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THE INTERACTION OF THE MONTHLY MEAN FLOW AND LARGE-SCALE TRANSIENT EDDIES IN TWO DIFFERENT CIRCULATION TYPES

Part III: Potential vorticity balance

by

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Abstract

The local budget of monthly mean potential vorticity was studied using NMC height and temperature analyses for the Northern Hemisphere in a high and a low zonal index case. The transient horizontal flux of potential vorticity implied a positive (but variable) mixing coefficient for the down-gradient component and a smaller but mainly negative cross-gradient component in the troposphere. The divergence of the transient flux is slightly smaller than that of the mean flux, but both are important in the mean budget. The differences of the flux divergences in the two cases were not large.

1. *Introduction*

The quasigeostrophic theory of atmospheric motion has been a useful tool in meteorology. One of the concepts within this theory is potential vorticity, which is conserved in adiabatic frictionless motion. This conservation law has been the basis for numerical short-range weather prediction before the era of modern primitive equation models. Consequently, it is interesting to ask how this quantity behaves for longer time scales.

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GREEN [4] presented the idea that a diffusion approach could be used for the parameterization of the meridional eddy transport of potential vorticity. Combining this with the corresponding temperature diffusion approach, the nondiffusive momentum transport can be constructed for zonally averaged conditions. PEARCE's review [10] describes the main features of this model. WIIN-NIELSEN and SELA [16] calculated the observed yearly meridional transport of potential vorticity and large-scale mixing coefficients for potential vorticity and temperature. These were used by SELA and WIIN-NIELSEN [13] and later by WIIN-NIELSEN and FUENZALIDA [15] in a quasigeostrophic zonally averaged model simulating yearly climate. Many studies have used potential vorticity as a conservative tracer, especially in the stratosphere (*e.g.* STALEY [14], DANIELSEN [2], HARTMANN [5]).

The present calculations for the monthly mean potential vorticity balance use the same data (NMC analyses) as Parts I and II (SAVIJÄRVI [11, 12]), where the local balance of kinetic energy, absolute vorticity and temperature were studied. The main problems studied here are the relative importance of the time-mean motion and the large-scale turbulence in the horizontal transfer of potential vorticity and the relationship between the turbulent flux and the gradient of mean potential vorticity.

2. *Potential vorticity concept*

In the following, pressure is used as the vertical coordinate. The thermodynamic energy equation for a unit mass of air is then written

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + V \cdot \nabla \theta + \omega \frac{\partial \theta}{\partial p} = Q \tag{2.1}$$

where Q is irreversible diabatic heating. Thus, in adiabatic flow ($Q = 0$) potential temperature θ is conserved. Combining (2.1) through vertical velocity $\omega = dp/dt$ with the vorticity equation

$$\frac{d\eta}{dt} - \eta \frac{\partial \omega}{\partial p} = S \qquad \eta = \zeta_p + f \tag{2.2}$$

where S includes the (normally) small twisting and friction terms gives, after some calculation:

$$\frac{d\eta}{dt} + \eta \frac{1}{\frac{\partial \theta}{\partial p}} \frac{d}{dt}\left(\frac{\partial \theta}{\partial p}\right) + \eta \frac{1}{\frac{\partial \theta}{\partial p}} \frac{\partial V}{\partial p} \cdot \nabla \theta = \eta \frac{1}{\frac{\partial \theta}{\partial p}} \frac{\partial Q}{\partial p} + S \tag{2.3}$$

Neglecting $\frac{\partial V}{\partial p} \cdot \nabla \theta$ as small (it disappears in a purely geostrophic flow) and multiplying (2.3) by $-\frac{\partial \theta}{\partial p}$ gives

$$\frac{d}{dt}\left(-\eta \frac{\partial \theta}{\partial p}\right) = -\eta \frac{\partial Q}{\partial p} - S \frac{\partial \theta}{\partial p} \tag{2.4}$$

In θ coordinates potential vorticity $(\zeta_\theta + f) \frac{\partial \theta}{\partial p}$ is strictly conserved for adiabatic and frictionless motion. In pressure coordinates the conservation law,

$$\frac{dq}{dt} = 0 \tag{2.5}$$

for potential vorticity

$$q = -\eta \frac{\partial \theta}{\partial p} = -(\zeta_p + f) \frac{\partial \theta}{\partial p} \tag{2.6}$$

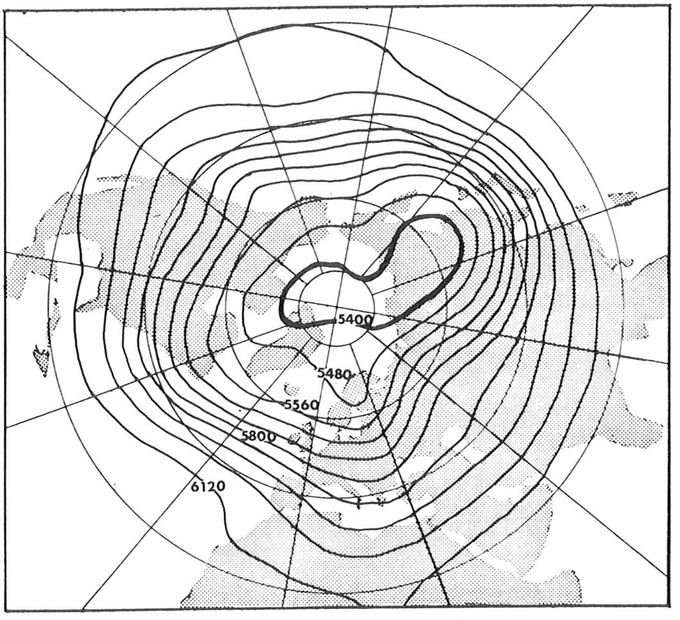
is a good approximation (derived from (2.4)) for adiabatic and frictionless flow, the twisting and $\frac{\partial V}{\partial p} \cdot \nabla \theta$ terms being neglected. Equation (2.6) is an approximation of ERTEL's [3] more exact potential vorticity. There are many other approximations of potential vorticity and also other complex combinations of hydrodynamic motion and temperature variables, which are conservative under adiabatic and frictionless conditions (HOLLMANN, [6], MONIN, [8]), but only few of them seem to be of any practical value.

The often-used quasigeostrophic potential vorticity q_g is

$$q_g = \zeta_p + f + f_0 \frac{\partial}{\partial p}\left(\frac{T}{\sigma}\right) \qquad \sigma = \frac{T}{\theta} \frac{\partial \theta}{\partial p} = \frac{\partial T}{\partial p} - \frac{\alpha}{c_p} \tag{2.7}$$

The conservation law $dq_g/dt = 0$ is obtained when the quasigeostrophic vorticity equation is used instead of (2.2) and, consistently, the stability factor σ is kept horizontally constant. WIIN-NIELSEN and SELA [16] used this formulation, but because large-scale stability varies both latitudinally and longitudinally, the more general form (2.6) and the associated balance equation (2.4) will be used here.

NMC numerical height and temperature analyses were used as data. Calculations are based on geostrophic winds, centred differences on NMC grid and simple arithmetic time means for two one-month periods; the high index case (December 1965, 62 analyses) and the low index or blocking case (February 1965, 63 analyses, 28.1–28.2). Fig. 1 shows the mean geopotential height fields averaged with respect to available



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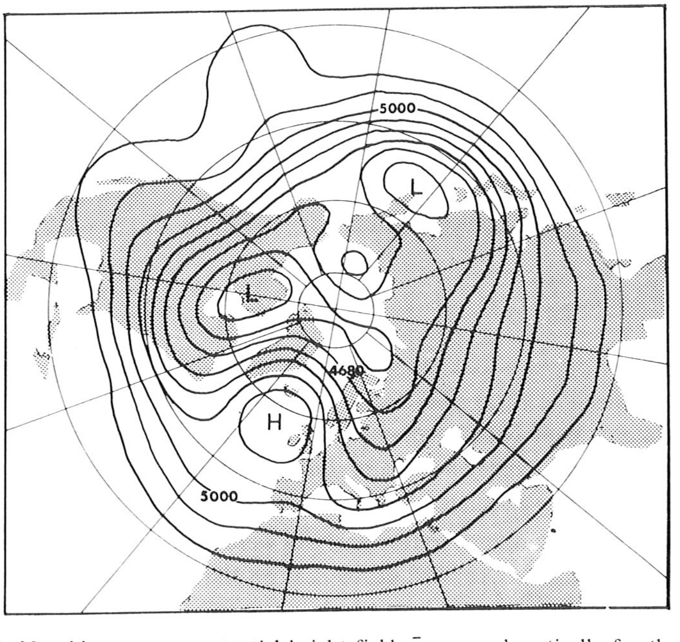

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Fig. 1. Monthly mean geopotential height fields $\bar{z}$ averaged vertically for the two periods in 1965. Unit: m.

pressure levels. A more detailed discussion of the data is given in Parts I and II. In the potential vorticity calculations, the stability $\frac{\partial \theta}{\partial p}$ was computed between available data levels: 850–700, 700–500, 500–300, 300–200 mb. Other variables are mean values between these levels. The hydrostatic stability factor in potential vorticity formulations is sensitive to errors. The present formulation, also used by HARTMANN [5], seems to be reasonably accurate in this sense.

The horizontal distributions shown are middle tropospheric values (700–500 mb) or vertical averages between 850 and 300 mb. Zonal averages, denoted by [ ], are basically similar in the two cases and only the high index case zonal averages are shown.

3. *Results*

Taking the time average of the potential vorticity budget (2.4) gives

$$\frac{\partial \bar{q}}{\partial t} = -\nabla \cdot \bar{q}\bar{V} - \nabla \cdot \overline{q'V'} - \frac{\partial}{\partial p}\overline{q\omega} - \overline{\eta \frac{\partial Q}{\partial p}} - \overline{S\frac{\partial \theta}{\partial p}} \tag{3.1}$$

In the two one-month cases investigated here, the left-hand side of Eq. (3.1) can be assumed to be approximately zero. The following discussion will consider the distribution of $\bar{q}$ and $\overline{q'V'}$ and the relative role of the first two terms on the right-hand side of (3.1).

The mean potential vorticity consists of three parts

$$\bar{q} = -\overline{\eta \frac{\partial \theta}{\partial p}} = -f\frac{\partial \bar{\theta}}{\partial p} - \bar{\zeta}_p \frac{\partial \bar{\theta}}{\partial p} - \overline{\zeta_p' \frac{\partial \theta'}{\partial p}} \tag{3.2}$$

scale				
$10^{-6}\,Ks^{-1}\,mb^{-1}$	6	5	1	0.1

where the scale is the rms value over the grid in the 700–500 mb layer in the two one-month periods studied. The covariance between vorticity and stability is negligible in the sum; thus $\bar{q} \approx -\bar{\eta}\frac{\partial \bar{\theta}}{\partial p}$.

Fig. 2 shows the zonal mean value $[\bar{q}]$ and the meridional transient transport $[\overline{q'v'}]$ in the high index case. The distribution of $[\bar{q}]$ is very similar to that given by NEWELL *et al.* ([9], Fig. 12.15) for yearly mean values; high stability in the stratosphere and the coriolis effect at high latitudes combine to give very large values of potential vorticity at stratospheric polar regions. The meridional transient flux of $\bar{q}$, $[\overline{q'v'}]$ (Fig. 2b) is negative (southwards) in the troposphere, thus transporting $\bar{q}$ from high values towards lower values at low latitudes along the meridional gradient of $[\bar{q}]$.

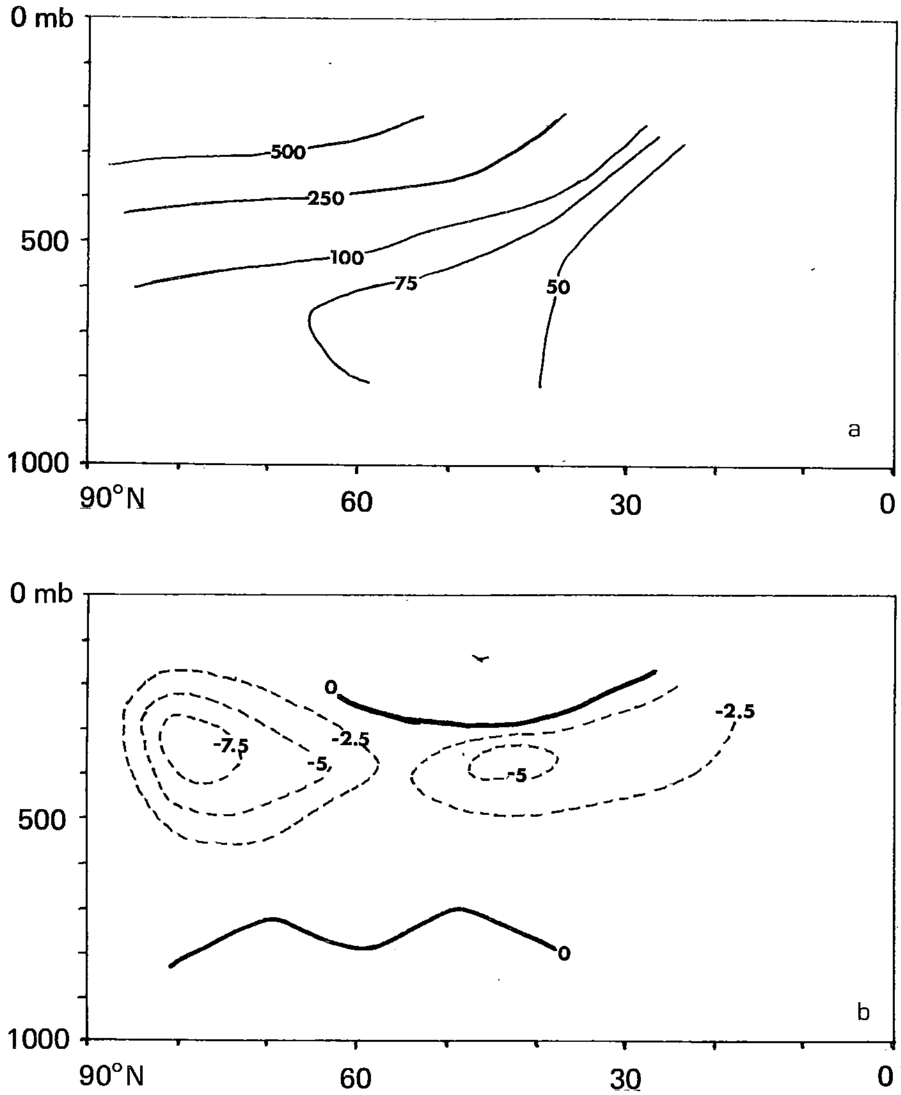
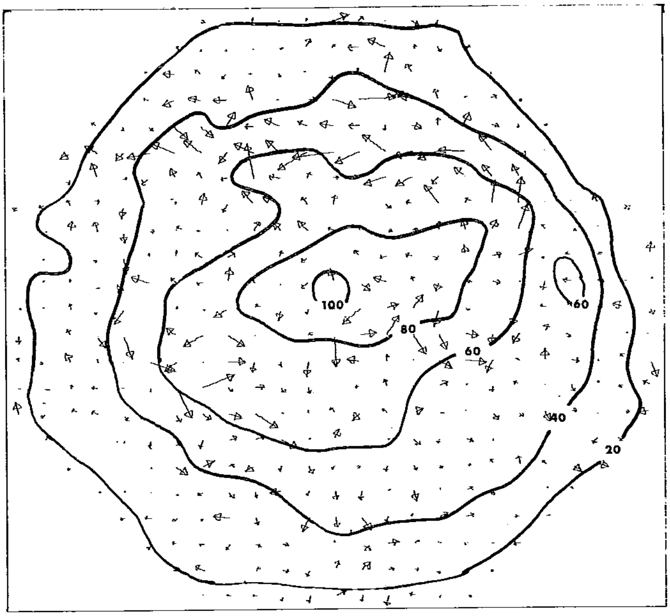


Fig. 2. Zonal averages for the high index case (12/1965).
a) Mean potential vorticity $[\bar{q}]$. Unit: $10^{-7}\,\mathrm{K\,s^{-1}\,mb^{-1}}$.
b) Transient meridional transport of potential vorticity $[\overline{q'v'}]$. Unit: $10^{-6}\,\mathrm{K\,m\,s^{-2}\,mb^{-1}}$.



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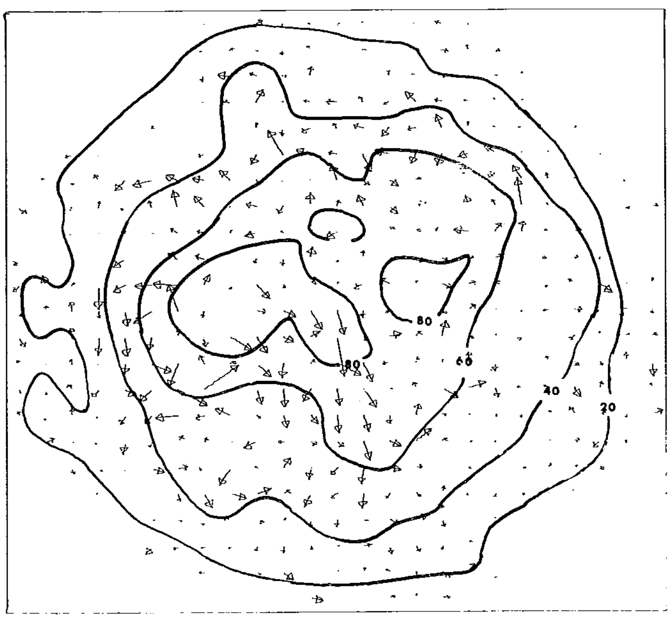

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Fig. 3. Monthly mean potential vorticity $\bar{q}$ (unit: $10^{-7}\,K\,s^{-1}\,mb^{-1}$) and transient potential vorticity flux vectors $\overline{q'V'}$ in the layer 700–500 mb (unit: grid length = $20 \cdot 10^{-6}\,K\,m\,s^{-2}\,mb^{-1}$).

The flux obtained is basically similar to the quasigeostrophic flux calculated by WIIN-NIELSEN and SELA [16], who used yearly and seasonally averaged data and a horizontally constant stability factor.

The local distribution of $\bar{q}$ is shown in Fig. 3 together with the transient flux vectors $\overline{q'V'}$. (The background is the same as in Fig. 1.) The flux is strongest in the cyclone activity areas. At most points the flux vectors have a component along the gradient of $\bar{q}$. There is often also a non-Fickian rotational (after CLAPP, [1]) component parallel to the isolines of $\bar{q}$. This division into along-gradient and rotational components

$$\overline{q'V'} = -K\nabla\bar{q} + R\hat{k}\times\nabla\bar{q} \tag{3.3}$$

may be further studied with the aid of the corresponding mixing (K) and rotational (R) diffusion coefficients.

The values of the mixing coefficient K (Fig. 4, 700–500 mb) are mainly positive, the mean value over the grid being $0.73 \cdot 10^6\ m^2s^{-1}$ in the high index case. Negative values are found in the south. However, K is far from constant and its variations are not easily connected with the mean variables of the flow. The same seems to be true for the local distribution of R (Fig. 5), which is mainly negative (mean value -0.52 $10^6\ m^2s^{-1}$) but more zonally symmetrical than K. Both K and R differ considerably in the two cases.

The meridional cross-sections of both $[K]$ and $[R]$ (Fig. 6) resemble the corresponding zonally averaged diffusion coefficients for transient heat flux (Part II, Fig. 10). Locally this similarity is much weaker. $[K]$ is mainly negative in the stratosphere and its maximum is in the upper troposphere between 60–70°N. The positive values of $[R]$ are concentrated at middle latitudes.

The horizontal flux divergences $-\nabla\cdot\overline{q'V'}$ in the two cases are shown in Fig. 7, averaged vertically between 850 and 300 mb. Their magnitudes are about $10^{-11}\ Ks^{-2}mb^{-1}$ at maxima and the distributions show basically similar features in the two cases. There seems to be no simple correlation between the transient flux divergence and mean potential vorticity; for example $r(-\nabla\cdot\overline{q'V'}, \nabla^2\bar{q}) \simeq 0$. Formally the transient flux consists of three parts

$$\overline{q'V'} = \bar{\eta}\overline{\frac{\partial\theta'}{\partial p}V'} + \frac{\partial\bar{\theta}}{\partial p}\overline{\eta'V'} + \overline{\eta'\frac{\partial\theta'}{\partial p}V'} \tag{3.4}$$

of which the triple covariance term was negligible in the two one-month periods considered. The transient stability part $-\nabla\cdot\bar{\eta}\overline{\frac{\partial\theta'}{\partial p}V'}$ (not shown) is larger and smoother (rms value over grid $3\cdot 10^{-12}\ Ks^{-2}\ mb^{-1}$ at the 700-500 mb layer) than the mean stability

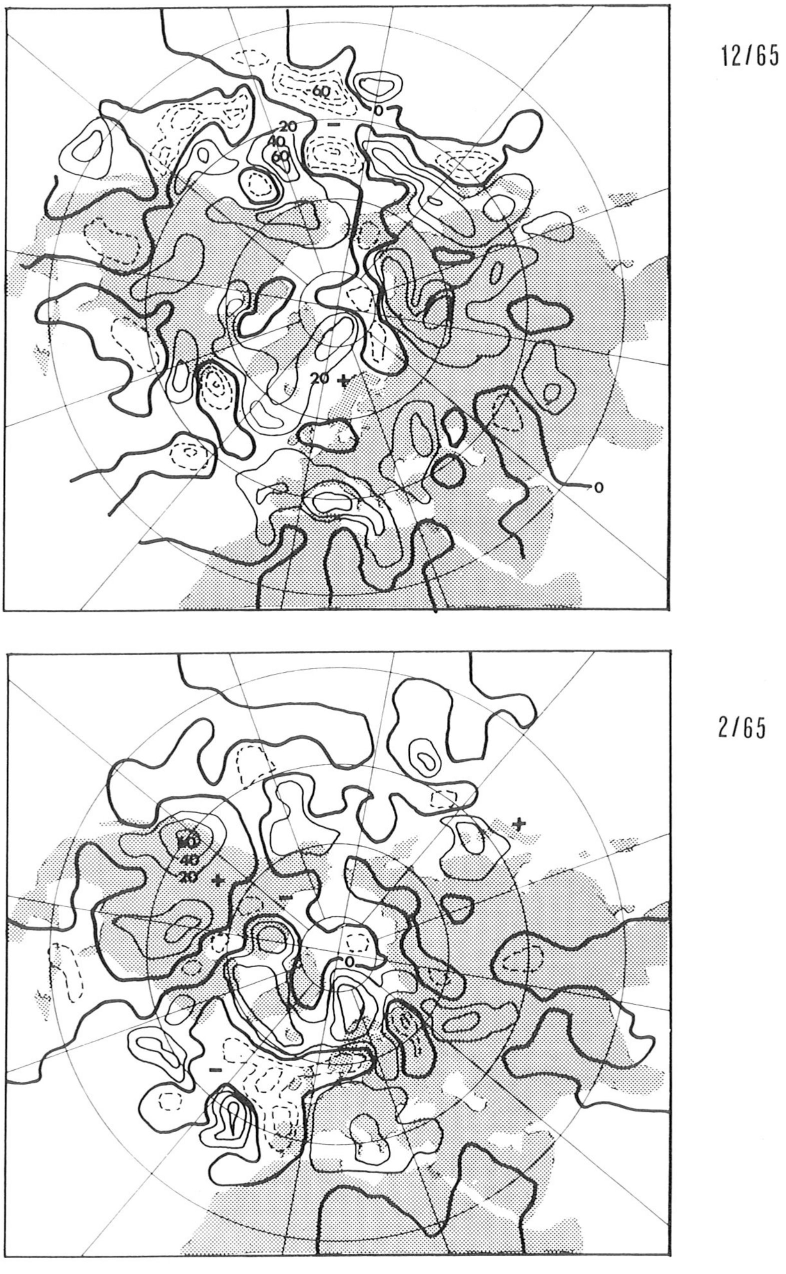

Fig. 4. Distribution of mixing coefficient K, 700–500 mb. Unit: $10^5\,m^2\,s^{-1}$.

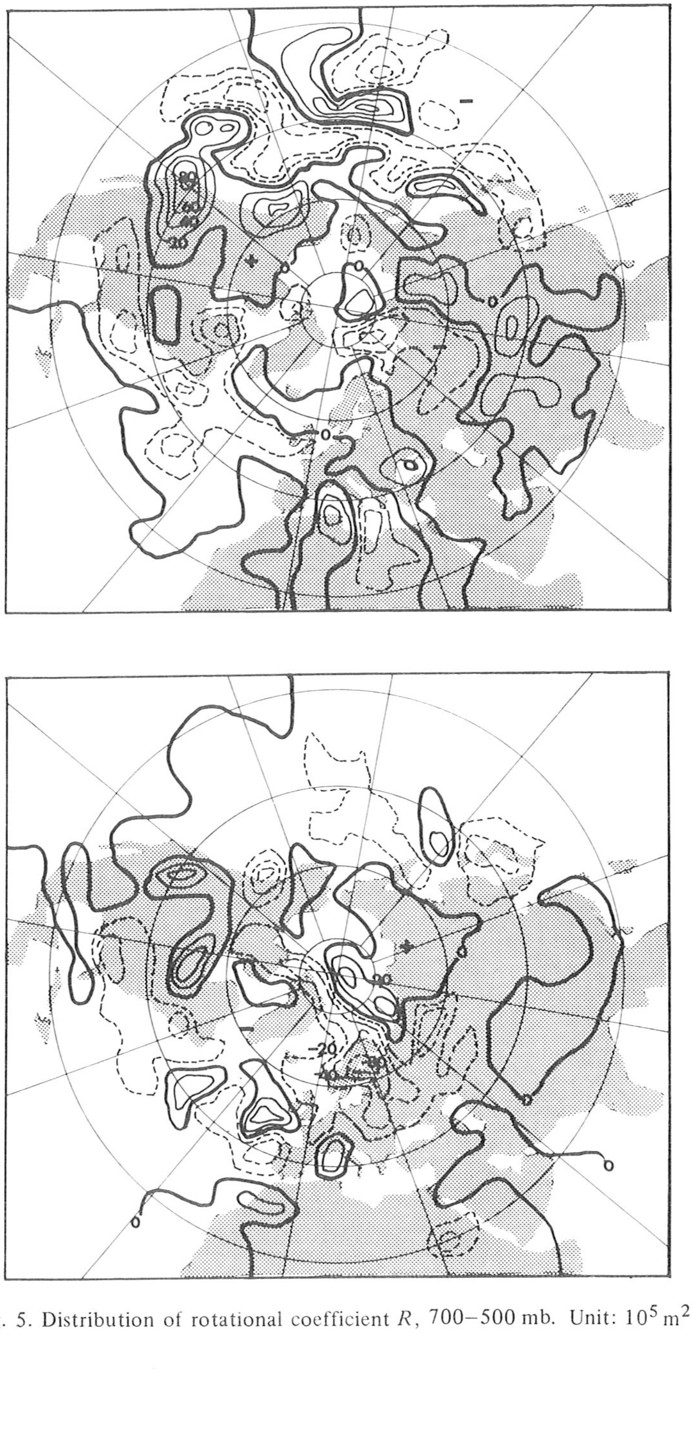

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Fig. 5. Distribution of rotational coefficient R, 700–500 mb. Unit: $10^5\ m^2 s^{-1}$.

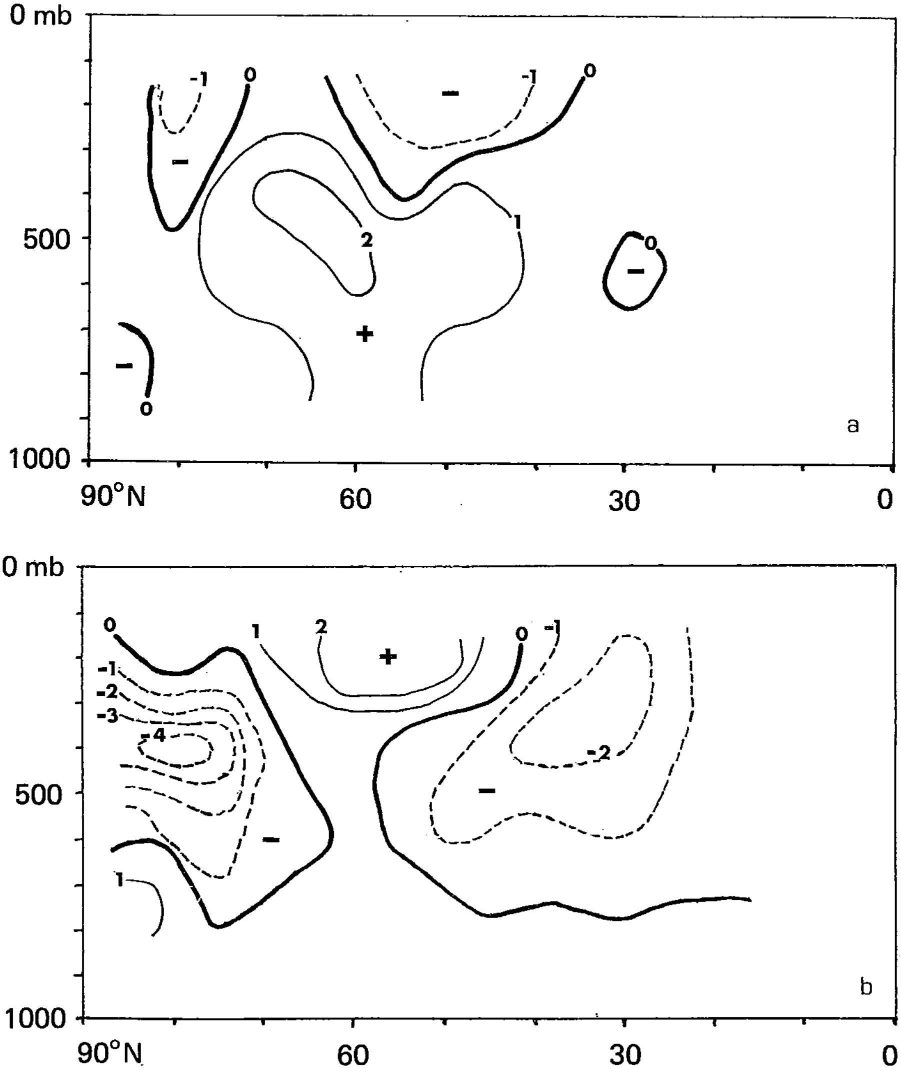

Fig. 6. Zonal averages for the high index case (12/1965). a) $[K]$ b) $[R]$. Unit: $10^6 m^2 s^{-1}$.

part $-\nabla\cdot\frac{\partial\bar{\theta}}{\partial p}\overline{\eta' V'}$ (rms value $0.8 \cdot 10^{-12}\,Ks^{-2}\,mb^{-1}$). The latter term is largely similar to the corresponding forcing term $\hat{k}\cdot\nabla\times A_H \approx -\nabla\cdot\overline{\eta' V'}$ in the mean absolute vorticity balance (Part II, Fig. 3), modified by the horizontal variations in time-mean stability.

The divergence of the mean flux $-\nabla\cdot\bar{q}\,\bar{V}$ is shown in Fig. 8 for comparison. It is

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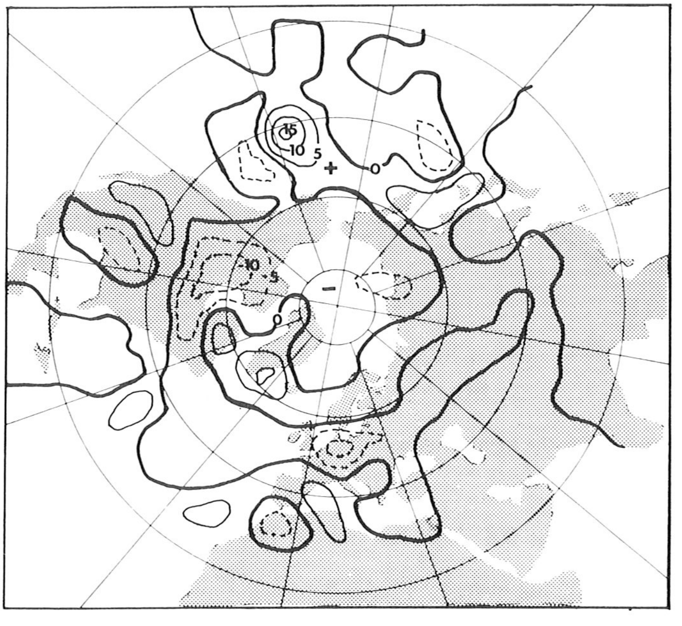

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Fig. 7. Distribution of $-\nabla \cdot \overline{q'V'}$ averaged between 850–300 mb. Unit: $10^{-12}\,K\,s^{-2}\,mb^{-1}$.

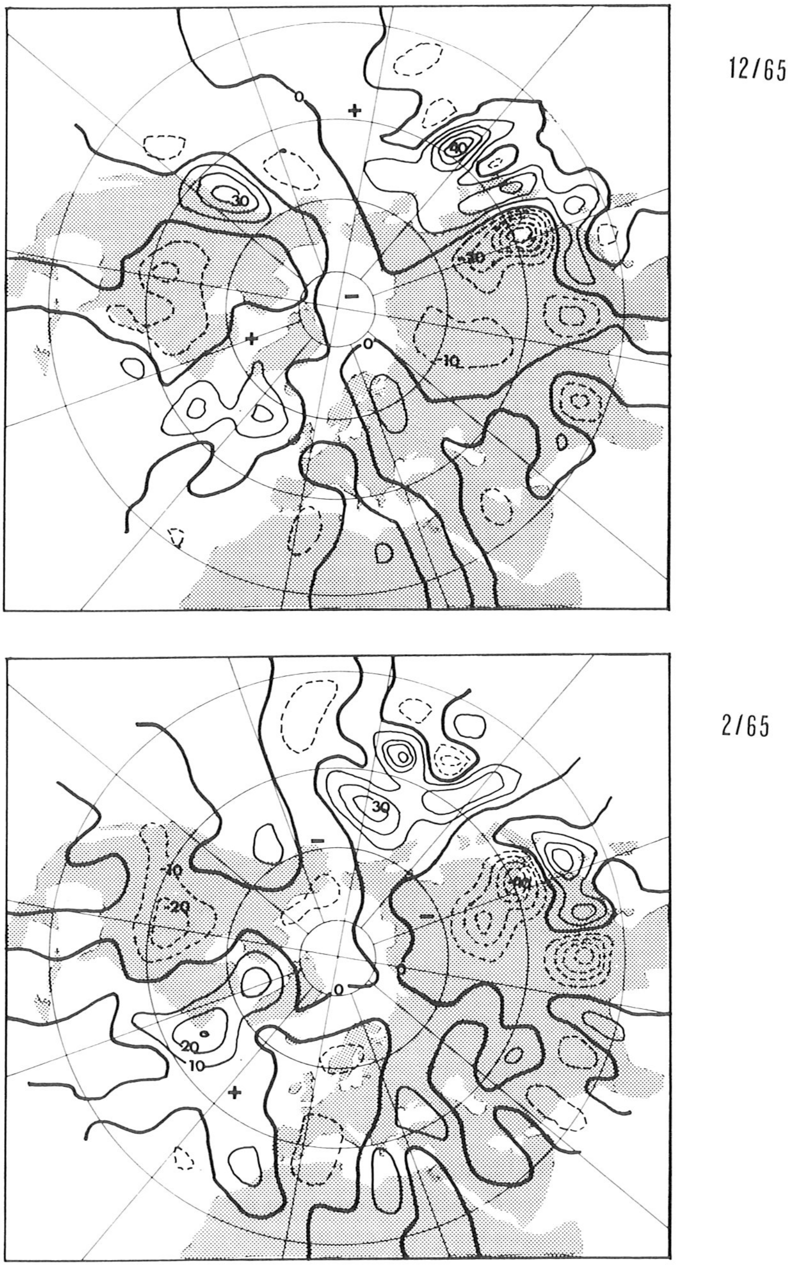

Fig. 8. Distribution of $-\nabla \cdot \overline{q}\,\overline{V}$ averaged between 850–300 mb. Unit: $10^{-12}\,K\,s^{-2}\,mb^{-1}$.

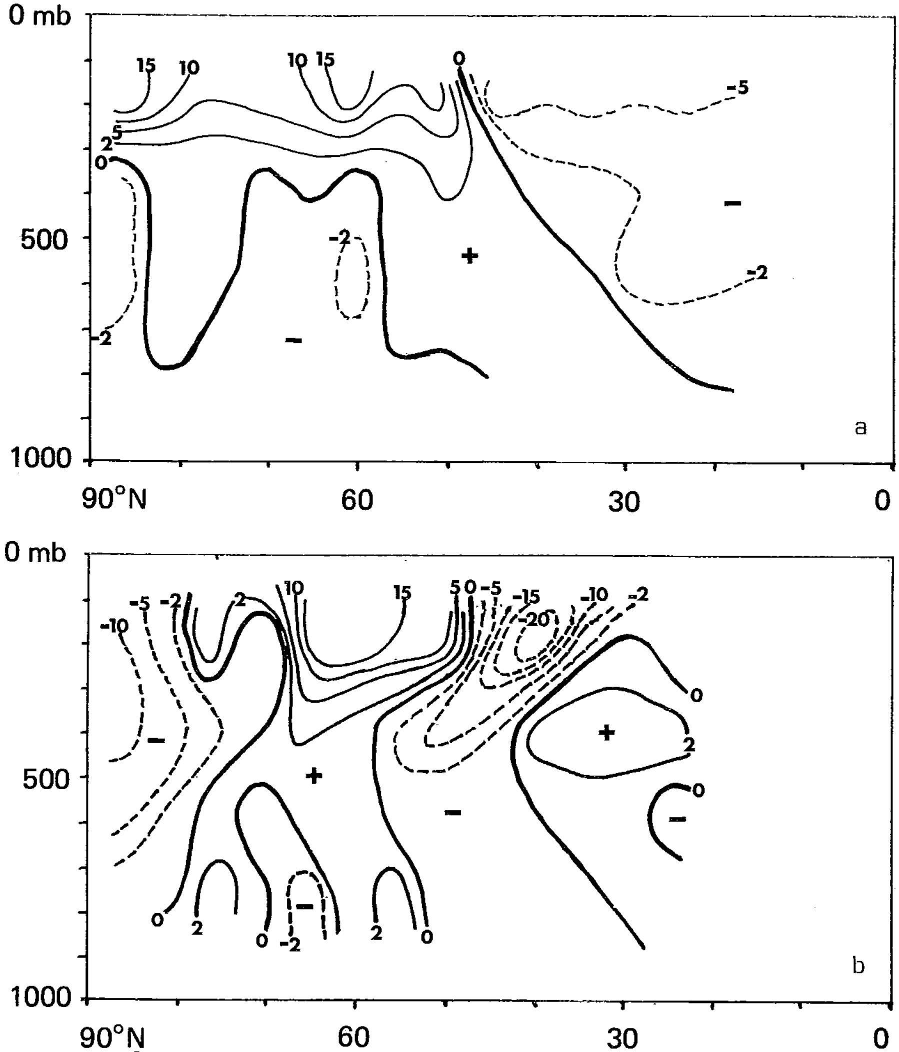

Fig. 9. Zonal averages for the high index case (12/1965). a) $[-\nabla\cdot\bar{q}\bar{V}]$ b) $[-\nabla\cdot\overline{q'V'}]$. Unit: $10^{-12}\,\mathrm{K\,s^{-2}\,mb^{-1}}$.

larger in absolute values than the transient flux divergence term. When averaged zonally (Fig. 9) the difference is smaller. However, in the mean potential vorticity balance the effect of transient eddies does not seem as important as in the mean absolute vorticity balance (HOLOPAINEN, [7]) or in the mean temperature balance (Part II).

4. *Conclusion*

In the two one-month periods studied here, the large-scale turbulence was important in the horizontal transfer of potential vorticity. Its relative importance was slightly less than that in the transfer of absolute vorticity or temperature for the same periods. This is surprising, as potential vorticity is a combination of absolute vorticity and temperature-dependent hydrostatic stability.

The divergence of the transient horizontal flux of potential vorticity $-\nabla \cdot \overline{q'V'}$ has quite a variable structure, not very different in the two cases. The flux itself is mainly down the gradient of time-mean potential vorticity, especially when zonally averaged. It is non-Fickian: there are transient flux components parallel to the mean isolines producing mainly negative cross-gradient diffusion coefficients. In zonal mean values the along and cross-gradient coefficients resemble those of the heat flux for the same data (Part II), indicating universal character for the large-scale mixing of passive quasi-conservative properties of the atmosphere. The resemblance is much weaker locally.

In the time-span of one month, with 12 hours between observations, the transient disturbances (defined as deviations from the arithmetic time-mean) may be due to different physical reasons. The main contribution is undoubtedly due to passing large-scale phenomena, but contributions are also made by the large-scale fluctuation and the trend. How much these motions of very low frequency affect the results is not known, but their existence can be one reason for the large variability often encountered in transient forcing fields.

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