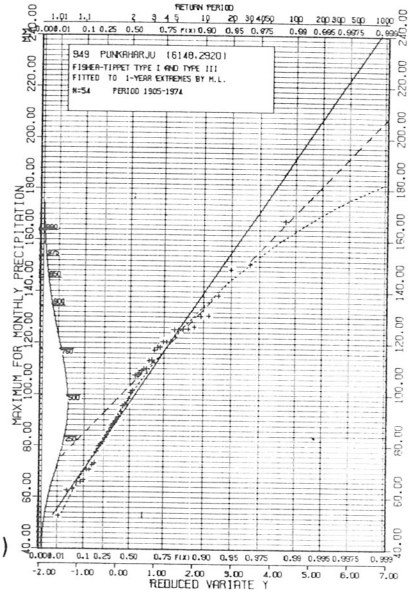
e)

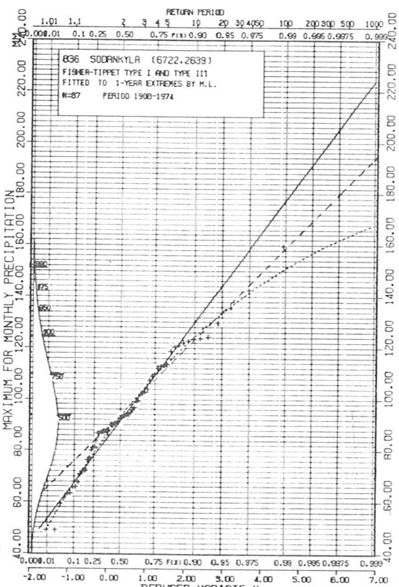


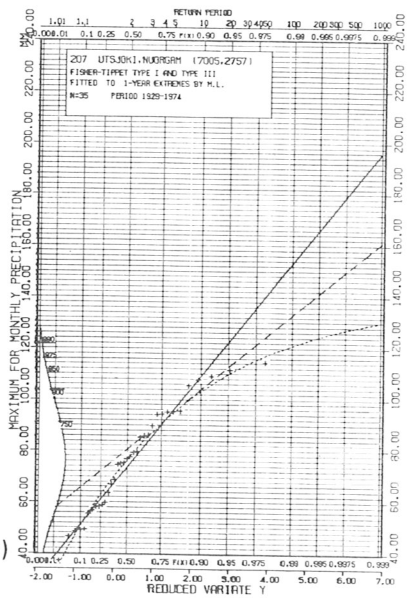
f)

Fig. 9. Extreme CDFs for monthly precipitation at 8 stations; a) Maarianhamina, b) Lappeenranta, c) Mäntyluoto, d) Punkaharju, e) Vaasa, f) Kuusamo, g) Sodankylä, h) Utsjoki. Fisher-Tippett Type I fitted from annual maxima (solid straight line); Fisher-Tippett Type I fitted

c)


d)


g)


h)


from 5-year maxima (broken straight line); Fisher-Tippett Type II (or III) fitted from annual maxima (broken curve); the »observed distribution» + marks. The PDF for Type II (or III) can be seen on the ordinate (x-axis).

Table 6. The parameters for the extreme Fisher-Tippett Type I and Type II (or III) CDFs fitted from annual monthly precipitation maxima with the respective D_n and K_n values.

		Type I		Type II/III			D_n		K_n	
	n	x_0	α	x_0	α	k	I	II/III	I	II/III
Maarianhamina	74	86.60	23.08	87.32	23.49	0.056	0.050	0.047	0.771	0.761
Helsinki	59	95.46	26.65	97.38	27.39	0.134	0.049	0.044	0.630	0.610
Turku	74	96.29	22.62	97.02	23.01	0.059	0.073	0.064	0.939	0.949
Lappeenranta	73	100.19	26.24	101.80	26.77	0.113	0.075	0.058	0.847	0.835
Heinola	66	94.55	22.57	94.80	22.70	0.020	0.050	0.053	0.887	0.896
Tampere	74	90.11	21.38	91.52	21.80	0.121	0.107	0.083	0.828	0.813
Mäntyluoto	63	80.71	23.44	80.73	23.45	0.001	0.064	0.063	0.873	0.872
Punkaharju	54	88.95	22.24	91.21	23.36	0.186	0.103	0.078	0.836	0.918
Niinisalo	33	99.66	21.25	99.76	21.31	0.008	0.053	0.053	0.649	0.653
Jyväskylä	73	100.85	27.97	102.81	28.64	0.128	0.079	0.059	1.210	1.114
Ilomantsi	24	98.27	27.58	101.68	29.70	0.226	0.082	0.086	0.690	0.688
Kuopio	56	94.60	22.65	94.75	22.79	0.012	0.064	0.063	1.031	1.026
Vaasa	74	89.84	21.29	88.72	20.49	−0.098	0.071	0.055	1.001	0.936
Kajaani	73	92.73	24.13	94.89	25.17	0.165	0.061	0.062	0.981	0.945
Oulu	74	80.66	19.44	80.46	1935	−0.018	0.038	0.041	0.716	0.728
Kuusamo	66	89.65	19.84	91.98	20.61	0.217	0.071	0.052	0.942	0.894
Sodankylä	67	83.93	20.17	85.89	20.72	0.179	0.126	0.090	1.028	1.020
Muonio	28	77.29	23.80	79.73	25.61	0.185	0.133	0.115	1.239	1.156
Ivalo	29	71.99	17.98	74.09	19.01	0.212	0.127	0.095	0.985	1.011
Utsjoki, Nuorgam	35	66.63	18.68	69.76	20.36	0.306	0.107	0.094	1.144	1.195

The extreme Type I CDFs have also been determined for 5-year maxima and the x_0 and α values are presented in Table 7.

In Fig. 9 these CDFs can be seen as broken straight lines. Among the largest maxima this CDF can also be considered a good compromise between Type I and Type III (or II) CDFs obtained from annual maxima.

An areal analysis of maximum monthly precipitation for the return periods of 50 and 200 years has been made in Fig. 11, based on these CDFs.

It can be seen that the difference between the analyses is about 30 mm for each isoline. It should be remembered that using calendar month maxima means that the real maxima for a one-month period are underestimated. During the period 1959–1974 all the calendar month maxima were over 60 % of the real maxima, 95 % of the calendar month maxima were over 75 % of the real maxima, 50 % of the calendar month maxima were over 88 % of the real maxima, 17 % of the calendar month maxima were over 95 % of the real maxima and 3.5 % of the calendar month maxima were over 99 % of the real maxima. This percentage is probably greater among the largest maxima, perhaps of the order of 95 %, and of the order of 60–75 % among the smallest maxima.

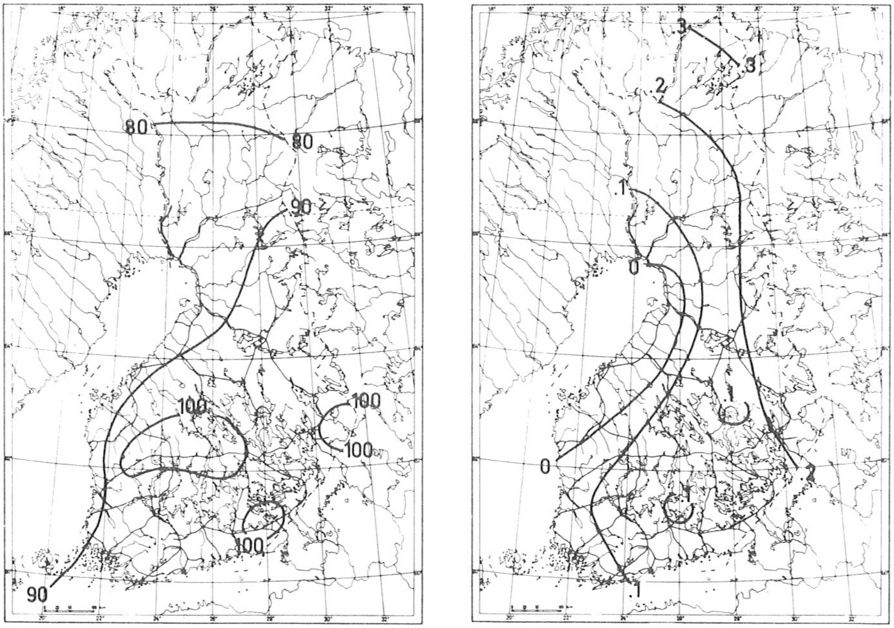


Fig. 10. An areal analysis of the parameters x_0 and k of Type II (or III) CDFs fitted from annual maxima of monthly precipitation.

Table 7. The parameters for the extreme Fisher-Tippett Type I CDFs fitted from 5-year monthly precipitation maxima.

	n	x_0	α
Maarianhamina	74	89.28	20.67
Helsinki, Kaisaniemi	59	100.35	22.22
Turku	74	99.03	19.93
Lappeenranta	73	103.28	22.07
Heinola	66	93.34	23.62
Tampere	74	93.36	16.93
Mäntyluoto	63	78.62	24.30
Punkaharju	54	97.78	15.79
Niinisalo	33	100.42	21.58
Jyväskylä	73	108.77	20.55
Ilomantsi	24	111.11	20.41
Kuopio	56	93.04	22.71
Vaasa	74	82.82	26.50
Kajaani	73	98.71	19.10
Oulu	74	79.11	19.57
Kuusamo	66	97.14	13.66
Sodankylä	67	88.87	15.25
Muonio	28	89.73	17.10
Ivalo	29	79.34	12.79
Utsjoki	35	75.93	12.38

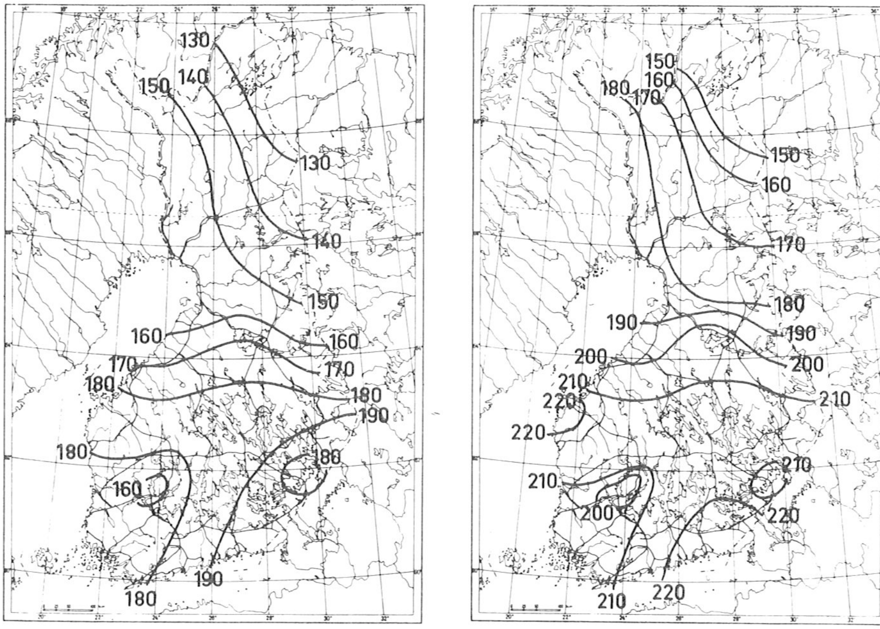


Fig. 11. An areal analysis of monthly precipitation maxima (mm) with return periods of 50 years and 200 years.

4. *Conclusion*

Although the problem of estimating the maximum probable amount of precipitation has been studied for years, it would still appear that better estimates for future maximum values could be found.

From the theoretical point of view the possibilities can be considered quite satisfactory. The basic problem perhaps is, as suggested by JENKINSON [7], the selection of the annual maximum values and deciding what are »real» extreme values. The maximum precipitation in one year may result from a continuous cyclonie type of precipitation and in another from a heavy cumulonimbus rainfall. Thus classifying of the maxima and determining extreme CDFs within each class may help achieve better estimates. It would also have importance for applying the extreme CDFs, because the effects are very much different.

Areal differences in the maximum amounts of daily and monthly precipitation are quite clear. The analysis of annual maxima shows that the CDFs in western Finland are of Type II, while in eastern Finland they are of Type III. Type of the CDFs for annual daily maxima changes in central Finland and for annual monthly maxima near the west coast. The reasons for this are the different intensities and water contents of small and large scale disturbances in western and eastern Finland, which in turn are due to the effect of Köli mountain and the effect of sea as an energy source.

According to this study the best estimates for the largest maxima can be obtained by using the Type I CDF fitted from 5-year maxima although solutions obtained from annual maxima should be used for analysis of the smallest maxima.

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