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AN ATTEMPT TO DETERMINE THE EFFECTS OF TURBULENT FRICTION IN THE UPPER TROPOSPHERE FROM THE BALANCE REQUIREMENTS OF THE LARGE-SCALE FLOW: A FRUSTRATING EXPERIMENT

by

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Abstract

All but the frictional terms in the equations of motion and the kinetic energy equation were evaluated at the 300 mb level on a daily basis using aerological data from six British stations (period 1961—65) and from four German stations (period 1962—69). The residual terms necessary to satisfy the equations were computed and their significance in representing the effects of turbulent friction due to subsynoptic motion systems was discussed.

Statistical analysis of the computed forces shows a high degree of balance in the equations of motion between the pressure gradient force and the Coriolis force: the geostrophic balance. A tendency toward a secondary balance was found between the horizontal advection of momentum and the sum of the pressure gradient and Coriolis forces. The local change of momentum is generally smaller than the horizontal advection of momentum. The terms representing vertical advection of momentum were computed using the thermal wind equation and the adiabatic vertical velocities. Both these and the »metric» acceleration terms appear to be relatively small.

An analysis of errors, inevitably involved in the numerical evaluation of the different terms, was made and the significance of the computed residual force discussed; this force has a large day-to-day variability and a relatively large long-term mean value. It is concluded that the variance of the residual force arises mainly from the random errors made in evaluation of the pressure gradient force. The time-mean values of the residual force probably do not represent the mean frictional force but rather a systematic error in the pressure gradient force. The probable distribution of the associated systematic error in height was determined. The question whether or not the mean residual force is due to systematic errors or at least partially represents the mean effect of the sub-grid scale friction, cannot, however, be definitely answered.

Analysis of the kinetic energy balance shows that on the average there is destruction of kinetic energy of the large-scale motion over the British Isles due to the work done by pressure forces. This destruction is largely compensated for by horizontal advection of kinetic energy. The residual term, which in principle represents the work done by frictional forces, has a statistically significant positive mean value; the physical significance of this (energy input from the subsynoptic to synoptic scales) is, however, questionable.

One must conclude that any attempt similar to this to determine the effects of turbulent friction in the upper troposphere or stratosphere from the balance requirements of the large-scale flow is likely to give inconclusive and frustrating results. The main contribution of such an attempt may be some information about observational errors.

1. *Introduction*

When one tries to construct a model of the large-scale behaviour of the atmosphere either for the simulation of the general circulation of the atmosphere or for the purpose of long-range numerical weather prediction, a question of major importance and difficulty is how to take forcing by the sub-grid scale motion systems or the turbulent frictional force into account. This force is known to be important, and can roughly be accounted for in the boundary layer of the atmosphere (*e.g.* Zilitinkevich [15]). During the last ten years some studies (Holopainen [4]; Kung [6], [7]; Trout and Panofsky [12]; Vinnichenko [13]) using different approaches have suggested that the turbulent frictional forces and the associated dissipation of (large-scale) kinetic energy may also be important in the free atmosphere, particularly in the upper troposphere close to the tropopause and the level of maximum wind.

Some numerical experiments on the general circulation of the atmosphere (*e.g.* Everson and Davies [2]) indicate that the statistical characteristics of a model circulation are very sensitive to the formulation of the internal friction due to sub-grid scale effects in the model. At present we do not know appropriate means for taking these effect into account. Even their order or magnitude is not very well known and practically nothing is known about how these forces should be parametrized in terms of the large-scale flow.

The turbulent frictional force acting on the large-scale flow can formally be written as the divergence of the momentum flux due to turbulent eddies, the space scale of which is smaller than in motion systems which are explicitly taken into account in the model. The total horizontal frictional force is the sum of lateral frictional force (due to the horizontal stress components) and the force due to vertical eddy stresses.

The calssical way of parametrizing the eddy stresses in terms of the large-scale flow is to write them in analogue with the molecular stresses proportional to the gradient of the corresponding component of the large-scale velocity. Very little is known about the proportionality coefficients which enter the picture. In principle these coefficients must depend on the size of the grid used to describe the large-scale flow. For a horizontal grid size of a few hundred kilometers, the coefficient for the horizontal exchange (lateral eddy viscosity coefficient) has been assigned value of the order of $10^5 - 10^6$ m^2 sec^{-1}. Values varying from 0 to 100 m^2 sec^{-2} have been suggested for the corresponding coefficients in the vertical direction in the free atmosphere.

Based upon the Heisenberg similarity theory SMAGORINSKY [11] derived a more sophisticated scheme which subsequently has been much used for the lateral friction force. In his formulation, which presupposes a »—5/3»-spectrum of kinetic energy at the scale of the grid size, the horizontal frictional force due to the lateral eddies (and also the appropriate lateral eddy viscosity coefficient involved) becomes dependent upon the field of horizontal deformation.

Empirical data on the spectrum of kinetic energy in the atmosphere do not, however, give any clear support to the »—5/3» law at the scales of a few hundred kilometers. In fact, some empirical evidence has recently been obtained for a »—3» law which can also be theoretically predicted for a two-dimensional flow. The appropriate coefficient of lateral eddy viscosity should be proportional (LEITH [9]) to the gradient of the magnitude of the relative vorticity for this kind of flow.

At present we do not have any direct empirical data from the free atmosphere, on the basis of which we could judge which one of the formulations so far presented is physically best.

The most straightforward way of empirically investigating turbulent friction would in principle be to undertake special field experiments to measure the sub-grid scale velocity fluctuations (both vertical and horizontal) and the associated flux of momentum. Due to instrumental and observational difficulties and the enormous cost of such experiments the application of this method has so far been confined mainly to the lower layers of the atmosphere, where the horizontal frictional force is almost entirely due to the vertical eddy stresses.

Another method of investigating the turbulent frictional force is to infer it from the balance requirements of the large-scale flow. If from observations we know the field of large-scale motion we can in principle at any time evaluate the field of pressure gradient and Coriolis forces and field of acceleration. The forcing effect of the sub-grid scale motion systems can then be obtained as a residual force, which is needed for the large-scale flow to satisfy the equations of motion.

The limitations and pitfalls of this residual technique are well-known. Due to the quasi-geostrophic character of the large-scale flow in the atmosphere the pressure gradient force and Coriolis force are relatively large terms and almost counterbalance one another. Errors necessarily arise in numerical evaluation of these forces and of the acceleration term for instance due to random observational errors. The quasibalance between the dominating terms then inevitably leads to large errors in the residual force, even though the percentage errors in each of the dominating terms were small. For this reason, the application of the residual technique is meaningful, if anywhere, only in a dense network of aerological stations which all use the same type of radiosonde. In a heterogeneous network of aerological stations systematic errors in the pressure gradient force can be expected, particularly in the upper layers. Furthermore, in order to have some hope of getting the systematic frictional forces separated from the noise caused by different kinds of errors, one should use a long series of data.

The residual techniques in an attempt to compute the vertical distribution of the zonal and meridional component of the time-averaged frictional force was first used by KURIHARA [8], who made a numerical analysis of the equations of motion using aerological data from three Japanese aerological stations for a period of 60 days. The magnitudes of the components of the mean residual (frictional) force turned out to be large throughout the whole troposphere. These forces cannot, however, be interpreted physically and probably are mainly to be considered (KURIHARA, personal communication) as systematic errors arising from the computational scheme. The present author (HOLOPAINEN [5]) made similar calculations for the area of the British Isles using aerological data from the 3-month period September—November 1954. The time-averaged residual force below about 800 mb could be explained as the mean frictional force in the planetary boundary layer. In the free atmosphere up to 200 mb, the zonal component of the mean residual force turned out to be negligibly small; for the meridional component, however, unexplainably large negative mean values were obtained in the upper troposphere. It was concluded that in these upper layers the mean residual force, obtained from the 3-months' material in question most likely did not represent the real mean frictional force.

By using North American aerological data from a five year period KUNG [6], [7] determined the mean dissipation of kinetic energy at different levels as a residual term in the equation of kinetic energy for the large-scale motion. Besides the boundary layer, his results indicate a secondary layer of energy loss in the upper troposphere. Whether the results obtained for the upper layers indeed represent the real mean energy dissipation or merely the effect of systematic errors is not known for sure.

The purpose of the present study was to study the balance of forces and kinetic energy at the level of the indicated secondary maximum of frictional effects in the upper troposphere. For this purpose numerical analysis of the equations of motion and the kinetic energy equation was done for the 300 mb level using long series of aerological data from two European regions. Besides the long-term mean values of different forces their time-variation was also considered. The object of primary interest was the sub-grid scale frictional effects, which are computed as residual terms. Considerable emphasis in the paper is given to error analysis and the subsequent discussion of the physical significance of the computed residual terms.

2. *Data and the scheme of numerical analysis*

The data used in this investigation are the observations (at 00 and 12 GMT) of height, wind and temperature at 300 mb from the six British and the four German aerological stations appearing in Fig. 1. The British data (taken from »Daily Aerological Record of the Meteorological Office, London») cover the 5-year period 1961—65, the German data (taken from »Täglicher Wetterbericht, Amtsblatt des Deutschen Wetterdienstes, Teil C: Aerologische Beobachtungen») the 8-year period 1962—69. The main part of the paper is concerned with results obtained for the area of the British Isles.

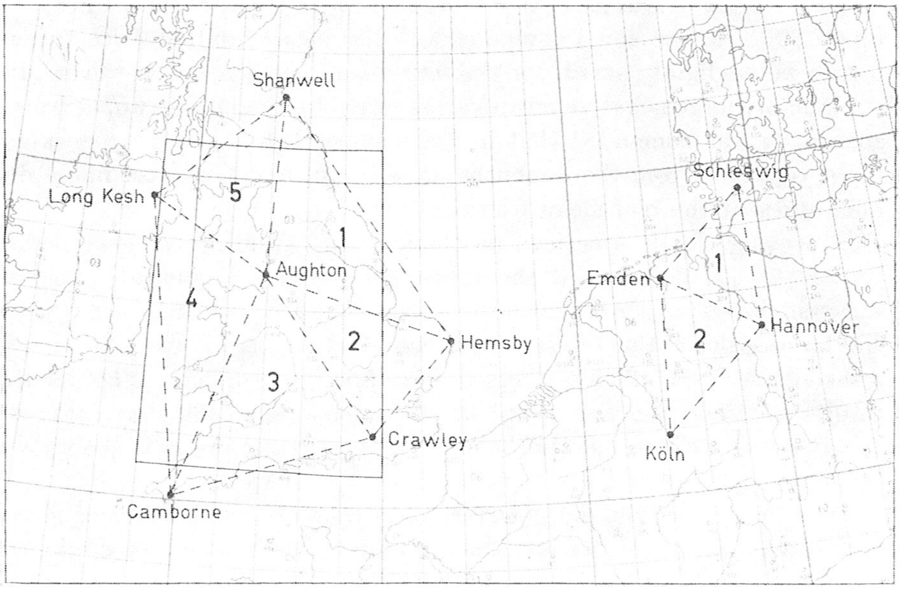

Fig. 1. The network of aerological stations used in calculations. The main part of the paper concerns the results obtained for the area of the British Isles.

The equations of horizontal motion in a spherical λ, φ, p-coordinate system can be written as

$$\underset{(1)}{\frac{\partial u}{\partial t}} + \underset{(2)}{\mathbf{v} \cdot \nabla u} + \underset{(3)}{\omega \frac{\partial u}{\partial p}} - \underset{(4)}{uv \frac{\tan \varphi}{a}} + \underset{(5)}{\frac{1}{a \cos \varphi} \frac{\partial \phi}{\partial \lambda}} - \underset{(6)}{fv} = R_\lambda \qquad (1)$$

$$\underset{(1)}{\frac{\partial v}{\partial t}} + \underset{(2)}{\mathbf{v} \cdot \nabla v} + \underset{(3)}{\omega \frac{\partial v}{\partial p}} + \underset{(4)}{uu \frac{\tan \varphi}{a}} + \underset{(5)}{\frac{1}{a} \frac{\partial \phi}{\partial \varphi}} + \underset{(6)}{fu} = R_\varphi \qquad (2)$$

where R_λ and R_φ denote, respectively, the zonal and meridional component of the turbulent frictional force; otherwise the notation in Eqs. (1) and (2) is conventional.

In this study all the terms (1)—(6) on the left-hand side of the above equations have been evaluated from observations; the sum of the six terms from Eqs. (1) and (2) is then, in principle, an estimate of the zonal and meridional component of the frictional force. Because R_λ and R_φ here contain in addition to the real frictional effects also the errors made in evaluation of the left-hand side terms it is better to refer to R_λ and R_φ as the zonal and meridional component of the residual force, which in our scheme and analysis is needed in order to satisfy the equations of motions.

For the purpose of numerical analysis Eqs. (1) and (2) are averaged in the present work over a 12-hour period and over the horizontal area formed by the six/four British/German stations (see Fig. 1). The first terms in Eqs. (1) and (2) are then easily computed from the change of the area-averaged zonal/meridional velocity component during the period. The 12-hour average of each of the terms (2)—(6) has been computed as an arithmetic mean of the values obtained from aerological data at the beginning and the end of the period. Only those 12-hour periods which had complete observations of geopotential, temperature and wind at both ends at all six/four stations were used.

Two methods of describing the horizontal distribution of different quantities were used. In the first one, called the second-degree method and applied over the British Isles, the horizontal distribution of an arbitrary quantity q is assumed to have the form

$$q(\lambda, \varphi) = c_1\lambda^2 + c_2\varphi^2 + c_3\lambda\varphi + c_4\lambda + c_5\varphi + c_6 , \tag{3}$$

where c_i $(i = 1, \ldots, 6)$ are constants to be determined from the observations. In matrix notation we can write

$$q = Ac , \tag{4}$$

where q is the vector formed by the observations of q at the six stations (see Fig. 1), c is the vector formed by the coefficients in Eq. (3), and A is a 6×6 matrix:

$$A = \begin{bmatrix} \lambda_1^2 \; \varphi_1^2 \; \lambda_1 \; \varphi_1 \; \lambda_1 \; \varphi_1 \; 1 \\ \cdots\cdots\cdots\cdots \\ \lambda_6^2 \; \varphi_6^2 \; \lambda_6 \; \varphi_6 \; \lambda_6 \; \varphi_6 \; 1 \end{bmatrix} \tag{5}$$

(The subindex in (5) refers to station.) From Eq. (4) we have

$$c = A^{-1}q\,. \tag{6}$$

The area averages of the different terms in Eqs. (1) and (2) except the third term can now be computed with the aid of coefficients c_i. The averaging has been done over the area confined in Fig. 1 by the solid line.

In the second method used for depicting the horizontal distributions, called the linear method, all variables were assumed to be linear functions of λ and φ in a triangle formed by three observing stations (see Fig. 1). All the terms in Eqs. (1) and (2) could then be evaluated separately for each triangle. The final values of the different terms used in the subsequent analysis were obtained by taking an area-weighted average over the five triangles in the case of the British data and over the two triangles in the case of the German data.

If not otherwise mentioned, the results presented in the following sections for the area of the British Isles are those obtained by the second-degree method.

The determination of the vertical advection terms (terms (3) in Eqs. (1) and (2)) presupposes the knowledge of vertical velocity $\omega\left(=\frac{dp}{dt}\right)$. In the earlier studies by KURIHARA [8], HOLOPAINEN [4], [5] and KUNG [6], [7] ω was computed from the horizontal wind observations with the aid of the continuity equation. Because data in this study were available only for the 300 mb level this »kinematic method» could not be applied. ω has here been determined by the »adiabatic» method. By using constant long-term monthly mean values of the static stability (computed from the mean vertical temperature profiles at Larkhill) an estimate of ω could be obtained with this method from the 300 mb data alone. The derivatives $\frac{\partial u}{\partial p}$ and $\frac{\partial v}{\partial p}$ appearing in the vertical advection terms cannot be directly determined from wind data for one level only. An approximate value of these terms was computed in the present study from the 300 mb temperature data using the thermal wind relationship. In the final determination of the values of the vertical advection terms the area-averaged values of the estimates of ω, $\frac{\partial u}{\partial p}$ and $\frac{\partial v}{\partial p}$ were used. Thus the covariance of the deviations of the vertical velocity and the wind inside the area was assumed to be zero. This is probably not a serious assumption. In fact, the earlier studies by HOLOPAINEN [4], [5] and KUNG [6], [7] indicate that the vertical advection terms in the equation of motion and the kinetic energy equation have a very small magnitude compared with other terms; therefore, even a large percentage error in the numerical evaluation of this term has very little influence on the results obtained for the residual terms.

3. *Results from the analysis of the equations of motion*

In this section the results obtained from the analysis of the equations of motion for the area of the British Isles are discussed. With regard to the terms (1)—(6) in these equations the results for the German area are similar to those obtained for the British Isles. Therefore for the German area only the computed residual forces will be given in section 5.

The total number of the 12-hour periods for which complete observations existed at 300 mb at all six British stations during 1961—65 was 3319. This is 91 per cent of all possible 12-hour periods during the five year period: an excellent performance of the observational system.

The statistics of the area-averaged flow over the British Isles during 1961—65 is typical of the conditions in the free atmosphere in the middle latitudes with almost westerly mean flow ($\bar{u} = 12.8$ m sec^{-1}, $\bar{v} = -2.1$ m sec^{-1}) and with large variability ($\sigma_u = 16.2$ m sec^{-1}, $\sigma_v = 16.1$ m sec^{-1}). The equipartitioning of kinetic energy between the zonal and meridional components in the fluctuations, as indicated by the similarity of the standard deviations σ_u and σ_v, is also typical of the middle latitude conditions.

The results from the analysis of the terms in Eq. (1) and (2) are given in Table 1 and Table 2 respectively. In the following only the results obtained for the entire 5-year period (lower part of Tables 1—2) will be discussed; the results for different months are seen to be rather similar except that the variability of the terms during winter/summer is somewhat larger/smaller than on the average.

The long-term mean balance of forces is quite different in zonal and meridional directions. For both directions the mean values of the first terms, representing the local change of momentum, are practically zero. In the zonal direction (Table 1) the mean pressure gradient force (—0.63 units), mean Coriolis force (2.45 units) and mean horizontal advection of zonal momentum (—1.35 units) are all important, whereas the mean vertical advection and the mean metric acceleration are negligibly small.

Table 1. Mean values (upper numbers) and standard deviations (lower numbers in italics) of the different terms in the equation of zonal motion at 300 mb over the British Isles during the 5-year period 1961—1965. N = number of the 12-hour periods for which the calculations were made. Unit: 10^{-4} m sec^{-2}.

Month	N	(1) $\frac{\partial u}{\partial t}$	(2) $\mathbf{v} \cdot \nabla u$	(3) $\omega \frac{\partial u}{\partial p}$	(4) $-uv \frac{\tan \varphi}{a}$	(5) $\frac{1}{a \cos \varphi} \frac{\partial \phi}{\partial \lambda}$	(6) $-fv$	R_λ
I	284	0.01 *2.41*	−1.60 *5.91*	−0.01 *0.39*	0.00 *0.73*	−3.11 *15.42*	5.14 *16.46*	0.43 *5.44*
II	255	−0.02 *2.25*	−1.69 *4.71*	0.01 *0.21*	0.13 *0.86*	−5.23 *19.46*	7.35 *19.04*	0.55 *5.44*
III	279	0.12 *2.22*	−0.74 *4.16*	0.00 *0.30*	0.10 *0.65*	−1.83 *16.46*	3.54 *16.29*	1.19 *4.61*
IV	278	0.03 *1.95*	−1.96 *5.32*	0.07 *0.33*	0.17 *0.67*	1.05 *18.31*	1.46 *19.57*	0.82 *4.68*
V	302	−0.06 *2.30*	−2.03 *4.70*	0.01 *0.30*	0.06 *0.68*	−1.09 *19.46*	4.23 *18.33*	1.13 *5.56*
VI	266	−0.02 *1.75*	−0.72 *4.97*	0.01 *0.65*	−0.13 *0.59*	2.94 *17.84*	−0.84 *17.17*	1.23 *5.23*
VII	296	0.05 *1.81*	−0.92 *4.95*	0.06 *0.52*	0.06 *0.64*	−2.23 *18.32*	2.39 *16.85*	−0.05 *4.88*
VIII	283	0.08 *1.92*	−1.16 *5.19*	0.09 *0.66*	−0.03 *0.80*	1.76 *18.41*	0.36 *17.37*	0.94 *5.62*
IX	276	−0.02 *1.93*	−0.97 *5.95*	0.04 *0.52*	−0.05 *0.85*	4.72 *22.33*	−2.79 *21.75*	0.93 *5.27*
X	282	0.01 *2.06*	−1.51 *4.80*	0.04 *0.47*	−0.01 *0.72*	3.20 *18.67*	−1.49 *18.85*	0.23 *5.03*
XI	247	0.04 *2.18*	−1.23 *5.12*	−0.02 *0.50*	0.09 *0.76*	−3.16 *18.63*	3.78 *19.15*	−0.49 *5.09*
XII	271	0.04 *2.44*	−1.65 *6.29*	0.04 *0.41*	0.09 *1.11*	−5.05 *19.94*	6.00 *22.04*	−0.53 *5.68*
Year	3319	0.01 *2.09*	−1.35 *5.29*	0.03 *0.46*	0.04 *0.77*	−0.63 *18.93*	2.45 *18.87*	0.54 *5.26*

Table 2. Mean values (upper numbers) and standard deviations (lower numbers in italics) of the different terms in the equation of meridional motion at 300 mb over the British Isles during the 5-year period 1961—1965. Unit: 10^{-4} m sec^{-2}.

Month	N	(1) $\frac{\partial v}{\partial t}$	(2) $\mathbf{V} \cdot \nabla v$	(3) $\omega \frac{\partial v}{\partial p}$	(4) $uu \frac{\tan \varphi}{a}$	(5) $\frac{1}{a} \frac{\partial \phi}{\partial \varphi}$	(6) fu	R_φ
I	284	0.01 *3.46*	−1.12 *6.86*	−0.04 *0.38*	1.12 *1.26*	−14.75 *18.65*	13.77 *20.35*	−1.01 *4.83*
II	255	0.08 *3.17*	−1.53 *5.78*	−0.02 *0.38*	0.99 *1.24*	−8.95 *20.62*	7.70 *21.86*	−1.71 *4.77*
III	279	0.04 *2.60*	−2.20 *5.51*	−0.01 *0.31*	0.84 *0.86*	−10.87 *16.12*	11.81 *17.40*	−0.39 *3.97*
IV	278	0.05 *2.84*	−1.13 *4.93*	−0.07 *0.30*	0.82 *0.86*	−14.25 *16.22*	13.55 *16.19*	−1.03 *4.31*
V	302	0.02 *2.78*	−1.21 *6.49*	−0.10 *0.39*	1.05 *1.09*	−16.30 *18.90*	15.66 *18.29*	−0.87 *4.00*
VI	266	0.07 *2.82*	−0.73 *5.79*	−0.11 *0.49*	1.10 *1.32*	−18.70 *19.15*	16.64 *18.13*	−1.73 *4.27*
VII	296	0.02 *2.54*	0.00 *4.90*	−0.06 *0.50*	0.98 *0.95*	−18.50 *17.61*	16.61 *15.64*	−0.96 *4.07*
VIII	283	−0.07 *2.89*	−1.08 *6.94*	−0.10 *0.43*	1.30 *1.16*	−25.21 *16.72*	22.83 *14.80*	−2.32 *4.70*
IX	276	−0.07 *3.06*	−0.98 *6.71*	−0.05 *0.36*	1.05 *1.14*	−18.20 *17.39*	17.15 *16.95*	−0.93 *4.57*
X	282	0.08 *5.08*	−0.71 *5.83*	−0.11 *0.63*	0.85 *0.98*	−15.47 *17.94*	13.48 *16.33*	−2.03 *4.60*
XI	247	−0.02 *3.02*	−1.72 *7.00*	−0.07 *0.94*	1.21 *1.42*	−14.24 *22.70*	12.94 *22.49*	−1.91 *4.34*
XII	271	−0.08 *3.61*	−1.73 *7.78*	−0.07 *0.42*	1.49 *1.59*	−17.99 *23.05*	17.00 *23.77*	−1.39 *5.16*
Year	3319	0.01 *3.00*	−1.16 *6.28*	−0.07 *0.49*	1.06 *1.19*	−16.19 *19.23*	15.01 *18.96*	−1.34 *4.51*

The significance of the relatively large mean residual force in the zonal direction (0.54 units) will be discussed in section 5.

The long-term average of the equation of motion in the meridional direction is (Table 2) essentially reduced to the equation of geostrophic balance: the mean pressure gradient force (—16.19 units) and the Coriolis force (15.01 units) are clearly the dominating terms. From the other terms (1)—(6) the mean horizontal advection of southerly momentum (—1.16 units), and the mean metric acceleration term (1.06 units) are next in magnitude; the mean vertical advection term is negligible also in the meridional direction. The residual force (—1.34 units) has a surprisingly large magnitude. The significance of this term, which according to Table 2 is negative not only in the 5-year average but also in all months, will be discussed in section 5.

Now considering the standard deviations of different terms in the entire 5 year sample, one can see in Tables 1—2 that these are rather similar in zonal and meridional directions for each term. This is natural in light of the energy equipartitioning referred to above between the two velocity components in the transient fluctuations. Therefore we can with the aid of these standard deviations and the correlation coefficients between different terms (Table 3) discuss the balance of forces in the

Table 3. Correlation coefficients between the dominating terms in the equations of motion at 300 mb over the British Isles, computed from a 5-year data sample (N = 3319). The numbers refer to Eqs. (1) and (2) on page 156. Unit for standard deviations is 10^{-4} m sec^{-2}.

	Term number	(1)	(2)	(5)	(6)	(5)+(6)	(2)+(5)+(6)	Standard deviation
Zonal component	(1)	1.000	−.337	−.048	.077	.093	−.225	2.09
	(2)		1.000	.082	−.251	−.534	.387	5.29
	(5)			1.000	−.951	.167	.262	18.93
	(6)				1.000	.147	−.083	18.87
	(5)+(6)					1.000	.573	5.95
	(2)+(5)+(6)						1.000	5.45
Meridional component	(1)	1.000	−.486	.015	.042	.208	−.393	3.00
	(2)		1.000	.096	−.271	−.633	.595	6.28
	(5)			1.000	−.963	.187	.315	19.23
	(6)				1.000	.085	−.251	18.96
	(5)+(6)					1.000	.246	5.21
	(2)+(5)+(6)						1.000	5.02

transient motion systems in general without referring to any of the two components in particular. We first notice that the geostrophic balance is also good for transient motion: the horizontal pressure gradient force and the Coriolis force clearly dominate in magnitude (both having a standard deviation of about 19×10^{-4} m sec^{-2}) and largely counterbalance each other as seen from the correlation coefficients (0.95 and -0.96 in the zonal and meridional directions, respectively).

The sum of the pressure gradient and Coriolis forces has a standard deviation (see Table 3) of 5.95 (5.21) units in the zonal (meridional) direction and is thus, in the transient motion systems, of the same order of magnitude as the horizontal advection of momentum. The correlation between the sum of pressure gradient and Coriolis forces and the horizontal advection of momentum is -0.53 (-0.63) in the zonal (meridional) direction according to Table 3. In addition to the high degree of the geostrophic balance the results thus indicate a tendency to a secondary balance in the equations of motion.

The degree of the secondary balance and the relative magnitude of the local change terms must depend upon the frequency of the oscillations which produce the main part of the transient variations: the balance is presumably better the longer the period of oscillation and it cannot be very good at high frequencies and at small (but still synoptic) wave lengths. Because the relatively low frequencies are known to contribute most to the transient variations in the middle latitudes, our result mainly reflects the balance of forces at these slowly varying components. In terms of the vorticity equation the secondary balance in the equations of motion means a quasibalance between the horizontal divergence and the horizontal advection of absolute vorticity. This is a wellknown feature of the long waves in the atmosphere.

4. *Error analysis*

Many sources of errors affect the results. At any station the observed value of an arbitrary quantity q, say q_O, can be considered the sum of the true large-scale component, q_L, which we would like to be present in the observations alone, the small-scale (but real) component q_S and an error component q_E, which is due to the natural limitations of the measuring systems and to the human errors involved in the preparation of the reported data. We thus have

$$q_O = q_L + q_S + q_E \,.$$

Due to the analysis scheme used, both of the components q_S and q_E produce errors in the computed values of the different terms even when the scheme itself is perfect. This is, however, not the case and an additional type of error, truncation error, arises essentially because of the evaluation of partial derivatives and mean values from observations made at discrete points in space and time. This error would arise, even if $q_S = q_E = 0$. A special kind of methodological error arises due to the drift of the radiosonde balloons, which means that in a strict sense a fixed grid system (distribution of stations) is not applicable.

All of the above errors can be partly random, partly systematic. The random errors contribute mainly to the standard deviations of the different terms, systematic errors to their mean values.

4.1. Random errors

A lower limit to the random errors in the results can be sought by considering the effect of the standard random errors of observation alone on the different terms.

A crucial problem is obviously the accuracy of the estimates of the pressure gradient force. The general expressions for calculation of the area-averaged components of the gradient of an arbitrary quantity q are:

$$\frac{1}{a \cos \varphi} \frac{\partial q}{\partial \lambda} = \sum_{i=1}^{6} a_{\lambda i} q_i \tag{7}$$

$$\frac{1}{a} \frac{\partial q}{\partial \varphi} = \sum_{i=1}^{6} a_{\varphi i} q_i \tag{8}$$

where $a_{\lambda i}$ and $a_{\varphi i}$ $(i = 1, \ldots, 6)$ are constants, and q_i is the reported value of q at the station i. By assuming that the standard observational error of q, say σ_q, is the same at all six stations, and that the errors at different stations are uncorrelated, we get the following for the standard error in evaluation of the gradient from Eqs. (7) and (8):

$$\sigma \left(\frac{1}{a \cos \varphi} \frac{\partial q}{\partial \lambda} \right) = \left(\sum_{i=1}^{6} a_{\lambda i}^{2} \right)^{1/2} \sigma_q \tag{9}$$

$$\sigma \left(\frac{1}{a} \frac{\partial q}{\partial \varphi} \right) = \left(\sum_{i=1}^{6} a_{\varphi i}^{2} \right)^{1/2} \sigma_q \tag{10}$$

For the second-degree method $(\sum a_{\lambda i}^2)^{1/2} = 3.18 \times 10^{-6}\text{m}^{-1}$ and $(\sum a^2_{\varphi i})^{1/2} = 2.10 \times 10^{-6}\text{m}^{-1}$.

In order to determine the random errors in determinations of the pressure gradient force from Eqs. (9) and (10) we need a measure of random observational errors of the isobaric height, σ_z. From an experiment with »twin» soundings with the British radiosonde (Mark 2B) HARRISON [3] obtained a value of $\sigma_z = 19$ meters at 300 mb.

By using this value we now obtain the following for the standard error in the evaluation of the pressure gradient force:

$$\sigma\left(\frac{1}{a\cos\varphi}\frac{\partial\phi}{\partial\lambda}\right) = 6.0 \times 10^{-4}\text{m sec}^{-2}$$

and

$$\sigma\left(\frac{1}{a}\frac{\partial\phi}{\partial\varphi}\right) = 4.0 \times 10^{-4}\text{m sec}^{-2}.$$

(These values correspond to standard errors of 4—6 m sec^{-1} in the evaluation of the area-averaged components of the geostrophic wind.)

In order to make an estimate of the random errors in the other terms of the left-hand side of Eqs. (1) and (2), we need a measure of the standard error in the upper-wind observations. In the measurements made by radar this error depends (Meteorological Office [10]) on errors of slant range, elevation and azimuth as well as on the ratio between the mean wind (below the height considered) and the rate of ascent of the balloon. With the typical wind condition over the British Isles one gets a value of 3 knots for the standard vector error of wind observations in the upper troposphere. This corresponds to $\sigma_u = \sigma_v = 1.1$ m sec^{-1}.

By using a value of 1.1 m sec^{-1} for the standard observational error in wind component the corresponding standard error of the area-averaged wind component is found to be 0.4 m sec^{-1} from the scheme of calculation. The standard error in the calculation of the components of the area-averaged Coriolis forces (terms (6)) is then 0.5×10^{-4} m sec^{-2} and thus much smaller than the corresponding error in the pressure gradient force. We then get a standard error of 0.3×10^{-4} m sec^{-2} for the standard error of the rate of the local change terms, measured over a 12-hour period: an insignificant error compared with the error in the pressure gradient force.

The error in the evaluation of the horizontal advection of an arbitrary quantity q can be written as

$$\delta(\mathbf{v} \cdot \nabla q) = V \frac{\partial}{\partial s} (\delta q) + \delta V \frac{\partial q}{\partial s}$$

where V is the wind speed and s is the distance in the direction of $\mathbf{V}$; δq and δV are the observational errors in q and V, respectively. Because generally $|V| >> |\delta V|$ and $\left|\frac{\partial}{\partial s} (\delta q)\right| \sim \left|\frac{\partial q}{\partial s}\right|$, we can expect the first term in the above expression to be the dominating one and we can therefore write

$$\sigma(\mathbf{v} \cdot \nabla q) \approx V \frac{\partial}{\partial s} (\delta q) .$$

For the standard error in the evaluation of the horizontal advection of q, we thus obtain, by assuming V and δq to be uncorrelated

$$\sigma(\mathbf{v} \cdot \nabla q) = b(\overline{V^2})^{1/2}\sigma_q$$

where σ_q is the standard error of observation of q and b is a quantity determined by the scheme of analysis (see Eqs. (9) and (10)). In the present case $(\overline{V^2})^{1/2} = 26$ m sec^{-1}. With $b = 2.7 \times 10^{-6}$m^{-1} and $\sigma_u = \sigma_v = 1.1$ m sec^{-1} we now obtain

$$\sigma(\mathbf{v} \cdot \nabla u) \approx \sigma(\mathbf{v} \cdot \nabla v) \approx 0.8 \times 10^{-4} \text{ m sec}^{-2} .$$

Comparing these values with the calculated values of the standard deviation of the terms $\mathbf{v} \cdot \nabla u$ and $\mathbf{v} \cdot \nabla v$, seen in Tables 1—2, it can be concluded that only a very small percentage of the variance of this term is due to random observational errors.

The errors in the vertical advection terms depend very greatly on the accuracy of the estimates of ω[1] and are difficult to assess. However, because this term is very small compared with other terms, even large percentage errors in it have little significance in the results obtained for the residual force. The standard error arising in the metric acceleration

[1]) The computed ω-values have, during the 5-year period considered, a mean value and a standard deviation of -0.06×10^{-4} cb sec^{-1} and 0.89×10^{-4} cb sec^{-1}, respectively; the corresponding values of the real vertical velocity $w = \frac{dz}{dt}$ are approximately $+1.3$ mm sec^{-1} and 2 cm sec^{-1}.

terms due to observational errors can be estimated to be not larger than 0.1×10^{-4} m sec^{-2}. Thus the accuracy in the evaluation of this term is high; this accuracy is insignificant for the residual force, however, because of the smallness of the term.

The residual force for each 12-hour period is determined as the sum of the terms (1)—(6) in the equation of motions. If we assume that the errors (due to random observational errors) in the evaluation of these six terms are independent of each other, we get for the standard error of the residual force:

$$\sigma(R_\lambda) = \left(\sum_{i=1}^{6} \sigma_{\lambda i}^2\right)^{1/2}, \tag{11}$$

$$\sigma(R_\varphi) = \left(\sum_{i=1}^{6} \sigma_{\varphi i}^2\right)^{1/2} \tag{12}$$

where $\sigma_{\lambda i}$ and $\sigma_{\varphi i}$ are the standard error associated with the i^{th} term in the equation for zonal and meridional motion, respectively. On the basis of the calculations made above the error associated with the pressure gradient force ($i = 5$) is by for the most dominating one. By using the estimates of $\sigma_{\lambda i}$ and $\sigma_{\varphi i}$ just derived we get

$$\sigma(R_\lambda) = 6.1 \times 10^{-4} \text{ m sec}^{-2}$$

$$\sigma(R_\varphi) = 4.1 \times 10^{-4} \text{ m sec}^{-2}$$

Comparing these numbers with the computed values of the standard deviations of R_λ and R_φ in Tables 1 and 2 (5.26×10^{-4} m sec^{-2} and 4.51×10^{-4} m sec^{-2}) we find an approximate agreement. The important conclusion to be drawn from this is that the variability of the residual force in our calculations is almost entirely due to error noise. Hence, we cannot infer anything from our results concerning the variability of the real sub-grid scale frictional forces.

4.2. Systematic errors

In order to avoid systematic errors in the results, calculations were made in this study for a network of stations which all use the same type of radiosonde. Despite this, systematic errors were possible due to systematic radiosonde errors, the systematic effect of meso-scale features on observations at the individual stations, or systematic truncation errors.

An essential question in the present investigation is whether the computed values $\bar{R}_\lambda$ and $\bar{R}_\varphi$ (the bar here denotes a time average) for the mean residual force are due to systematic errors or whether they represent a real mean forcing effect of the sub-grid scale motion systems. It is quite obvious that the mean values of the local change terms in the equations of motion cannot have any large error. It is also likely that the very small vertical advection terms cannot have a systematic error with a magnitude as large as $|\bar{R}_\lambda|$ and $|\bar{R}_\varphi|$. Furthermore, even though the metric acceleration term in the meridional equation of motion is systematically underestimated, the (systematic) error involved is very small (HOLOPAINEN [5]). No source of systematic error is known in the British wind-measuring system (HARRISON, personal communication).

Table 4. Mean values and standard deviations (SD) of the dominating terms in the equation of motion at 300 mb over the British Isles during the 5-year period 1961—65. N = 3319. The term numbers refer to Eqs. (1) and (2) on page 156. Unit = 10^{-4} m sec^{-2}.

A: terms computed by the linear method[1)]

B: terms computed by the second-degree method

C: difference of simultaneous values given by the second-degree method and the linear method.

			(1)	(2)	(4)	(5)	(6)	Residual
Zonal component Eq. (1)	A	Mean	0.01	−1.33	0.04	−0.81	2.49	0.40
		SD	*1.90*	*4.69*	*0.73*	*18.61*	*18.16*	*4.25*
	B	Mean	0.01	−1.35	0.04	−0.63	2.45	0.54
		SD	*2.09*	*5.29*	*0.77*	*18.93*	*18.87*	*5.26*
	C	Mean	0.00	−0.02	0.00	0.18	−0.04	0.14
		SD	*0.46*	*1.50*	*0.10*	*2.99*	*1.57*	*3.10*
Meridional component Eq. (2)	A	Mean	0.01	−1.08	1.05	−16.10	14.89	−1.24
		SD	*2.74*	*5.93*	*1.13*	*18.99*	*18.43*	*4.17*
	B	Mean	0.01	−1.16	1.06	−16.19	15.01	−1.34
		SD	*3.00*	*6.28*	*1.19*	*19.23*	*18.96*	*4.51*
	C	Mean	0.00	−0.08	0.01	−0.09	0.12	−0.10
		SD	*0.51*	*1.58*	*0.16*	*1.65*	*1.76*	*2.26*

1) The vertical advection term is assumed to be zero.

Therefore, the estimates of the mean Coriolis force and the mean horizontal advection of momentum are not likely (due to systematic observational errors) to be so far wrong as to explain the computed values of $\bar{R}_\lambda$ and $\bar{R}_\varphi$.

Some idea about the order of magnitude of the truncation errors in the results can be obtained by comparing the results obtained by the second-degree and the linear method (Table 4). It is seen from this table that the 5-year mean values of the different terms are practically the same obtained by the two methods. Even in the case of the horizontal advection terms, for which one might have expected the largest difference, both methods give practically the same results.

In the numerical analysis performed here one should in principle use the actual location of the radiosondes and not the fixed coordinates of the aerological stations. The location of the radiosondes changes all the time, and especially in cases of strong winds the internal distances of balloons launched from a network of stations may already at 300 mb depart considerably from those at the time of the launch. Because data in this study were compiled only for the 300 mb level, the use of a fixed grid system was necessary. In order to get some idea about the effect of balloon drifting on the results, the dominating terms in the equations of motion were computed using both the correct »floating grid» and »fixed grid» for the 3-month period September—November 1954, for which

Table 5. Mean values and standard deviations (SD) of some terms appearing in the equations of motion at 300 mb over the British Isles during the period 1 September to 31 November, 1954 (N = 175). Unit: 10^{-4} m sec^{-2}.
A: terms computed using a »floating grid»
B: terms computed using a »fixed grid»
C: difference of the simultaneous values given by methods A and B.

		$\mathbf{v} \cdot \nabla u$	$\mathbf{v} \cdot \nabla v$	$\frac{1}{a\cos\varphi}\frac{\partial\phi}{\partial\lambda}$	$\frac{1}{a}\frac{\partial\phi}{\partial\varphi}$
A	Mean	−2.55	−0 89	2.70	−30.40
	SD	*6.66*	*10.11*	*20.03*	*15.94*
B	Mean	−2.61	−0.90	2.64	−30.62
	SD	*6.63*	*10.05*	*19.90*	*15.93*
C	Mean	0.06	0.01	0.06	0.22
	SD	*0.26*	*0.25*	*1.07*	*0.68*

data from all tropospheric levels were available from the author's earlier work (HOLOPAINEN [5]). The results for 300 mb are given in Table 5. The difference between the »fixed grid» and »floating grid» results is very small. The characteristics of atmospheric circulation during September—November 1954 were qualitatively the same as on the average during 1961—65 (with, for example, advection of both zonal and meridional momentum into the area) except that the circulation in 1954 was more intense. The mean error caused by using a fixed grid in the calculations for the period 1961—65 must therefore be smaller than during September—November 1954, and therefore completely negligible.

On the basis of the above considerations we can conclude that the systematic errors possibly arising from the scheme of numerical analysis are too small to explain the relatively large mean values of R_λ and R_φ. It thus appears that the only possible source of systematic error that could produce spurious mean values as large as those computed for the residual force, is the pressure gradient force. However, we cannot yet exclude the possibility that the computed values of $\bar{R}_\lambda$ and $\bar{R}_\varphi$ at least partly represent real mean frictional forces.

5. *Significance of the computed mean residual force*

The results for the mean residual force over the British Isles for each of the five years during 1961—65 are given in Table 6. For every year the zonal component of the residual force is positive, which should physically mean an energy input from the sub-grid scale motion systems to the large-scale westerly flow. The meridional component is syste-

Table 6. Mean values and standard deviations (SD) of the zonal and meridional components of the residual force in the equations of motion (p. 00) over the British Isles for different years 1961—65. N = number of instantaneous (*i.e.* 12-hour mean) values. Unit 10^{-4} m sec^{-2}.

		1961	1962	1963	1964	1965	1961—65
R_λ	Mean	0.15	0.77	0.44	0.85	0.47	0.54
	SD	*6.05*	*5.46*	*5.28*	*4.66*	*4.75*	*5.26*
R_φ	Mean	−0.58	−1.20	−1.67	−1.97	−1.25	−1.34
	SD	*4.94*	*4.68*	*4.14*	*4.10*	*4.54*	*4.51*
N		624	666	675	678	676	3319

matically negative with a magnitude much larger than that of the zonal component; the same was also found by the author (HOLOPAINEN [5]) to be the case during September—November 1954.

The computed mean residual force for each year is statistically significantly different from zero. The real question is whether or not they have any physical relevance with regard to the turbulent friction or merely are systematic errors arising in the evaluation of the pressure gradient force. Using the classical formulation we may write for the mean frictional force an expression $A\nabla^2\bar{\mathbf{v}} + K\frac{\partial^2\bar{\mathbf{v}}}{\partial z^2}$, where A and K are constants. A numerical evaluation of this expression from data (for example, with $A = 10^5$ m^2 sec^{-1}, $K = 1$ m^2 sec^{-1}) gives a vector, which is very much smaller in magnitude than the computed mean residual force, and oriented almost opposite to it. Thus, at least in classical terms, the computed $\bar{R}_\lambda$ and $\bar{R}_\varphi$ values cannot be explained as friction.

The positive sign of $\bar{R}_\lambda$, if assumed to be physically realistic, would mean convergence of the eddy flux (either vertical, lateral or both) of westerly momentum at 300 mb. One mechanism that possibly could produce this effect is the internal gravity waves, produced by mountains, which can be associated with a considerable vertical flux of momentum (*e.g.* BRETHERTON [1]). WOOLDRIDGE [14] studied the vertical distribution of the zonal component R_λ of the residual force over the southwestern United States using the same method as employed here during a period when very pronounced mountain waves were present. In this case (when an easterly wind component prevailed in the lower troposphere) positive values of R_λ as large as $+ 10\times10^{-4}$ m sec^{-2} were obtained at the levels of the westerly jet stream in the upper troposphere. The height of the mountains of the British Isles are not at all comparable with the height of mountains in the southwestern United States but their ability to produce vertical momentum flux is in any case remarkable in certain weather situations (BRETHERTON, *loc. cit.*). It is doubtful, however, that the residual forces obtained in the present study can have anything to do with the mountain-induced internal gravity waves, for two reasons. First, the areal distribution of the mean residual force (Fig. 2) does not correlate with the distribution of mountains in any clear way. Secondly, the meridional component of the computed residual force has a larger magnitude than the zonal component; this cannot be explained as the effect of internal gravity waves on the westerly current.

The topography of the British Isles is heterogeneous and it is possible that the radiosonde data may contain some mesoscale features which produce systematic errors in our calculations. For this reason, numerical analysis of the equations of motion was made also using aerological data from four German stations (see Fig. 1) for the 8-year period 1962—69. The linear method of analysis (section 2) was used. The total number of 12-hour periods, for which complete observations at 300 mb existed at all the four stations, was 3314. The magnitude and the mutual relationship between the different terms (1)—(6) was found to be about the same as over the British Isles. This is natural because of the closeness of the two regions. The 8-year mean residual forces for the two triangles over the German area appear in Fig. 2. These forces are too large to represent real frictional forces: other forces being zero, they would completely stop a westerly flow of 10 m sec^{-1} in about 9 hours.

It is thus likely (but not certain) that the mean residual forces obtained in the present study mainly represent systematic errors arising in the calculation of the pressure gradient force. Assuming this to be

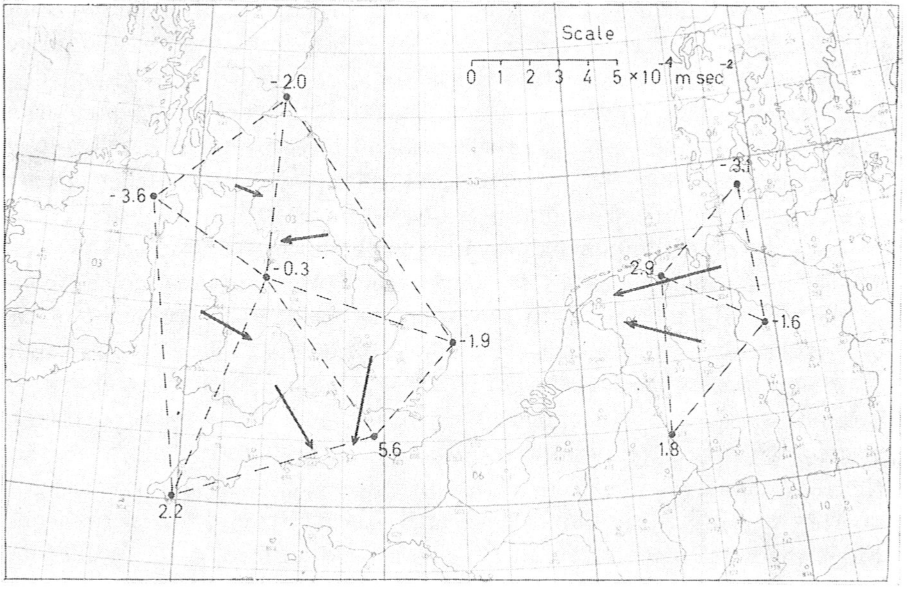

Fig. 2. The long-term mean residual force at 300 mb for the different triangles used in the analysis with the linear method. The numbers at the stations provide the best estimate (see text) of the mean height errors in meters that could explain the computed residual force.

the case one may now ask for the distribution of the mean 300 mb height error that can produce the computed residual force. The possible height errors at the different stations were computed by the least square technique. An area-averaged mean of the square of the vector difference between the computed residual force (obtained by the linear method) and the spurious pressure gradient force (resulting from the possible height errors) was defined as

$$E = \sum_{i=1}^{N} w_i(\overline{\boldsymbol{R}}_i - \nabla(\overline{\delta\phi_E}))^2$$

where $\overline{\boldsymbol{R}}$ is the computed mean residual force (with components $\bar{R}_\lambda$ and $\bar{R}_\varphi$), w_i is the weighting factor defined as the percentage of the area of the i^{th} triangle from the whole area and N is the number of the triangles (5 for the British, 2 for the German area). The value of $\overline{\delta\phi_E}$ at each station was then computed by minimizing E and requiring that the mean height error of all six British (four German) stations disappears. The systematic 300 mb height errors so obtained are seen in Fig. 2. It is noticed that rather small systematic height error can explain the computed mean residual forces.

Differences in the systematic height error between stations all using the same type of radiosonde must be connected with the different origin and history of the radiosondes and with the different practices at the different stations (HARRISON [3]). The results presented in Fig. 2 can thus be useful in the national intercomparison of the performance of the different aerological stations.

In general, an analysis of the equations of motion, using the method employed here, can possibly be used in a study of the compatibility of observations of wind, temperature and pressure in the free atmosphere. This method is obviously better than the comparison of geopotential and wind data merely on the basis of the geostrophic relationship.

6. *Analysis of the kinetic energy equation*

The equation of kinetic energy of the horizontal motion can be written as

$$\frac{\partial k}{\partial t} + \mathbf{v} \cdot \nabla k + \omega \frac{\partial k}{\partial p} + \mathbf{v} \cdot \nabla \phi = \mathbf{v} \cdot \boldsymbol{R} \tag{13}$$

All the terms on the left-hand side of this equation can be evaluated from the same data and by the same methods which were used in the analysis of the equations of motion. The sum of these estimates gives a value of $\mathbf{v} \cdot \boldsymbol{R}$ for each 12-hour period; in principle this term represents the work done by frictional forces but in practice also represents all the errors made in computation of the left-hand side terms.

The results obtained from the analysis of the kinetic energy equation for the area of the British Isles are given in Table 7.

Table 7. Mean values and standard deviations (SD) of the different terms in the kinetic energy equation at 300 mb over the British Isles during the period 1961—65. N = 3319. Unit: Watts m^{-2} (100 mb)$^{-1}$.

	$\frac{\partial k}{\partial t}$	$\mathbf{v} \cdot \nabla k$	$\omega \frac{\partial k}{\partial p}$	$\mathbf{v} \cdot \nabla \phi$	$\mathbf{v} \cdot \boldsymbol{R}$
Mean	0.01	−2.97	0.02	3.99	1.05
SD	*7.17*	*17.66*	*1.97*	*16.02*	*1.73*

On the basis of this table it is apparent that the area of the British Isles isa sink of kinetic energy on the average $(- \mathbf{v} \cdot \nabla \phi < 0)$. The loss of kinetic energy due to work done by pressure forces is to a large extent compensated for by the horizontal advection of kinetic energy into the area $(- \mathbf{v} \cdot \nabla k > 0)$; the mean values of the local change term and the vertical advection term are completely negligible. The tendency to compensation between the work done by pressure forces and the horizontal advection of kinetic energy occurs not only in the mean values but also in the transient fluctuations of these terms. It is a reflection of the secondary balance in the equations of motion between the horizontal advection of momentum and the sum of the pressure gradient and Coriolis forces discussed in section 3. This tendency toward compensation has a clear synoptic interpretation: in a diffluent/confluent flowpattern ageostrophic crossisobaric components occur which are directed horizontally towards higher/lower pressure.

According to Kung [6], [7] the North American continent is, on the average, an area where kinetic energy is generated $(- \mathbf{v} \cdot \nabla \phi > 0)$ and from where there is a horizontal outflux of kinetic energy, a situation just opposite to that over the British Isles. It thus appears that in the middle latitudes there are both source regions and sink regions of kinetic energy and from results for some limited areas, one cannot draw conclusions concerning the balance of kinetic energy in the whole extra-tropics.

The value for the term $\mathbf{v} \cdot \boldsymbol{R}$ is of interest because in principle it represents the work done by friction forces; a positive sign means an energy input from the sub-grid scale motion systems to the large-scale motion and negative sign dissipation of the large-scale kinetic energy. The 5-year (1958—63) mean value obtained by KUNG [6] for the North American continent is at 300 mb -0.5 watts m^{-2} (100 mb)$^{-1}$. The corresponding value obtained for the area of the British Isles is $+1.1$ watts m^{-2} (100 mb)$^{-1}$ in Table 7. The positive value is statistically highly significant. However, because of the possibility that systematic errors may arise in the evaluation of the pressure gradient force, an interpretation of the value in physical terms is probably not justified.

7. *Concluding remarks*

From the point of view of the original objectives this study has been a frustrating experiment: it has added practically nothing to our knowledge about the effects of the sub-grid scale turbulence in the upper troposphere. One result of this investigation (at least for the author himself) is a warning: in diagnostic studies where the quantities of main interest are evaluated as residual terms from some balance relationships, a careful error analysis has to be made before physically interpreting the results for these residual terms.

It might be of interest to make a numerical analysis of the equations of motion for all those areas of the globe where a reasonably dense network of observations exist. If the residual forces obtained cannot be interpreted as real frictional forces (as is most likely), they can possibly be used, in the way indicated in the present study, to provide information about the errors of the observation system.

REFERENCES

1. Bretherton, 1969: Momentum transport by gravity waves. *Quart. J. Roy. Met. Soc.*, **95**, 213–263.
2. Everson, P. J. and D. R. Davies, 1970: On the use of a simple two-level model in general circulation studies. *Ibid.*, **95**, 404–412.
3. Harrison, D. N., 1962: The errors of the Meteorological Office Radiosonde, Mark 2B. *Meteorological Office, Scientific Paper No.* **15**.
4. Holopainen, E. O., 1963: On the dissipation of kinetic energy in the atmosphere. *Tellus*, **15**, 26–32.
5. —»—, 1964: Investigation of friction and diabatic processes in the atmosphere. *Soc. Scient. Fenn. Comment. Phys.-Math. XXIX* **9**.
6. Kung, E. C., 1967: Diurnal and long-term variations of the kinetic energy generation and dissipation for a five-year period. *Mon. Weath. Rev.*, **95**, 593–606.
7. —»—, 1969: Further study on the kinetic energy balance. *Ibid.*, **97**, 573–581.
8. Kurihara, Y., 1960: Numerical analysis of atmospheric motion. *J. Met. Soc. Japan*, **38**, 288–304.
9. Leith, C. E., 1969: Two dimensional eddy viscosity coefficients. *Proceedings of the WMO/IUGG Symposium on Numerical Weather Prediction in Tokyo Nov. 26–Dec. 4, 1968.* Japan Met. Agency, Tokyo.
10. Meteorological Office, 1961: *Handbook of meteorological instruments. Part II.* Her Majesty's Stationery Office, London.
11. Smagorinsky, J., 1963: General circulation experiments with the primitive equations I. The basic experiment. *Mon. Weath. Rev.*, **91**, 99–164.
12. Trout, D. and H. A. Panofsky, 1969: Energy dissipation near the tropopause. *Tellus*, **21**, 355–358.
13. Vinnichenko, N. K., 1970: The kinetic energy spectrum in the free atmosphere — 1 second to 5 years. *Ibid.*, **22**, 158–166.
14. Wooldridge, G. L., 1972: Effects of internal gravity waves on energy budgets and vertical transport of angular momentum over mountainous terrain. *Mon. Weath. Rev.*, **100**, 177–188.
15. Zilitinkevich, 1970: *Dynamics of the atmospheric boundary layer* (in Russian). Hydrometeorological Publishing House, Leningrad.