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EXPERIMENTS ON THE MEASUREMENT OF AREAL RAINFALL BY RADAR

by

ERKKI JATILA and TIMO PUHAKKA

Department of Meteorology
University of Helsinki

Abstract

An X-band radar applying the stepped gain method was used to measure precipitation amounts over an area of 180 sq. km with 15 recording rain gauges. It has been shown that for various types of precipitation in Finland the following radar reflectivity factor—rainfall intensity relationships seem to be valid:

Continuous rain: $Z = 196\ R^{1.6}$
Showers: $Z = 360\ R^{1.6}$
Drizzle: $Z = 56\ R^{1.6}$

Radar measurements were also calibrated by a reference rain gauge within the check site. In continuous rains this procedure offered no advantage over the use of the MARSHALL-PALMER relationship ($Z = 200\ R^{1.6}$) but in showers the mean daily error drops from 42.9 % (M-P coefficients) to 25.2 %. Correspondingly the percentage of correct 15 minute average intensities (within 50 % error limits) increases from 45 % (M-P coefficients) to 52 %.

The filter paper technique was applied to obtain the average drop size distribution for each storm. Samples were gathered in three places within the check site at time intervals of 2.5 minutes. Radar estimates of areal precipitation were calculated using the coefficients of drop size measurements for each storm. In continuous rains the mean daily error was 21.2 % and 86 % of the 15 minute observations were inside the error limits. The corresponding figures for showers were 98.2 % and 28 %.

1. *Introduction*

1.1 General

The quantitated measurement of rainfall by radar is based on the fact that the average power $\bar{P}_r$ scattered by rain back to the radar aerial is related to a quantity depending on the drop size distribution. This quantity is the s.c. radar reflectivity factor Z, which is the sum of sixth powers of raindrop diameters in the unit volume. The relationship between back scattered power and the radar refelectivity factor can for instance be expressed as follows (after PROBERT-JONES [26]):

$$\bar{P}_r = \left[\frac{\pi^3}{1024 \ln 2}\right] \left[\frac{P_t\, h\, G^2\, \Phi\, \Theta}{\lambda^2}\right] \left[|K|^2 \varkappa \frac{Z}{r^2}\right] \tag{1}$$

where

$\bar{P}_r$ = the average scattered power gathered by the radar aerial
P_t = peak power of a pulse transmitted by the radar
h = length of a pulse
Φ, Θ = beam width to 3 dB points vertically and horizontally, respectively
G = gain of radar aerial
K = $(m^2-1)/(m^2+2)$, where the complex refractive index of water $m = n - in'$. n is the real refractive index and n' the absorption coefficient of water
$\varkappa$ = attenuation
r = distance

The first term on the right hand side of the equation (1) is always constant, the second depends entirely on the radar parameters, while the third is wholly a function of the characteristics of scatterers.

All of the parameters in the second term can be easily and accurately determined with the exception of gain G. Following SMITH [29], we measured the gain of the radar aerial by producing a known field of power over the radar aerial and measuring the amount of power the radar is capable of gathering. A signal generator with a standard antenna was used as the known source of power. The gain of the radar aerial was calculated using the equation:

$$G = 16\,\pi^2 \frac{P_r\, r^2}{P_s\, \lambda^2 G_s} \tag{2}$$

where

P_r = power gathered by the radar aerial
P_s = power transmitted by the signal generator
r = range between the signal generator and the radar
G_s = gain of the standard antenna

The measurements yielded the value of 4000 for the gain. This is about 4 dB less than the gain calculated from the geometrical dimensions of the radar aerial.

In the last term of the equation (1) the factor $|K|^2$ depends on the state of the scatterers and also to some extent on temperature. Generally it is considered a constant: $|K|^2 = 0.93$ for raindrops and $|K|^2 = 0.197$ for ice crystals and snowflakes [1].

When it penetrates a medium the electromagnetic radiation is always attenuated. Only the attenuation caused by precipitation is noticeable when the wavelength is 3 cm or more. The attenuation is a function of drop size distribution and hence a function of rainfall intensity. For a typical drop size distribution the attenuation can be written [3, 27]:

$$\varkappa = 10^{-0.2\int_0^r K'R^\gamma dr} \tag{3}$$

where K' and γ are experimental coefficients. In this paper the values given by GUNN and EAST [11] were used: $K' = 0.0074$ and $\gamma = 1.31$.

Using the equation (1) it is possible to calculate the radar reflectivity factor Z by measuring $\bar{P}_r$. If we suppose that the shape of drop size distribution is known, we can use the value of Z to arrive at all the meteorological quantities which are functions of drop size distribution. In this paper we are interested in the relationship between the radar refelectivity factor Z and the rainfall intensity R.

The Z-R relationship is generally accepted to be of the form $Z = aR^b$, where a and b are coefficients. Putting this relationship into the equation (1) we can compute the intensity R if $\bar{P}_r$ is measured and the coefficients a and b are known. The greatest uncertainty in radar rainfall measurements is the variation of the coefficients a and b with the variation in drop size distribution in various types of rain. As a result radar meteorologists have had to focus their attention on this problem in order to find out the proper Z-R relationship in various types of rain.

1.2 Methods for finding out the proper Z-R relationship

At the very beginning of radar meteorology MARSHALL and PALMER [24] defined the Z-R relationship based on relatively large amounts of observations from different sources. They gave the relationship $Z = 220\,R^{1.6}$ which has been simplified into the form $Z = 200\,R^{1.6}$. This equation has since been applied widely and especially in continuous rain has given good radar estimates of rainfall intensity. The Marshall-Palmer relationship (abbreviation throughout this paper M-P) presumes a certain type of drop size distribution. If the drop size distribution in rain is different from that assumed by Marshall-Palmer, a change in the Z-R relationship is a natural consequence.

The actual measurement of drop size distribution can be made for instance by photographing a known volume in rain and measuring the diameters of the drops (this technique was originally developed by JONES and DEAN [17] and used later for instance by JONES and MUELLER [18], STOUT and MUELLER [31], and many others) or by using dyed filter papers (DIEM [9]). Electrical recording instruments have also been developed for drop size measurements. The distrometer by JOSS and WALDVOGEL [22] is the most advanced of these.

STOUT and MUELLER [31] and DIEM [9] made drop size measurements in a number of places in the Northern Hemisphere. They found that the drop size distribution varies widely, also resulting in variations in the Z-R relationship. The same value of Z in one place can correspond to a rainfall rate of many times larger in another place.

CATANEO [5] has introduced a method for calculating coefficients a and b from climatological parameters:

1) mean annual per cent of rain days that are thunderstorm days, and
2) mean annual relative humidity at 0.5 km above ground.

In evaluating values obtained for independent material, Cataneo found that his coefficients in most cases led to a somewhat better estimate of radar rainfall than the M-P coefficients.

Later CATANEO and VERCELLINO [7] tried to predict the actual Z-R relationship from close upper air soundings. Their variables were: 1) precipitable water content between the surface and 500 mb level, 2) lifting condensation level, and 3) freezing level. They found that the coefficient a was correlated to these quantities but the exponent b had no significant correlation. In conclusion they stated that the application of the model revealed a significant improvement over the use of the M-P equation.

Seasonal classification of measurement material has been done by Diem [9] and Joss *et al.* [21]. Both of these investigations came to the conclusion that on the average clearly different *Z-R* relationships exist for various seasons, although Joss mentions that interstorm variations in the *Z-R* relationship are often larger than interseasonal variations.

Johnson [15] and Stout and Mueller [31] tried to use a stability index as a predictor for the *Z-R* relationship. Due to the small number of soundings (two times a day), success was limited.

The statistical properties of the echo pattern on the radar scope were used to specify the *Z-R* relationship by Austin [2] and Wilson [33]. For instance the following statistics were investigated: average echo intensity, intensity variance, average echo length, ellipticity of pattern, orientation of pattern, and echo coverage. Unfortunately both of these papers state that there should be more extensive material before a proper *Z-R* relationship can be determined from the echo statistics.

Many authors have classified their data after a synoptic type of precipitation (for example Stout and Mueller [31], Cataneo and Stout [6] and Mueller [25]). They had 6—10 synoptic classes (air mass, cold frontal, warm frontal, overrunning, easterly wave, trough aloft, occlusion, etc.), which were determined afterwards using weather charts. Some improvement in radar rainfall measurements was found.

Many radar meteorologists have realized that the drop size distribution is typically different in showers from distribution in stable rain. For this reason they have grouped the data into 3—4 classes, depending on whether the rain is caused by a front or pure convection. Table 1 presents some of these studies made during the 1960's. Earlier works have been referred to, for example, by Battan [3], Atlas [1] and Borovikov *et al.* [4]. One can easily see in Table 1 that in all but a couple of studies the coefficient *a* has lower value in continuous (frontal) rains than in showers or thunderstorms.

All of the methods discussed above are based on the determination of a proper *Z-R* relationship utilizing some criterion and this relationship has then been applied to large areas and long periods of time. The *Z-R* relationship may, however, vary widely during one single storm. In order to take this variation into account one should know the shift in *Z-R* relationship in real time. This can be done by using two methods: either by measuring the drop size distribution continuously in rain and cal-

Table 1. *Z-R* relationships for various types of precipitation.

Author	Thunder-storm	Shower	Continuous rain	Drizzle	Site
Hardy & Dingle [12]		$Z = 188R^{1.48}$			Michigan
Jones & Mueller [18]	$Z = 263R^{1.42}$	$Z = 144R^{1.60}$	$Z = 217R^{1.41}$		Miami
Srivastava & Kapoor [30]		$Z = 197R^{1.70}$	$Z = 277R^{1.54}$		New Delhi
Fujiwara [10]	$Z = 450\mathrm{R}^{1.46}$	$Z = 300\mathrm{R}^{1.37}$	$Z = 205\mathrm{R}^{1.48}$		Illinois
Sims [28]	$Z = 446\mathrm{R}^{1.43}$		$Z = 439\mathrm{R}^{1.46}$		Illinois
Harrold [13]	$Z = 350\mathrm{R}^{1.6}$	$Z = 280\mathrm{R}^{1.6}$	$Z = 200\mathrm{R}^{1.6}$		London
Jones [16]	$Z = 435\mathrm{R}^{1.48}$	$Z = 370\mathrm{R}^{1.31}$	$Z = 311\mathrm{R}^{1.43}$		Illinois
Joss et al. [20]	$Z = 500\mathrm{R}^{1.5}$		$Z = 250\mathrm{R}^{1.5}$	$Z = 140\mathrm{R}^{1.5}$	Locarno
Stout & Mueller [31]	$Z = 224\mathrm{R}^{1.51}$	$Z = 250\mathrm{R}^{1.47}$	$Z = 322\mathrm{R}^{1.33}$		Florida
—»—		$Z = 146\mathrm{R}^{1.42}$	$Z = 226\mathrm{R}^{1.46}$		Marshall Islands
—»—	$Z = 339\mathrm{R}^{1.64}$	$Z = 327\mathrm{R}^{1.66}$	$Z = 295\mathrm{R}^{1.59}$		Oregon

culating the respective *Z-R* relationship or by recording *R* with a rain gauge and simultaneously measuring *Z* above the gauge by radar. JOSS *et al.* [19] used both of these methods and WILSON [34] the latter method. If the drop size distribution did not vary remarkably in space these methods could be used to essentially improve radar rainfall measurements.

The purpose of the present paper is to discover the proper *Z-R* relationships in various types of rain in Finland. This was done by calculating the *Z-R* relationship which leads to a correct total amount of precipitation measured by radar for continuous rain, showers, and drizzle. Areal radar rainfall amounts were compared with the corresponding amounts measured by a network of 15 recording rain gauges over an area of 180 sq. km. Further more, in order to take into account daily variations in the *Z-R* relationship, two methods were applied: 1) the radar was calibrated to show the same daily amount of rainfall as a fixed reference rain gauge in the check site and 2) the drop size distribution was measured in three places inside the check site (filter paper technique) and the average value of the coefficients *a* and *b* obtained for each storm were applied to radar measurements.

2. *Measurement procedures*

2.1 Radar technique

In the present investigation an X-band radar (Selenia Meteor RMT-

1L, described by Jatila *et al.* [14]) was used. Radar measurements were carried out by altering the gain of the radar amplifier on the average of 6 dB steps and filming the radar scope at each gain setting. The minimum detectable power was —95 dBm and the maximum power scattered by rain generally —60 to —40 dBm. The distribution of scattered power over the check site was determined at a frequency of 1 per 5 minutes. Each measurement took 2—3 minutes depending on the number of gain steps required. A detailed description of the radar technique used has been made by Jatila *et al.* [14]. Using radar measurements the average rainfall rate over a time interval of 15 minutes was computed. Because the radar measured the power scattered by raindrops about 400 meters above the ground, a certain time is needed for drops to fall on to the ground. This time was arranged by calculating the average radar rainfall intensity for a 15 minute period using the equation

$$R_{r/15} = \frac{0.5\, R_{-5} + R_0 + R_{+5} + 0.5\, R_{+10}}{3} \tag{4}$$

where R_0 is the radar-measured intensity at the time t_0. R_{-5} is the intensity 5 minutes before this moment and R_{+5} 5 minutes afterwards, etc. The equation (4) gives an advance of 5 minutes, but because of the time required (2—3 min.) to film the scope at 8—10 gain values, the real advance decreases to about 3 minutes.

2.2 Rain gauge measurements

Radar measurements were compared with the recordings of the rain gauge network shown in Fig. 1. The area limited by the solid line was used as the check site. The »true» areal rainfall was determined by using rain gauges nos. 1—15 and applying Thiesen's method. If any of the gauges was temporarily out of order for any reason, new polygons required by the method were drawn to get the best possible estimate of areal rainfall. Due to inaccuracies in the estimation of time with rain gauges (although daily drums were used) only average values of 15 minutes for areal rainfall were considered reliable.

2.3 Measurements of drop size distribution

The filter paper technique (see Diem [8], for example) was used to get the Z-R relationship from drop size measurements. These measure-

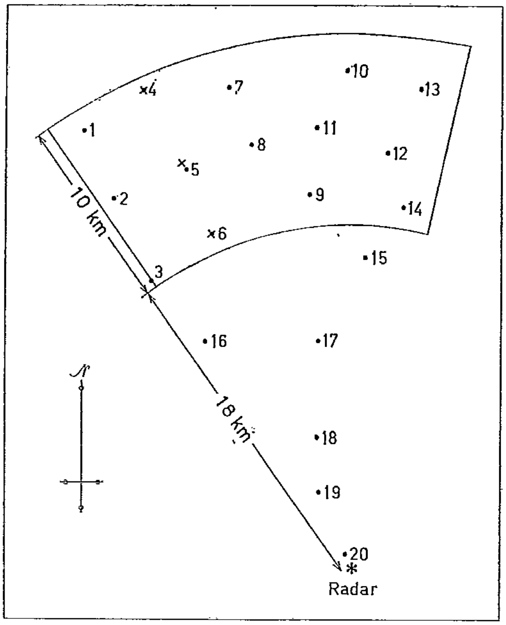

Fig. 1. The check site (180 sq.km) with the locations of recording rain gauges. To estimate the areal rainfall gauges nos. 1—15 were used, to calculate the attenuation the observations of gauges nos. 9 and 15—20 were applied. Places denoted by (x) are those for drop size distribution measurements. Gauge no. 8 was used when the radar was calibrated with the aid of a reference gauge.

ments were carried out in the check site on three places marked with (x) in Fig. 1. Each station took samples at 2.5 minute intervals. The exposure time varied from a few seconds to one minute depending on the rainfall intensity. Each sample was composed of 100—200 raindrops. The radar refelectivity factor Z and the rainfall rate R were computed applying the equations:

$$Z = \sum_{i=0}^{19} \frac{N_i D_i^6}{\bar{v}_i tA} \tag{5}$$

$$R = \frac{\sum_{i=0}^{19} N_i \pi D_i^3}{6\,tA} \tag{6}$$

where i denotes drop size intervals ($<$ 0.25 mm, 0.25—0.50 mm, 0.50—0.75 mm, 0.75—1.00 mm, etc.). N_i is the number of drops in a size interval, where D_i is the mean diameter of drops and $\bar{v}_i$ is the average terminal falling velocity. t is the exposure time and A the area of the filter paper. The Z-R relationship for each day was calculated applying the least square technique. Fig. 2 is an example of the average Z-R relation measured during a day (14 Sept. 1969). Each dot presents the Z-R relationship of one sample.

The greatest disadvantage of the filter paper technique is that the sample does not perfectly represent the volume where the radar measures the scattered power [23]. Another defect is the variation of the drop size

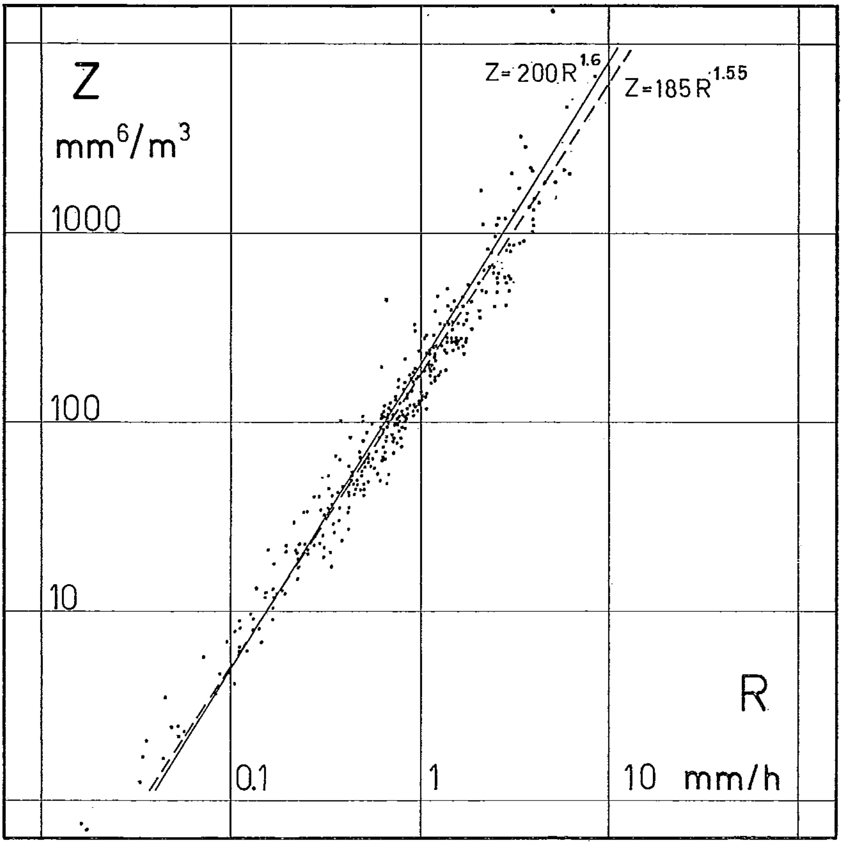

Fig. 2. Z-R relationship obtained by using the filter paper method for one day (14th September 1969). The solid line is the Marshall-Palmer relation ($Z = 200\ R^{1.6}$) and the broken line presents the regression line ($Z = 185\ R^{1.55}$) for the present samples.

distribution in space. In order to minimize these sources of error the measurements of the drop size distribution were carried out simultaneously in three places.

2.4 Attenuation

The attenuation of the electromagnetic radiation caused by rain between the radar and the check site was estimated by using the equation (3). The equation is based on the knowledge of the rainfall rate distribution along the path penetrated by the radiation. This distribution was approximated using the average 15 minute intensities of the recordings of gauges nos. 15—20 and 9 (Fig. 1). It was thus possible to compute only the average 15 minute attenuation between the radar and the check site (24 km to the middle of the check site).

Fig. 3 shows the effect of the attenuation on the precipitation rate, when the rate is calculated by combining the equation (1) (with various values of $\bar{P}_r$) and the relation $Z = aR^b$. The curves in the figure have been drawn for the values $b = 1.4$, $b = 1.6$, and $b = 1.8$.

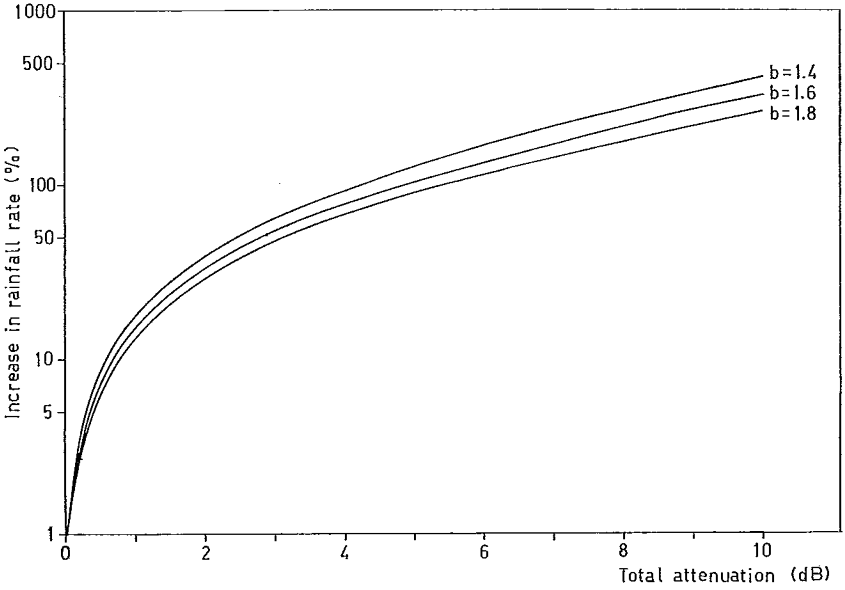

Fig. 3. The increase (%) in rainfall intensity for various values of b (in Eq. $Z = aR^b$) as a function of total attenuation (dB).

The maximum average attenuation observed during a 15 minute period was 2.77 dB. If this amount of attenuation is taken into account, the radar-derived rainfall rate will increase by 48 % (see Fig. 3), if the value $b = 1.6$ is applied. The mean attenuation for the whole observation time (56 h 15 min) was 0.36 dB (5 % increase in rainfall rate). The attenuation was larger than 1 dB (15 % increases in rainfall rate) during 12.4 % of observation time. Only during 1.3 % of the time the attenuation was larger than 2 dB (33 % increase in rainfall rate). Thus in Finland the attenuation in a 3 cm wavelength does not seem to be a serious problem. On the countrary, in districts where larger rainfall intensities occur frequently, the attenuation considerably restricts radar rainfall measurements.

3. *Results*

According to the studies made elsewhere (see Table 1), it was concluded that the material has to be divided into various types of rainfall in order to find out the best radar method to estimate rainfall rates by radar in Finland. Three main types of rainfall were considered: continuous (frontal) rain, showers, and drizzle. The determination of the rainfall type of each storm was based on the observations made by the Meteorological Office at Helsinki Airport (station no. 15 in Fig. 1).

The total amount of areal precipitation measured by radar for each storm was calculated applying Z-R relationships obtained by four different ways:

1) simply applying the Marshall-Palmer relation $Z = 200\,R^{1.6}$
2) calibrating the radar in such a way that the total daily amount of areal precipitation measured by radar became equal to the total amount of rainfall measured by the network of the gauges. Only the value of the coefficient a was calculated, while the exponent b was assumed to be a constant equal to 1.6.
3) calibrating the radar so that the total daily amount of rainfall measured by radar above a fixed reference gauge became equal to the rainfall amount measured by the gauge (gauge no. 8 in Fig. 1). Again the exponent b was held as a constant equal to 1.6.
4) measuring the drop size distribution in the check site on three places and for each storm calculating the average Z-R relationship, which was assumed to be valid for the whole check site.

Methods 2) and 3) require a constant value equal to 1.6 for the exponent b. At the very beginning of the project it was the intention to compute both coefficients a and b for each storm applying the least square method, which would lead to the best correlation between the average areal 15 minute rainfall rates measured by the radar and the network. Because the time period over which the integrated rainfall amounts were calculated was as long as 15 minutes, the great majority of the average intensity observations were concentrated in the interval $1 \text{ mm/h} < R < 5 \text{ mm/h}$. Even a great change in the value of the exponent did not considerably effect the correlation and the best result was frequently obtained with unrealistic values of the coefficients. For this reason the exponent was fixed and the radar was calibrated only by changing the coefficient a. The exponent has also been attached by JOSS *et al.* [19] and BOROVIKOV *et al.* [4], who have taken a value $b = 1.5$. According to Borovikov the accuracy of radar rainfall measurements does not alter greatly as long as the exponent is between 1.3 and 2.5. He even suggests that b could be fixed universally to make comparisons of radar rainfall measurements easier. In the present study the value $b = 1.6$ was only taken because it is the value in the M-P relationship.

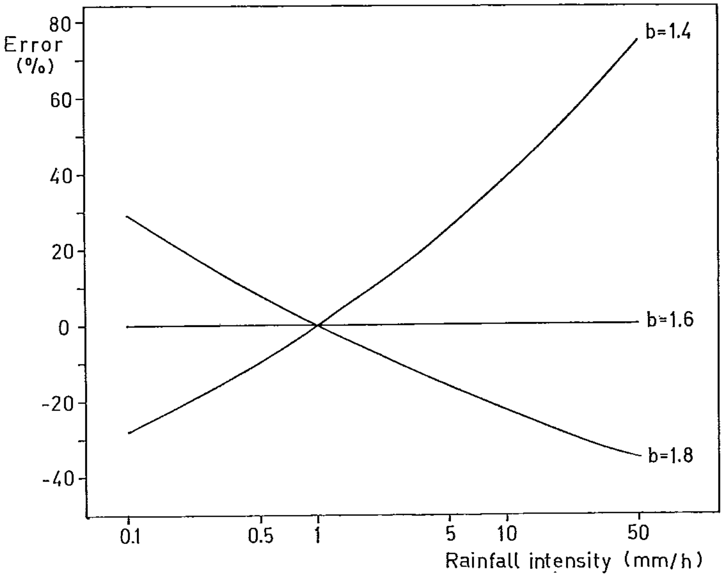

Fig. 4. The error (%) in rainfall intensity for various values of the exponent b (in Eq. $Z = aR^b$) as a function of rainfall intensity. The correct intensity is assumed to have been obtained with the value $b = 1.6$.

Figure 4 shows the error that will arise if instead of the value $b = 1.6$ (which is assumed correct in the figure) the values $b = 1.4$ or $b = 1.8$ are applied. In Finland the rainfall rate in continuous rains seldom exceeds the value $R = 5$ mm/h. If this intensity is measured correctly with $b = 1.6$, it will be overestimated by 26 % if $b = 1.4$ and underestimated by 16 % if $b = 1.8$. In convective showers the rate of 20 mm/h is very seldom exceeded, if the average 15 minute values are considered. This rainfall rate will be overestimated by 53 % if $b = 1.4$ and underestimated by 28 % if $b = 1.8$. Measuring the areal rainfall by radar we have to notice that the radar generally overestimates the intensity in one part of the area and underestimates it in another part. Accordingly errors will partly compensate for each other.

3.1 Coefficients *a* and *b* using various methods

The results of the gauge network observations for each storm are presented on the left hand side of Table 2. The observations are classified according to the type of rainfall. Column Q_g is the daily areal precipitation amount and R_{gmax} is the maximum 15 minute average intensity observed within the check site by a single rain gauge during the storm. In continuous rains this value changes between 1.0—7.4 mm/h., while in showers values as high as 30.0 mm/h have been obtained.

3.1.1 Radar calibrated by rain gauge network

The best possible value for the coefficient a can be obtained for each storm by calibrating the radar so that the daily amount of areal rainfall measured by the radar equals that observed by the gauge network. These values of a have been presented in the column »Coefficient a when radar was calibrated by network» of Table 2. The value of a in continuous rains changes between 106—402. In convective showers a is generally considerably greater especially for storms with thunder. In drizzle the coefficient has very low values. The temporal existance of larger rain drops during the storm on 16 Sept. has clearly led to a higher value of a.

Even a general look at the value of the coefficient a given above makes it apparent that it is quite natural to use different values of a for different types of rainfall. The mean value of a weighted by the daily amount of precipitation was computed to get the best possible value of a for each rainfall type:

continuous rain: $a = 196$
showers: $a = 360$
drizzle: $a = 56$

Table 2. Results of daily measurements for each type of rainfall.
Q_g: daily amount of areal precipitation measured by the network.
R_{gmax}: the maximum 15 minute intensity measured by any of the gauges.

Type of rainfall	Date 1969	Duration h	Q_g mm	R_{gmax} mm/h	Coefficient a when radar was calibrated by		Coefficients a and b given by the drop size distribution measurements		Remarks
					network (see Fig. 1)	fixed reference gauge (N:o 8 in Fig. 1)	a	b	
Continuous rain	12 July	6.75	6.58	4.40	256	322	484	1.54	
	18 July	4.00	4.31	7.40	106	212	290	1.35	
	26 Aug.	2.75	0.75	2.16	294	822	376	1.66	
	6 Sept.	1.75	1.86	4.28	200	170	—	—	
	13 Sept.	2.00	0.44	1.00	402	290	381	1.67	
	14 Sept.	10.25	11.11	6.72	204	150	185	1.55	
	15 Sept.	1.00	1.51	6.00	117	84	—	—	
	22 Sept.	4.00	8.35	4.48	180	92	—	—	
	Σ	32.50	34.91						
Showers	25 July	5.75	4.33	17.40	415	1115	285	1.31	
	23 Aug.	4.50	2.00	12.60	480	703	314	1.38	Thunder
	25 Aug.	1.50	5.12	29.96	243	222	201	1.38	
	30 Aug.	2.00	0.56	3.60	515	534	271	1.40	Thunder
	2 Sept.	1.50	0.66	4.48	308	2200	—	—	
	3 Sept.	2.00	0.24	2.40	575	—	—	—	
	Σ	17.25	12.91						
Drizzle	27 Aug.	3.50	2.37	2.08	10	10	32	1.28	
	16 Sept.	2.50	9.19	8.28	68	32			Rain and drizzle
	Σ	6.00	11.56						

The value of a obtained for continuous rain is almost indentical to the value of a given by Marshall-Palmer ($a = 200$). It thus seems reasonable to simply apply the M-P relationship $Z = 200\,R^{1.6}$ in Finland for continuous rain.

Many authors have calculated (see Table 1) different coefficients for showers with and without thunder, because it is probable that the mechanism producing rain in thunder clouds is stronger than in other convective clouds. This will increase the relative number of bigger drops in rain from thunder clouds leading to a higher value of the coefficient a in the Z-R relation. It is, however, difficult to recognize the thunder clouds among other convective clouds with radar, if for instance, no observation of the heights of the clouds is made. The present material contains two cases of thunderstorm (23 and 30, Aug., see Table 2) and in both of them fairly high values of a (480 and 515) were obtained. However on 3 September an even greater value ($a = 575$) was attained although no thunder was observed on the ground.

Measurements include only two cases with drizzle. On this basis it can be only said that the best a seems to be lower than 100.

3.1.2 Radar calibrated by a reference rain gauge

The method described above to calibrate the radar using a dense network of rain gauges is not applicable in routine measurements. On the other hand, it is easy to arrange the transmission of information from one rain gauge to the radar. Rainfall measured by a reference gauge within the check site can be compared with the radar data either point by point or so that the average rainfall over a larger area above the gauge is computed from the radar data. Wilson [34] has used this latter technique and states that the radar-measured rainfall for a 145 sq.km area was better correlated to the gauge-measured rainfall than that from a 15.5 sq.km area above the gauge. We changed the size of the comparison area in radar data from a point to 25 sq.km around the gauge. On the average, the best correlation was obtained between the amounts measured by the gauge and those computed from a point in radar data. This result contrary to Wilson's finding is probably influenced by the fact that we had a radar data collection frequency of $1/5\ \text{min}^{-1}$, while Wilson had only $1/10-15\ \text{min}^{-1}$.

The value of the coefficient a utilizing the reference gauge technique was computed so that the gauge-measured rainfall was equal to the radar-measured rainfall calculated from one single point above the gauge for each storm. This value was then applied to the whole check site. Table 2, column »Coefficient a when radar was calibrated by fixed reference gauge», includes these values for each day.

The deviation of the values of the coefficient is larger than the deviation of the values calculated using network observations. A natural explanation for this is that when a larger area is considered as a whole then the spatial variation of drop size distribution is smoothed out. It is worthwhile noticing that here in continuous rains the coefficient a also has a generally lower value than in convective showers. Furthermore in drizzle a very low value of a seems to be valid.

The mean values of the coefficient a weighted by the total amount of areal precipitation of each storm for various rainfall types are as follows:

continuous rain: $a = 191$
showers: $a = 720$
drizzle: $a = 28$

In continuous rains the mean value of a is very close to that obtained by network calibration indicating small variations in horizontal drop size distribution. In showers, on the other hand, the average value of a is two times larger than the value obtained by calibrating the radar with the network. A probable reason for this is that the most intensive clouds were frequently located over the reference gauge (orographic phenomenon?) giving rise to high values of a. On 3 September no rain was observed in the reference station and hence no value for a could be computed.

3.1.3 Coefficients with the aid of drop size distribution measurements

Using drop size distribution measurements, both coefficients a and b were calculated. The results are presented in Table 2 for each storm. These values were applied daily to the radar observations. The value of the exponent b is on the average near the M-P value $b = 1.6$ in continuous rains, but during showers it is always less. On 27 August (drizzle) both a and b were quite low.

3.2 *Reliability of radar measurements using various radar methods*

3.2.1 Continuous rain and showers

The question of which of the radar methods described above is best was solved by computing for continuous rains and showers:

1) the error in total areal precipitation amount during the whole summer,
2) the mean value of the absolute daily errors of each storm weighted by the daily amount of areal rainfall,
3) the percentage of the radar intensity estimates which scattered less than ± 50 % of the average areal 15 minute intensity values observed by the network.

All of these quantities are presented in Table 3. The values in the parentheses are those calculated only for the rains with drop size distribution measurements.

Table 3. Errors in radar rainfall estimates using various radar methods. The values in parentheses are those calculated for the storms with drop size distribution measurements only.

Type of rainfall	Method of radar measurement	Error in total precipitation amount %	Mean daily error in precipitation amount %	Percentage of »correct» 15 min observations %
Continuous rain Total precipitation amount: 34.91 mm (23.19 mm)	M-P	−2.0 (1.1)	11.5 (13.2)	82 (81)
	ref. gauge technique	13.1 (−1.4)	28.9 (22.3)	81 (80)
	drop size measurements	(−11.7)	(21.2)	(86)
Showers Total precipitation amount: 12.91 mm (12.01 mm)	M-P	42.9 (42.5)	42.9 (42.5)	45 (42)
	ref. gauge technique	−20.4 (−17.7)	25.2 (22.7)	52 (64)
	drop size measurements	(98.2)	(98.2)	(28)

The error in total precipitation amount was calculated using the expression $100\ (\Sigma Q_r - \Sigma Q_{\mathrm{net}}) / \Sigma Q_{\mathrm{net}}$, where Q_r and Q_{net} are the daily precipitation amounts measured by the radar and the gauge network, respectively. A positive error thus means overestimated radar precipitation and a negative error underestimation by the radar. These errors in continuous rains are rather small with any of the radar methods. In showers, the M-P coefficients led to an overestimation of 42.9 %. It is worth noticing that the coefficients obtained from drop size distribution measurements were unable to bring the radar estimates close to the true value. On the contrary, the error in total precipitation amount is as high as 98.2 %. The calibration of the radar by a rain gauge was rather successful (underestimation of 20.4 % in radar rainfall amount).

The weighting in the calculation of the mean daily error in the precipitation amount was done to avoid the excessive effect of the storms with a small amount of precipitation. The M-P coefficients have given a fairly small value for the mean daily error 11.5 % (see Table 3.) in continuous rains, while in showers the corresponding figure 42.9 % is less satisfactory. It is interesting to notice that when using a reference gauge the mean daily errors are almost equal both in continuous rains (28.9 %) and in showers (25.2 %). The drop size distbirution measurements were valuable in continuous rains but in showers the value of the mean daily error (98.2 %) is far too high. While examining these results one has to remember that the precipitation amount measured by 15 rain gauges over an area of 180 sq.km was taken as the true value of the daily precipitation amount. Especially in convective showers even this dense network does not give an exact estimate of the daily amount of precipitation (for example, see Borovikov *et al.* [4]).

The errors discussed above refer to the accuracy of the daily precipitation amount or the total precipitation amount of several storms measured by the radar. These errors do not, however, give any indication of the representativeness of radar observations of short duration. Because of the small amount of observations, only a very simple statistical parameter, the percentage of the radar observations which deviate less than ± 50 % from the true areal intensity (»correct» observations), was considered. An observation of short duration was taken to be the average 15 minute intensity. The last column to the right in Table 3 gives these percentages. In continuous rains more than 80 % of observations were »correct» with each radar measurement method. Considerably lower values were obtained for showers.

The best method to measure continuous rains by radar is the application of the M-P coefficients: the mean daily error has a lower value than the other radar methods. The next best method is the utilization of drop size measurement coefficients and the largest errors were obtained using a reference gauge. The highest percentage of »correct» 15 minute observations has, however, been obtained by using drop size measurement coefficients. Other methods have also led to almost equally good results.

In showers, when the errors of seasonal or daily precipitation amounts are considered, the order of superiority is: 1) the use of a reference gauge, 2) the M-P coefficients, 3) drop size measurement coefficients. The best number of »correct» observations was also obtained using a reference gauge. Both the M-P coefficients and the drop size measurement coefficients gave such low percentages of »correct» observations that their usefulness in showers is rather doubtful.

3.2.2 Drizzle

Because drizzle was measured only during a two day period, and one of these days included temporary rain and drizzle, there is no reason to give combined results for drizzle. Table 4 shows the calculations for both days separately. The M-P coefficients led to a clear underestimation of precipitation amount, which is to be expected due to the great relative number of small raindrops in drizzle. The utilization of a reference gauge gave almost correct results on 27 August but on 16 September it resulted in a serious overestimation. Drop size distribution measurements were carried out only on 27 August. The error in the precipitation amount using these coefficients was clearly less than the error using the M-P coefficients.

Table 4. The error in precipitation amount of each storm and the percentage of »correct» observations for drizzle.

Date 1969	Error in precipitation amount in each storm %			Percentage of ”correct” 15 min. observations %		
	M-P	ref. gauge technique	drop size measurements	M-P	ref. gauge technique	drop size measurements
27 Aug.	−85.3	−3.8	−56.4	0	100	29
16 Sept.	−49.2	60.1	—	54	18	—

In drizzle the drop size distribution (and the rainfall rate) varies generally only slightly in space and time. Hence it is probable that the use of a single reference gauge would greatly improve the radar measurements. This assumption cannot be proved correct here due to the small amount of data.

4. *Summary and conclusions*

During 16 storms in summer 1969 the areal precipitation over a check site (180 sq.km) was measured by an X-band radar. The radar estimates were compared with the observations of 15 recording rain gauges. The data were classified into three types of rainfall: continuous rain, showers, and drizzle. Such a value of the coefficient a (in Eq. $Z = aR^b$) was determined for each rainfall type, which makes the radar estimate of areal rainfall amount equal to the amount measured by the rain gauges during all the storms of a certain rainfall type. The exponent b was kept as a constant: $b = 1.6$. This procedure gave the following Z-R relationships:

Continuous rain: $Z = 196\ R^{1.6}$
Showers: $Z = 360\ R^{1.6}$
Drizzle: $Z = 56\ R^{1.6}$

The Z-R relationships given above have to be considered approximate, if no method exists to determine the actual Z-R relationship for each storm. If the computations of the radar-measured rainfall amount can be calibrated with even one rain gauge located within the area, it is worth doing. In continuous rains the gain obtained is small, but in convective showers the mean daily error drops from 42.9 %(M-P coefficients) to 25.2 %. Correspondingly the percentage of »correct» 15 minute average intensities (within ± 50 % error limits) increases from 45 % (M-P coefficients) to 52 %. Drizzle measurements were carried out during only two days. In one case the utilization of a reference gauge led to a nearby correct result but during the other day the error was even larger than when using the M-P coefficients.

To determine the average drop size distribution within the check site, the filter paper technique was applied. Measurements were carried out in three places and samples gathered at a time interval of 2.5 minutes. The average drop size distribution was computed from all samples for each storm. The Z-R relationships obtained in this way were applied to radar measurements. In continuous rain the mean daily error in the precipitation amount was 21.2 % and 86 % of the 15 minute observations were inside the error limits. The corresponding figures for showers were 98.2 % and 28 %. The drop size distribution measurements are thus not capable of improving radar estimates in showers due to the great variability of drop size distribution both in space and time. Unfortunately the drop size distribution measurements were carried out only during one case of drizzle, and hence it has not been possible to prove their usefulness in drizzle.

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