

## VERTICAL EDDY DIFFUSIVITY OF WATERS IN THE BALTIC SEA

by

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### A b s t r a c t

Using the temperature observations made on the Finnish coastal and island stations at all the Baltic standard depths, the vertical reduction of the annual range and the vertical delay time of annual extreme temperatures were computed and by means of them the vertical component of the eddy diffusivity calculated for each layer. The vertical distributions of the said coefficient were divided into three types: the high seas type, the Archipelago type and the Gulf type. The effects of the distorting assumptions were estimated. It is believed that for layers ten metres thick the mean coefficient amounts to  $1 \text{ cm}^2 \text{ sec}^{-1}$  at the depth of the seasonal thermocline but is one or two orders of magnitude higher both above and below.

### *Historical*

GEHRKE [3] probably earlier than anyone else presented a number of basic ideas, including the equation (7) below, on the effects of eddy motion upon the vertical component of conductivity of heat. His »mixing intensity» was actually the coefficient of eddy conductivity. JACOBSEN [5] showed, among others, that the eddy viscosity occurs at least one order of magnitude faster than the eddy diffusivity. Furthermore, he showed also that the eddy conductivity of heat takes place similarly to the eddy diffusivity of other concentrations, which fact makes it

possible to draw conclusions on the eddy diffusivity from the distribution of heat or any other concentration in the sea. JACOBSEN gave values for the mean coefficient of vertical eddy diffusivity at Schulz's Grund in the narrow Sound (Öresund) with a very strong vertical stability of the water column, the highly diluted surface waters flowing into Kattegat and the saline deeper waters penetrating the Baltic Sea. His coefficient amounts to  $0.3 \text{ cm}^2 \text{ sec}^{-1}$  near the surface, only to 0.04 at 12.5 metres at the mean depth of the halocline, and to 2.1 at the bottom. His approach was based upon the equation (7) below, while the actual computations were based upon data on the vertical distribution of salinity.

SIMOJOKI [8] investigated the coefficient of vertical eddy conductivity at Bogskär lighthouse ( $\varphi = 59^\circ 31' \text{ N.}$ ,  $\lambda = 20^\circ 31' \text{ E.}$ ) and obtained, among others, values given in Table 1. Although his computations were based upon observations made in the years 1899—1914, a satisfactory compatibility can be seen between them and the high seas type of this article. It is also significant to mention that WITTING [9] carried out in the summer of 1932 experimental investigations on the eddy diffusion in the Baltic Sea. The order of magnitude of his values corresponds quite well to that of the present results.

The same problems were discussed in an instructive way, among others, by DEFANT [2], by SVERDRUP [7] who presented a number of numerical results (p. 484) on the coefficient of vertical eddy diffusivity by various authors, and recently by PROUDMAN [6]. In the following summary on the mathematics involved, special attention is paid to the significance of the necessary assumptions to be made. For this reason, the deduction analogous to that by PROUDMAN has to be repeated partly in the following.

Table 1. Numerical values of the coefficient of eddy conductivity at Bogskär lighthouse, in  $\text{cm}^{-1} \text{ g sec}^{-1}$ , as given by SIMOJOKI [8].

Depth: (m)	March	July	November
5	8.3	3.2	74.1
30	5.7	0.8	2.6
60	2.1	5.2	0.2

*Deduction of the Formula*

First of all, we have to assume that any particular quantity of water moves without any change of its heat content. This means that we assume that the eddy conductivity of heat dominates over the molecular conductivity. This certainly is the case inside any water column of the seas, especially in the upper layers. The same assumption also means that we neglect the effects of the energy exchange between the sea and atmosphere, this assumption certainly being more serious. However, this assumption can be made, since in the following consideration not particular attention will be paid to the magnitude of eddy diffusivity near the sea surface.

The above assumption makes it possible to introduce the continuity of heat in the form

$$\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + w \frac{\partial Q}{\partial z} = 0. \quad (1)$$

Now it is assumed that the water movements are turbulent. Let  $\Theta$  denote the »normal» temperature at a certain depth  $z$ , at a certain site  $x, y$ , and at a certain time  $t$ , in the sea. Let  $\Theta + \vartheta$  denote the actual temperature at the same  $t, x, y, z$ , so that the mean values at a particular point  $x, y, z$  over a short time interval centred on a particular instant  $t$ , or else the mean value, at a particular instant  $t$ , through a fundamental volume centred at a particular point  $x, y, z$ , will be

$$[\Theta + \vartheta] = \Theta, \quad [\vartheta] = 0.$$

Analogously, the components of water movement will be denoted by

$$U + u, \quad V + v, \quad W + w,$$

where  $U, V, W$  stand for the mean motion and  $u, v, w$  for the corresponding components of the turbulent motion, for the »eddy components».

The time-rate at which heat flows through a unit area in the direction of the x-axis equals to

$$c\rho(U + u)(\Theta + \vartheta)$$

since  $\Delta Q = c\rho\Delta\vartheta$  and, within the limits possible in the sea,  $Q = c\rho\vartheta$ , where  $c$  stands for the specific heat and  $\rho$  for the density of sea water. Finally, the mean value of the above time-rate equals to

$$c\rho(U\Theta + [u\vartheta]).$$

The above equation (1) may be written

$$c_{\rho} \left\{ \frac{\partial}{\partial t} + (U + u) \frac{\partial}{\partial x} + (V + v) \frac{\partial}{\partial y} + (W + w) \frac{\partial}{\partial z} \right\} (\Theta + \vartheta) = 0. \quad (2)$$

Now

$$\left[ \frac{\partial}{\partial t} (\Theta + \vartheta) \right] = \frac{\partial}{\partial t} [\Theta] + \frac{\partial}{\partial t} [\vartheta] = \frac{\partial \Theta}{\partial t}.$$

And furthermore

$$\left[ (U + u) \frac{\partial}{\partial x} (\Theta + \vartheta) \right] = U \frac{\partial \Theta}{\partial x} + \left[ u \frac{\partial \vartheta}{\partial x} \right]$$

with similar expressions for the other two components. It follows that the above equation (2) may be written for the average conditions

$$c_{\rho} \left\{ \frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial x} + \left[ u \frac{\partial \vartheta}{\partial x} \right] + V \frac{\partial \Theta}{\partial y} + \left[ v \frac{\partial \vartheta}{\partial y} \right] + W \frac{\partial \Theta}{\partial z} + \left[ w \frac{\partial \vartheta}{\partial z} \right] \right\} = 0$$

or

$$\begin{aligned} c_{\rho} \left\{ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} + W \frac{\partial}{\partial z} \right) \Theta + \right. \\ \left. + \frac{\partial}{\partial x} [u\vartheta] + \frac{\partial}{\partial y} [v\vartheta] + \frac{\partial}{\partial z} [w\vartheta] \right\} = 0. \end{aligned} \quad (3)$$

Now define three functions of time and position,  $K_x$ ,  $K_y$ ,  $K_z$ , by means of the equations

$$\left. \begin{aligned} c_{\rho} [u\vartheta] &= -K_x \frac{\partial Q}{\partial x} = -c_{\rho} K_x \frac{\partial \Theta}{\partial x} \\ c_{\rho} [v\vartheta] &= -K_y \frac{\partial Q}{\partial y} = -c_{\rho} K_y \frac{\partial \Theta}{\partial y} \\ c_{\rho} [w\vartheta] &= -K_z \frac{\partial Q}{\partial z} = -c_{\rho} K_z \frac{\partial \Theta}{\partial z} \end{aligned} \right\} \quad (4)$$

and call them coefficients of eddy conductivity of heat, numerically equal to those of the eddy diffusivity. On substituting (4) into (3)

$$\begin{aligned} \frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial x} + V \frac{\partial \Theta}{\partial y} + W \frac{\partial \Theta}{\partial z} = \\ = \frac{\partial}{\partial x} \left( K_x \frac{\partial \Theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial \Theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial \Theta}{\partial z} \right). \end{aligned} \quad (5)$$

Now it is assumed that there is no mean current, that is,  $U = V = W = 0$ . This assumption is bound to distort some of our results in connection with the existing advection or upwelling. Due attention must be paid to this obvious source of distortion. Furthermore, only the effect of vertical mixing will be considered. This means that the equation to be used takes the form

$$\begin{aligned} \frac{\partial \Theta}{\partial t} &= \frac{\partial}{\partial z} \left( K_z \frac{\partial \Theta}{\partial z} \right) \\ &= \frac{\partial K_z}{\partial z} \frac{\partial \Theta}{\partial z} + K_z \frac{\partial^2 \Theta}{\partial z^2}. \end{aligned} \quad (6)$$

Furthermore, the assumption is made that the first term of the right side in the above equation (6) is always negligible when compared with the second term. Actually, inside of a homothermal water-mass

$$\frac{\partial \Theta}{\partial z} \sim 0$$

while at the discontinuity layers of the temperature and, consequently, of the density, the vertical mixing is reduced to a minimum

$$\frac{\partial K_z}{\partial z} \sim 0.$$

This certainly is not very correct for layers which are neither homothermal nor actual thermoclines. Nevertheless, the introduction of this assumption leads us to the equation given already by GEHRKE [3].

$$\frac{\partial \Theta}{\partial t} = K_z \frac{\partial^2 \Theta}{\partial z^2}. \quad (7)$$

Finally, once more the observation must be made that the coefficients for vertical eddy conductivity equal to those for the vertical eddy diffusivity, since the purely molecular effects are negligible in comparison to those of the turbulentic ones. This fact makes it possible to use the temperature distributions for conclusions about the eddy diffusivity, which has broader applications.

*Application of Data*

The type-distribution for our purpose is fully analogous to that of PROUDMAN [6], equation (7) of §73 (p. 214)

$$\Theta = \Theta_0 + \Theta_1 e^{-\frac{z}{d}} \cos\left(\frac{2\pi t}{T} - \frac{z}{h}\right) \quad (8)$$

where  $T$  is a constant period of the fluctuations, that is, one year,  $d$  and  $h$  are two constant lengths, while  $z$  denotes the depth and  $t$  the vertical time delay of extreme temperatures.

The depth value  $d$  is related to the reduction of the annual range of temperature towards greater depths:

$$e^{-\frac{z}{d}} = \frac{\Theta_{v+1}}{\Theta_v}$$

or

$$d = \frac{z}{\ln \Theta_v - \ln \Theta_{v+1}} \quad (9)$$

where  $\Theta_v$  stands for the temperature range on top of a layer and  $\Theta_{v+1}$  for the temperature range at the bottom of the same layer.

The other constant depth value  $h$  is for each part related to the delay of extreme annual values at different depths.

$$\frac{2\pi t}{T} = \frac{z}{h}$$

or

$$h = \frac{zT}{2\pi t} \quad (10)$$

Assuming now that  $K_z$  is constant and uniform for each layer, and on substituting (8) into (7)

$$\begin{aligned} -\Theta_1 e^{-\frac{z}{d}} \frac{2\pi}{T} \sin\left(\frac{2\pi t}{T} - \frac{z}{h}\right) &= K_z \Theta_1 e^{-\frac{z}{d}} \left\{ \frac{1}{d^2} \cos\left(\frac{2\pi t}{T} - \frac{z}{h}\right) \right. \\ &\quad - \frac{2}{hd} \sin\left(\frac{2\pi t}{T} - \frac{z}{h}\right) \\ &\quad \left. - \frac{1}{h^2} \cos\left(\frac{2\pi t}{T} - \frac{z}{h}\right) \right\} \quad (11) \end{aligned}$$

On equating the coefficients of sin and cos terms in (11), it follows that

$$h^2 = d^2 \quad (12)$$

$$K_z = \frac{\pi h d}{T} \quad (13)$$

Equation (12) will be used to check the reliability of the individual values for  $h$ , while the equation (13) will then be used for the computation of  $K_z$  values.

### *Data*

For the estimation of vertical eddy diffusivity of waters in the Baltic Sea, long series of temperature observations are available from different depths at different sites. GRANQVIST [4] presented mean values of the temperature (and salinity) at fifteen different fixed hydrographical stations on the Finnish coast and islands, based upon observations from the years 1921 — (or 1926 —) 1930. Starting from the temperatures observed on the 1st, 11th and 21st day of every month, a smoothed annual cycle for every depth was found by means of harmonic analysis, using three waves only. (The obvious disadvantage of this method is the misleading appearance of a temperature rise in the beginning of the year, usually in February, which does not correspond to the actual conditions in the sea.)

AHLNÄS [1] compiled the corresponding temperature observations from the years 1948—1957. The same fifteen fixed hydrographical stations on the Finnish coast and islands, where observations were made every tenth day at standard depths, were used for the purpose. An error was introduced by the fact that in storms the observations were made one or more days later than intended, but never before the assigned datum. Starting from the observed temperatures, means were calculated for the 1st, 11th and 21st day of every month for each standard depth, for the decade 1948—1957. No correction was applied to the data to correct the systematic bias of the period of observations, the effect of which obviously is negligible in comparison to the other errors. From the mean temperatures thus computed a curve characterizing the annual cycle of temperature was drawn for each depth by the present author. From the temperature curves thus smoothed, the normal temperature values for each depth and for every tenth day were read. Finally, all the individual differences between GRANQVIST'S and AHLNÄS' mean

values were computed and found more or less logical and systematic corresponding to the differences in the method of treatment of the original data. Thus GRANQVIST's data, in spite of the numerous systematic inequalities, provided a useful controll for the reliability of the present data.

From the above temperature curves for 1948—1957 for each depth the annual range of temperature and the datum of appearance of the annual maximum temperature were recorded. Reference will be made below to the datum of appearance of the annual minimum temperature.

The data finally used for the purpose were from the thirteen fixed hydrographical stations, given in Table 2. (At Ulkokalla and Säppi — Säbbskär the observations did not cover the whole annual cycle.) In Table 3 the basic data are given,  $t_{\max}$  indicating the appearance of the annual maximum temperature, while  $\Theta_v$  indicates the annual range of temperature. Before listing the  $t_{\max}$  values for each depth, their original values were plotted against the depth and in some cases slightly smoothed graphically (as shown in Table 3 by means of the symbol  $i$ ).

Now the vertical time delay  $t$  was supposed to be the average delay of the annual maximum and of the annual minimum, while above only the dates of the appearance of the maximum were given under  $t_{\max}$ . The reason for the omittance of the dates of the minimum values was simply the fact that their estimation appears in most cases too difficult because of the lack of marked changes in the water temperature during the coldest

Table 2. Fixed Hydrographic Stations Used in the Computations.

Station	Position	Depths
Tankar	63° 57' 03" N., 22° 51' 03" E.	0—10 m
Valassaaret — Valsörarna	63° 25' 17" N., 21° 04' 05" E.	0—10 m
Storbrotten lightship	60° 25' 48" N., 19° 12' 48" E.	0—30 m
Märket	60° 18' 00" N., 19° 09' 00" E.	0—100 m
Utö	59° 46' 58" N., 21° 22' 42" E.	0—90 m
Jungfruskär	60° 08' 31" N., 21° 04' 20" E.	0—40 m
Lohm	60° 06' 38" N., 21° 40' 42" E.	0—50 m
Bengtskär	59° 43' 24" N., 22° 30' 20" E.	0—40 m
Russarö	59° 46' 33" N., 22° 57' 05" E.	0—30 m
Tvärminne	59° 51' 00" N., 23° 15' 09" E.	0—30 m
Harmaja — Gråhara	60° 06' 18" N., 24° 58' 36" E.	0—30 m
Söderskär	60° 06' 34" N., 25° 24' 50" E.	0—50 m
Tammio — Stamö	60° 25' 00" N., 27° 25' 22" E.	0—20 m



Table 3. The Basic Data Used for the Computation of Eddy Diffusivity.

Station	Depth	$t_{\max}$	$\Delta t$	$\Theta_v$
Tankar	0 m	July 30		15.3°
	10 m	Aug. 28	29	12.7°
Valassaaret — Valsörarna	0 m	Aug. 1		15.6°
	10 m	Aug. 23	22	13.1°
Storbrotten lightship	0 m	Aug. 5		15.4°
	10 m	Aug. 17	12	14.3°
	20 m	Sep. 10	24	10.6°
	30 m	Sep. 26	16	6.8°
Märket	0 m	Aug. 11		15.7°
	10 m	Aug. 17	6	14.3°
	20 m	Sep. 10	24	9.8°
	30 m	Oct. 9 <sup>i</sup>	29	6.2°
	40 m	Oct. 26 <sup>i</sup>	17	5.05°
	50 m	Nov. 8 <sup>i</sup>	13	4.95°
	60 m	Nov. 16 <sup>i</sup>	8	4.85°
	70 m	Nov. 21	5	4.75°
	80 m	Nov. 25	4	4.60°
	90 m	Nov. 27	2	4.10°
Utö	100 m	Nov. 29	2	3.70°
	0 m	Aug. 4		16.1°
	10 m	Aug. 6	2	15.1°
	20 m	Aug. 15 <sup>i</sup>	9	11.4°
	30 m	Sep. 30 <sup>i</sup>	46	7.8°
	40 m	Oct. 19 <sup>i</sup>	19	6.7°
	50 m	Nov. 1	13	5.9°
	60 m	Nov. 12 <sup>i</sup>	11	5.3°
	70 m	Nov. 21.5 <sup>i</sup>	9.5	4.25°
	80 m	Nov. 27	5.5	4.20°
Jungfruskär	90 m	Nov. 29.5 <sup>i</sup>	2.5	4.15°
	0 m	July 31		16.0°
	10 m	Aug. 13	13	15.1°
	20 m	Sep. 9	27	12.7°
	30 m	Sep. 23	14	11.2°
Lohm	40 m	Oct. 1	8	9.9°
	0 m	Aug. 1		17.8°
	10 m	Aug. 14	13	15.3°
	20 m	Sep. 11	28	12.0°
	30 m	Sep. 27 <sup>i</sup>	16	10.2°
	40 m	Oct. 3 <sup>i</sup>	6	9.5°
	50 m	Oct. 7	4	9.3°

Table 3. Cont'd.

Station	Depth	$t_{\max}$	$\Delta t$	$\Theta_v$
Bengtskär	0 m	Aug. 5	6	16.3°
	10 m	Aug. 11	15	14.9°
	20 m	Aug. 26	37	10.9°
	30 m	Oct. 2	8	9.0°
	40 m	Oct. 10		7.7°
Russarö	0 m	Aug. 10	4	16.3°
	10 m	Aug. 14	4	14.5°
	20 m	Aug. 18	14	11.3°
	30 m	Sep. 1		9.8°
Tvärminne	0 m	Aug. 16	4	15.7°
	10 m	Aug. 20	12	12.6°
	20 m	Sep. 1	30	9.8°
	30 m	Oct. 1		8.1°
Harmaja — Gråhara	0 m	Aug. 12	8	15.7°
	10 m	Aug. 20	19	12.8°
	20 m	Sep. 8	21	10.0°
	30 m	Sep. 29		7.9°
Söderskär	0 m	Aug. 9	5	15.9°
	10 m	Aug. 14	27	14.0°
	20 m	Sep. 10	21	11.3°
	30 m	Oct. 1	20	8.1°
	40 m	Oct. 21	21	5.1°
	50 m	Nov. 11		3.7°
Tammio — Stamö	0 m	Aug. 1	12	17.8°
	10 m	Aug. 13	23	15.3°
	20 m	Sep. 5		10.5°

part of the year, with all the uppermost layers having then temperatures not very much different from that of the freezing-point. Nevertheless, somehow the delay of the annual minimum must be taken into account. Estimates from the graphically smoothed annual temperature curves seem to show that the vertical delay of the annual minimum is approximately 56 per cent smaller than that of the annual maximum.

The same estimate can be checked by means of the equation (12), according to which in each case the value of  $h$ , based upon the time delay of extreme temperatures, and that of  $d$ , based upon the reduction

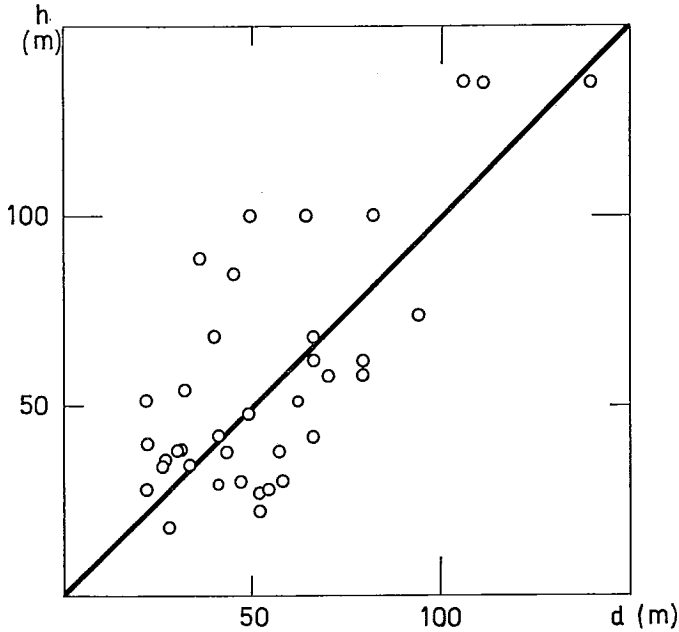


Fig. 1. The corrected values for  $h = \frac{zT'}{2\pi t}$ , in metres, plotted against the values for  $d = \frac{z}{\ln \Theta_r - \ln \Theta_{r+1}}$ , also in metres.

of the annual range of temperature towards greater depths, should equal. If, for the computation of  $h$  values, only the delays of maximum temperatures are used, systematically too small values are obtained in comparison to those for  $d$ . However, if the maximum delay data are reduced with 28 per cent, which corresponds to the above apparent inequality of 56 per cent, the  $h$  and  $d$  values are on an average equal, as shown in Fig. 1, where all pairs with values less than 150 metres for both  $h$  and  $d$  have been plotted.

### Results

Using the vertical delay data, thus reduced to correspond to the average between the delay of maximum and that of minimum, the results shown in Table 4 are obtained. In order to be able to estimate the minimum coefficient of eddy diffusivity, with its depth, and also to make a

Table 4. Coefficients for Vertical Eddy Diffusivity, in  $\text{cm}^2 \text{sec}^{-1}$ .

Depth	Tankar	Valassaaret Valsöarna	Storbotten	Märket	Utö	Jungfruskär	Lohm	Bengtskär	Russarö	Tvärninne	Harnaaja Gråhara	Söderskär	Tammio Starnö
0													
10	1.5	2.2	13.4	14.3	64.1	10.7	4.1	14.9	17.0	9.0	4.9	12.7	4.5
20			1.1	0.9	3.2	1.7	1.2	1.7	8.0	2.7	1.7	1.4	0.9
30			1.1	0.6	0.5	4.6	3.1	1.1	4.0	1.4	1.6	1.1	
40				2.3	2.8	8.2	18.8	6.4				0.9	
50				29.9	4.9		96.2					1.2	
60				48.2	6.9								
70				77.3	3.8								
80				61.8	125.5								
90				36.0	279.6								
100				39.7									

Table 5. The Minimum Values of the Coefficients for the Vertical Eddy Diffusivity.

	Depth (m)	$K_z$ (min) ( $\text{cm}^2 \text{sec}^{-1}$ )
High seas stations:		
Storbotten	20	0.97
Bengtskär	20.5	1.00
Märket	21.5	0.99
Utö	25	0.99
Archipelago stations:		
Lohm	17.5	1.10
Jungfruskär	16.5	1.67
Stations of the Gulf of Finland:		
Söderskär	36	0.87

distinction between the different types of vertical distribution of the eddy diffusivity, the results were plotted against the depth and connected simply graphically, as shown in Fig. 2 for the high seas stations, in Fig. 3 for the Archipelago stations and in Fig. 4 for the islands of the Gulf of Finland (and of the Gulf of Bothnia). In Table 5 the minimum coefficients are given for each station for which a minimum could be determined. Furthermore, it seems reasonable to divide the stations into three types corresponding to the grouping of stations in Figs. 2 to 4.

The high seas type is characterized by a strong vertical turbulence in the uppermost layer, by a distinct minimum layer at a depth between 20 and 25 metres, corresponding to the summer thermocline, and with a strong turbulence again below the thermocline.

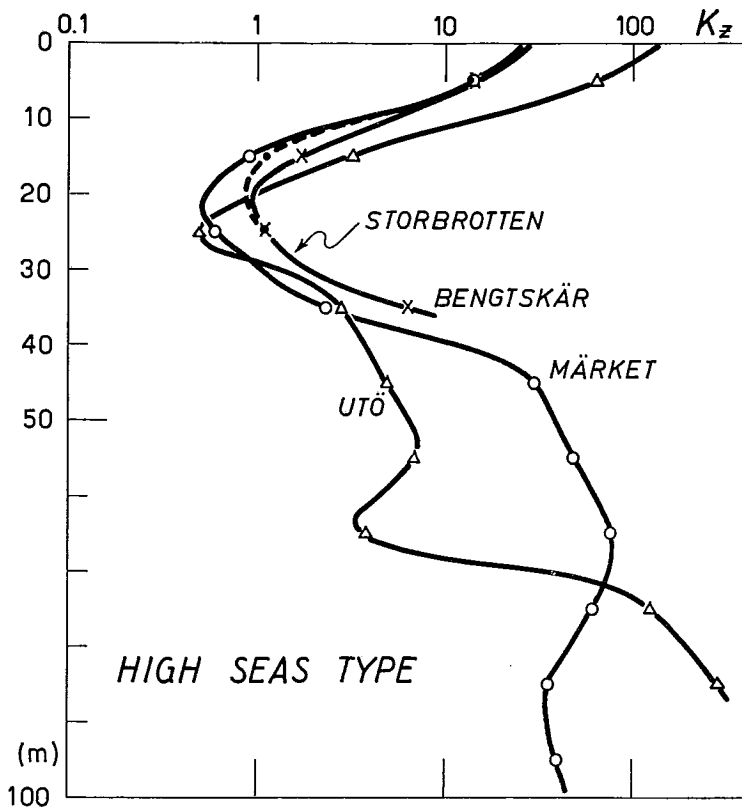


Fig. 2. The distribution of coefficient for average vertical eddy diffusion in the high seas type.

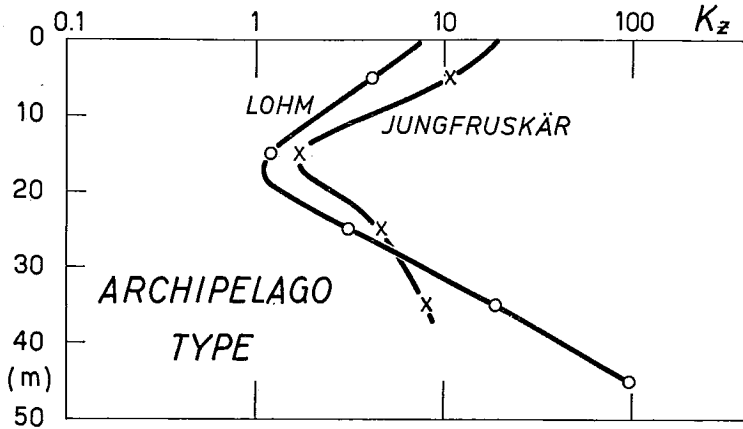


Fig. 3. The distribution of coefficient for average vertical eddy diffusion in the Archipelago type.

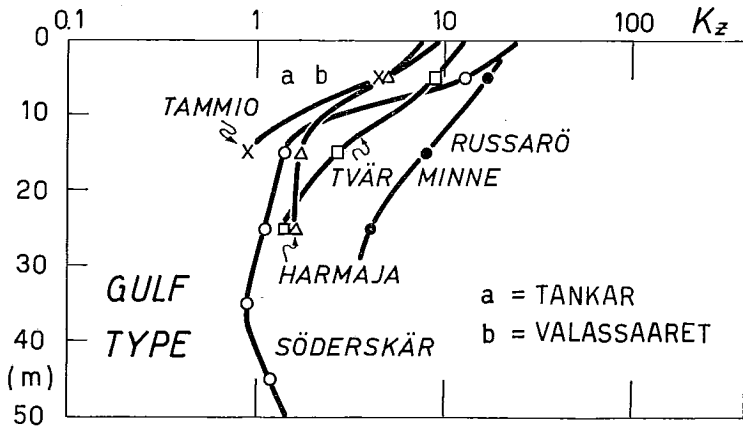


Fig. 4. The distribution of coefficient for average vertical eddy diffusion in the Gulf type.

The Archipelago type is characterized by a much more limited vertical turbulence in the uppermost layer, by a distinct minimum layer at a depth of some 17 metres and with a strong turbulence below the thermocline.

The Gulf type is characterized by a limited vertical turbulence in the uppermost layer and by a thick, less well developed minimum layer. Obviously this feature is brought about at least partly by the internal seiches which appear stronger towards the ends of a fluctuating basin than closer to its middle.

*Evaluation of the Results*

The order of magnitude of the values obtained for the vertical component of the eddy diffusivity appears reasonable when compared with the numerous earlier results from different sea areas. (Cf. SVIERDRUP, [7], p. 484). Values from  $10^0$  to  $10^1$   $\text{cm}^2 \text{sec}^{-1}$  seem to correspond to moderate stability of the water column, values  $10^1$  to  $10^2$  to smaller stability, and those larger than  $10^2$  to nearly homogeneous water layers. JACOBSEN [5] showed that coefficients as low as  $10^{-1}$  are connected with very great stability, typical for the Sound.

When taking into account that the vertical stability of the Baltic waters shows a great annual fluctuation at the depths where the effect of the seasonal thermocline is felt, one has to use only with great care the minimum coefficient, being roughly  $1 \text{ cm}^2 \text{sec}^{-1}$ . It is reasonable to assume that this minimum value is the combined result of a still better developed summer minimum, being perhaps of the order of magnitude of  $10^{-1} \text{ cm}^2 \text{sec}^{-1}$ , while during the winter the coefficient at the same depths reaches values larger than  $10 \text{ cm}^2 \text{sec}^{-1}$ . This fact has to be taken into account when making use of the presented results.

The data used in this article were those from the years 1948—1957. Experience shows that different periods yield results which are not identical, as far as the details are concerned. This shows indirectly that not too much attention must be paid to the exact numerical values of the results. Only their differences vertically and horizontally, including their orders of magnitude, are significant.

Finally, further attention must be paid to those applied assumptions, the effects of which obviously were significant. First of all, it was assumed that there is no mean current, that is, neither advective water movements nor upwelling. Since the advective water movements obviously are not negligible, the results are somewhat distorted in the uppermost layer of the water column and also in those deeper layers where advective processes are common. The error certainly is less serious at the middle depths. Also the Figures 2 to 4 show that this error apparently has affected the upper parts of the coefficient curves and also their lower ends, that is, at depths greater than some 50 metres. It is believed that the character of the curves resembles the actual conditions in the sea.

Secondly, the assumption was made that water-masses consist of homothermal layers and discontinuity layers of density only. This assumption is rather bold, since obviously in the water column there

are layers which are neither homothermal nor actual thermoclines or, rather, pycnoclines. We have to admit that this assumption in connection with some exceptional water layers significantly distorts the values obtained for the eddy diffusivity. It is believed here again, however, that the actual character of the vertical distribution of the eddy diffusivity has not changed too much because of this error.

Another error was brought about when assuming that  $K_z$  is constant and uniform for each layer. However, since the observation depths were numerous enough, the effect of this error probably was relatively limited only. Finally, minor errors were caused by the rude method of statistical introduction of the vertical delay of the annual minimum temperatures.

It is believed that when taking into account the possible distorting effects of the above four assumptions, the obtained values may be used successfully for the estimation of the mean vertical eddy diffusivity.

*Acknowledgement.* The author wishes to record his indebtedness to Miss AHLNÄS for her painstaking work in the basic treatment of data, upon which this article is based, and to the Woods Hole Oceanographic Institution for financial support for the above treatment of data.

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