551.465.73:551.467

HEAT FLOW FROM WATER TO ICE

by

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Abstract

Starting from the known equations for heat transfer and ice formation, a numerical method has been developed to calculate the heat flow coming from water to the ice cover. The method has been used in one case.

1. *Conditions prevailing below the ice*

It is well known that in waters of low salinity, as in the Baltic, a convection column exists below the ice cover. (Neumann—Pierson [6].) It extracts heat from the warm layers lying underneath. The mixing process has a tendency to extend downwards until new water masses involved have the same density and temperature as the column itself.

Heat can also be transported to areas below the ice by advection, *i.e.* by horizontal currents. Thus, the convection and advection phenomena together are liable for the heat that interferes with the growth of the lower ice surface.

2. *Earlier works*

In a few papers the heat flow through the ice and snow cover has been treated starting from physical principles. Simpson [8] and Michel [5] have written equations, which govern as well the heat flow in the ice and snow as the formation of ice. Both assume the air temperature and the properties of ice and snow to be known. SIMPSON adds the measurable heat content below the ice to the heat budget and derives then an equation for predicting the time integral of the air temperature (often called »degree days»), which is necessary to create a certain amount of ice.

MICHEL assumes that there is no heat storage below the ice and transforms the resulting differential equation to a finite difference formula to predict the ice growth. Because his equation is implicite, the method is iterative.

Our aim is to calculate the heat flow coming from the water to the ice using an iterative method.

3. *Formula for heat flow*

The temperature gradients in black ice, white ice and snow depend primarily on the difference of the air temperature just above the ice or snow and the freezing temperature of the water and on the corresponding heat conductivities. The influence of the specific heat is only temporary and is therefore discarded here. The formation of ice at lower ice surface depends again as well on the heat flows coming to the interface from the water and going to the ice as on the latent heat of fusion of the ice. The melting and freezing events on top of the ice sheet are disregarded because their effects during the period of ice formation are largely smoothed out during longer spells of time. Thus the heat flow in the black ice, white ice and snow is (*Cf.* SIMPSON [8], MICHEL [5].)

$$\lambda_i \frac{\Delta t_i}{s_i} = \lambda_{si} \frac{\Delta t_{si}}{s_{si}} = \lambda_s \frac{\Delta t_s}{s_s} = \zeta \varrho_i \left(b + \frac{ds_i}{d\tau} \right), \quad (1)$$

where

$\lambda_i, \lambda_{si}, \lambda_s$ = heat conductivity coefficients for black ice, white ice and snow,

$\Delta t_i, \Delta t_{si}, \Delta t_s$ = temperature differences for the lower and upper surfaces of the black ice, white ice and snow respectively,

s_i, s_{si}, s_s = thicknesses of the black ice, white ice and snow,

b = thickness of ice per unit time melted away by the heat flow coming from water to the ice,

ζ = latent heat of fusion of the black ice,

ϱ_i = density of the black ice,

τ = time.

The Eqns. (1) are easely transformed into

$$\frac{ds_i}{d\tau} = \frac{a}{s} - b \,, \tag{2}$$

$$s = s_i + \frac{\lambda_i}{\lambda_s} s_s + \frac{\lambda_i}{\lambda_{si}} s_{si} \,, \tag{3}$$

$$a = \frac{\lambda_i}{\zeta \varrho_i} \Delta t \,, \tag{4}$$

$$\Delta t = \Delta t_i + \Delta t_{si} + \Delta t_s \,. \tag{5}$$

The equation (2) can be integrated to give

$$s_i^{(n+1)} = s_i^{(n)} + \int\limits_{\tau_n}^{\tau_{n+1}} \left(\frac{a}{s} - b\right) d\tau \,, \tag{6}$$

where n is the number of a time step. According to Eqn. (3), s depends on s_i. Therefore the formula (6) is implicite for $s_i(n+1)$. If the ice thicknesses are measured at times τ_h and τ_k, and a mean value for b is used, then

$$s_i^{(k)} = s_i^{(h)} + \sum_{n=h}^{k-1} \int\limits_{\tau_n}^{\tau_{n+1}} \frac{a}{s} \, d\tau - \bar{b}(\tau_k - \tau_h) \,. \tag{7}$$

The ice thicknesses s_i, and thus s too, depend on $\bar{b}$. Therefore Eqn. (7) is implicit for the heat flow $\bar{b}$.

4. *Numercial evaluation method for the heat flow*

It is assumed that all integration steps $\Delta\tau = \tau_{n+1} - \tau_n$ are equal. The integral in Eqn. (7) can then be approximated by a three-point formula

$$\int\limits_{\tau_n}^{\tau_{n+1}} \frac{a}{s} \, d\tau = \frac{\Delta\tau}{12}\left(- \frac{a_{n-1}}{s_{n-1}} + 8\frac{a_n}{s_n} + 5\frac{a_{n+1}}{s_{n+1}}\right), \tag{8}$$

where n refers to the time step as before. After introducing into Eqn. (6), it ensues

$$s_i^{(n+1)} = s_i^{(n)} + \frac{\Delta\tau}{12}\left(-\frac{a_{n-1}}{s_{n-1}} + 8\frac{a_n}{s_n} + 5\frac{a_{n+1}}{s_{n+1}}\right) - \bar{b}\Delta\tau\,. \tag{9}$$

From this $s_i^{(n+1)}$ is determined by a method of NEWTON—RAPHSON. (KUNZ [3].) The iteration formula will be

$$s_i^{(n+1)} = \frac{s_i^{(n)} + \dfrac{\Delta\tau}{12}\left[-\dfrac{a_{n-1}}{s_{n-1}} + 8\dfrac{a_n}{s_n} + 5\dfrac{a_{n+1}}{s_{n+1}}\left(1 + \dfrac{s_i^{(n+1)}}{s_{n+1}}\right)\right] - \bar{b}\Delta\tau}{1 + \dfrac{5\Delta\tau}{12}\dfrac{a_{n+1}}{s_{n+1}^2}}\,. \tag{10}$$

In using this formula, a rough guess for $\bar{b}$, often $= 0$, is made. Values for $s_i^{(n)}, s_i^{(n-1)}$ are thought to be evaluated earlier, but a guessed value for $s_i^{(n+1)}$ must be given. The values for thicknesses of snow and white ice are determined from measured data by linear interpolation. Then all data on the right hand side of Eqn. (10) are either known or they can be obtained from Eqns. (3), (4). As soon as the value for $s_i^{(n+1)}$ has been determined, calculations for the next time step are initiated, etc.

Normally a series of thickness calculations is begun by a given thickness $s_i^{(h)}$ and terminated by a thickness value $s_i^{(k)\prime}$, which corresponds to the given value $s_i^{(k)}$. To reduce the difference of these values to (approximately) zero, Eqn. (7) is used. The correction for $\bar{b}$ will be

$$\delta\bar{b} = -\frac{s_i^{(k)} - s_i^{(k)\prime}}{\tau_k - \tau_h}\,, \tag{11}$$

which should be added to the last value of $\bar{b}$ used. The contribution from the integral in Eqn. (7) is small and is therefore discarded. Only now, a new iteration, *i.e.* a series of thickness calculations, will be undertaken with the values just determined. If $s_i^{(k)\prime}$ and $s_i^{(k)}$ do not agree, then a new correction for $\bar{b}$ will be made, and the iteration procedure will be carried out once more, etc.

5. *Data*

Ice and snow thicknesses were measured by the Institute of Marine Research at Ajos pilot station in the northern Bothnian Bay on Fridays during the winter 1970—71. (See Fig. 1.) First measurement was made on Dec. 11, 1970 and the last one on May 21, 1971. Some of the data were missing and the gaps were filled by values obtained by linear

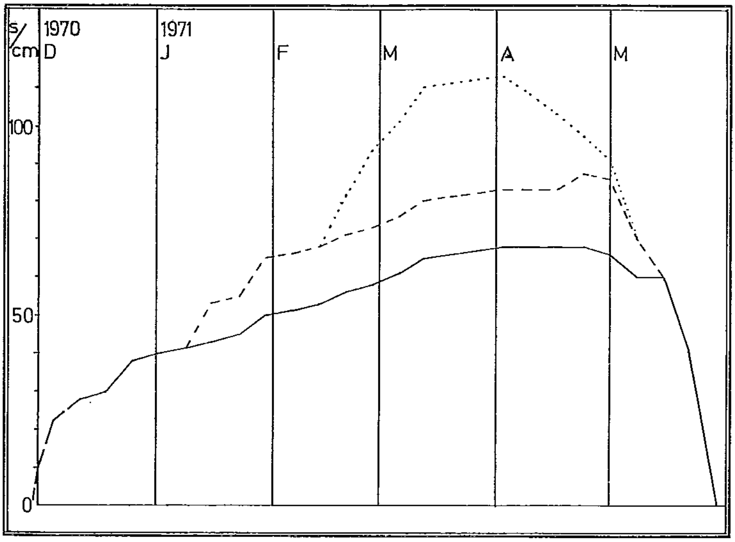

Fig. 1. Ice and snow observations at Ajos. The solid curve shows the thickness of black ice. The dashed line marks the upper surface of the white ice and the dotted line indicates the snow surface. Furthermore, backward calculations for the very first ice thickness have been plotted using long dashes.

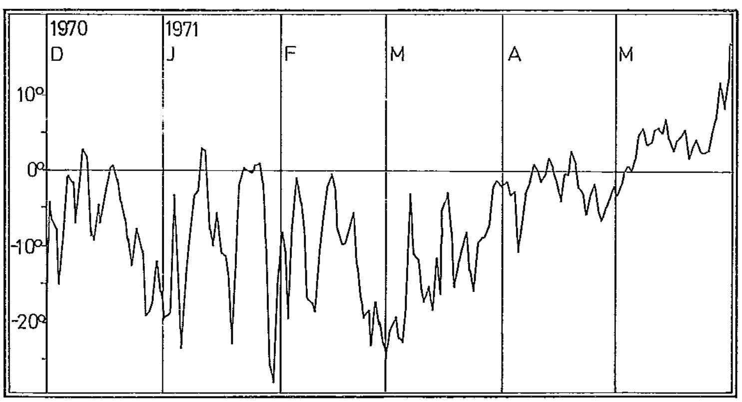

Fig. 2. Daily mean temperatures for Kemi from Dec. 1, 1970 to May 31, 1971.

interpolation. The interval of measurements, *i.e.* one week, was used as the value of $\tau_k - \tau_h$.

Temperature data for Ajos are not available. Instead, we could use daily mean temperature values of Kemi (See Fig. 2.), a city lying nearby*. The interval of temperature data, *i.e.* one day, was used for $\Delta\tau$.

As latent heat of fusion of the ice a value 68.6 cal/g was adopted corresponding to a salinity of 2‰. (LANDOLT—BÖRNSTEIN [4].) As heat conductivity of the black ice and snow the figures 389 cal/(cm °C d) and 62 cal/(cm °C d) were used. (SIMPSON [8].) The heat conductivity of white ice was chosen to be 195 cal/(cm °C d). For the density of the black ice a value 0.9 g cm^{-3} was taken.

6. *Results*

Calculated heat flows are reproduced in Fig. 3. At the beginning of the ice winter, the heat flow from the water to the ice was relatively large, but became smaller during the melting period in the spring. The heat flow from water to ice was averaged to 22.5 cal/(cm^2d) during the ice formation time of 20 weeks. It is learned from the figure that some negative values for the heat flow were obtained too.

It is believed that at the begin of the ice formation some thickness measurements were missing. Therefore no heat flow calculations were performed for this time, but instead, a tentative calculation to obtain

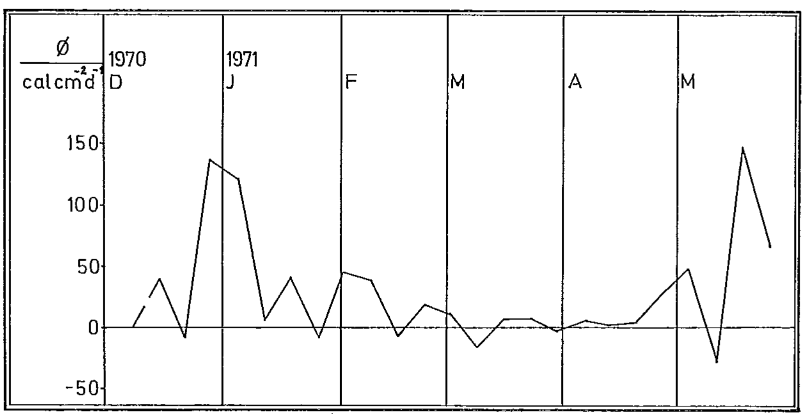

Fig. 3. Calculated heat flows from water to ice at Ajos. The first segment indicated by dashes is a rough guess only, used for the backward calculations of the ice thickness.

* Temperature observations were made by the FINNISH METEOROLOGICAL INSTITUTE, Helsinki, Finland.

the missing thicknesses was carried out on the assumption that there was no heat flow whatsoever from the water. This calculation shows that the first ice could have been formed on Oct. 29, 1970, or earlier.

7. *Discussion*

The large heat pulses at the begin of the ice formation period (See Fig. 3.) may indicate that convection is not the only source for the heat flow from the water to the ice, but a large amount of it may due to advection. The heat pulses smooth out, when time goes on. This is understandable though, because the heat storages become more and more depleted at least in areas nearby, where the heat to a large extent originates from. When the melting period arrives, then the heat flows obtained are larger, but they are probably spurious for reasons explained later.

The negative heat flow values obtained, which seem to indicate that heat may have been transferred from the ice to the water, may be resulted from »errors» in the thickness measurements. To obtain a new thickness value, a new hole is namely borred and, because of slight variations of the thickness, the observed values have statistical differences. An error of 1 cm in a weekly thickness measurement causes an error of about 8.8 cal/(cm^{2}d) in a heat flow value. Also a possibility for too large conductivity values must be accounted for. The numerical experiment performed to reduce the conductivities was not too encouraging.

One factor affecting the heat flow to the atmosphere is the constitution of the ice and snow. The conductivity of sea ice is a function of the brine content (Anderson [1]), which changes with time. Salt has a tendency to move downwards during temperature variations, but new salt may be introduced on top of the ice, when it is loaded by so much snow that sea water will be percolated through pores in ice. (Palosuo [7].) Snow on the ice will then be wetted and its water content may still be increased by thawing snow. By cold weather, layers of different constitution may be formed. The new formations are of white ice. This icing event is not introduced into our model, but because of the good conductivity properties of the ice, it does not make any large difference wheather the icing takes place above or beneath of the ice sheet. The conductivity of white ice is indeed smaller than the one of black ice, but we do not know exact values because of the inhomogeneity of white ice.

Above the white ice layer, there is mostly snow, which may also have different characteristics depending on its age: conductivity of just fallen snow is low compared to that of older snow.

Weather is the primary factor influencing on the heat flow through ice and snow. Those temperature data, which we have used, are for Kemi, a city a few kilometers north of Ajos. Temperature varies indeed heavily in the direction across the coastline at least if open sea lies in the neighbourhood. Rather large differences may also be observable with the height depending primarily on the cloudiness of the sky. Temperatures, appropriate for our theory, should have been measured just at the interface of the ice or snow and the air above. Therefore the temperature data used may not be very suitable.

The radiation from the sun and the radiation back into the space depend largely on the cloudiness. When there is no snow on the ice, as it was in December 1970 and most of January 1971, the radiation can in principle penetrate into the ice depending heavily on the quality of the ice surface. Indeed, the sun lies then during daytime but a few degrees above the horizon and most of the radiation will then be reflected back from the ice surface. The maximum value of the total radiation for the snow-free period was estimated to less than 20 cal/(cm^{2}d). (Ilmatieteen laitos [2].) Therefore, the existed radiation was really of no harm. Later on in the spring, when the radiation already is appreciable, the albedo of the snow determines the amount of heat generated by the absorbtion. For just fallen snow, the albedo may be as high as 90% and thus only 10% of the radiation penetrates into the snow. During most of the spring, the snow is relatively pure and consequently the albedo is high. The heat absorbed forms in any case a new heat source at the top layers of the snow and will therefrom be conducted in every direction. Because of the low heat conductivity of the snow, most of the heat will be transferred back to the atmosphere and a small fraction only will penetrate downwards. During the late spring again, the total radiation falling onto the snow surface is rather large and the albedo of the snow becomes low. Furthermore, the sun rises high above the horizon and the ambient temperature melts snow. Therefore, the conductivity properties of the snow mixed with water suffer serious changes, and the method becomes unreliable from this point onwards.

The influence of humidity and wind is small, when the air temperature and therefore also the evaporation are low. Yet, the evaporation grows larger with the rising temperature, i.e. primarily in the late spring and its cooling effect adds to other effects caused by the high temperature.

Acknowledgments: This work has been initiated at the Institute of Marine Research and completed at the Geophysical Department of the University of Helsinki. The author is greatly indepted to Drs. E. Palosuo and J. Virta for valuable criticism of the work.

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