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A NOTE ON THE INFLUENCE OF RANDOM ERRORS ON SOME CONCEPTS OF THE STATISTICAL THEORY OF TURBULENCE

by

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Abstract

The influence of random errors on the correlation coefficient and further on the scale of turbulence is discussed. An example is given of the estimation of random errors and of the correction of results.

It is suggested that the resolution and repeatability of recorders and evaluators used in fluctuation studies should be of an order of 0.01°C for temperature and 1 cm/s for wind speed to avoid the influence of random errors, in particular during stable atmospheric conditions near the ground.

In the classical papers of TAYLOR, the Lagrangian and Eulerian concepts of fluid flow are shown to lead to definite lengths l_1 and l_2, where l_1 is analogous to the mixing length and l_2 might be regarded as the avarage size of an eddy and is now commonly referred to as the scale of turbulence. These lengths are defined as [1]:

$$\begin{cases} l_1 = \sqrt{v^2} \int\limits_0^T R_t dt \\ l_2 = \int\limits_0^\infty R_y dy \end{cases} \tag{1}$$

where v = particle velocity, R_t = correlation between the velocity of a particle at any instant and that of the same particle after an interval of time t, R_y = correlation between the values of turbulent velocity at two points separated by a distance of y along the y-axis. In continuous turbulent movements R_t and R_y are = 1 when T or y are = 0 whereas the correlations fall to a level of insignificance when the time interval or spacing becomes large enough.

Let us suppose, however, that our data for computing R_t or R_y are subject to small random errors. It can be shown that in such a case we do not get the theoretical value of $R = 1$ for zero time delay or zero separation. It is possible to estimate the influence of random errors on the correlation coefficient in certain cases, of which the resolution of the measuring device or smoothing of the figures are the most common.

We can assume that our series of observations for computing R contains variable quantities X_i, Y_i (*e.g.* values of wind velocity or temperature) which are subject to small random errors ϑ_i, ε_i. These random errors are supposed to be independent of each other and of X and Y. Instead of the correct fluctuations x_i, y_i we thus record the fluctuations α_i, β_i where:

$$\begin{cases} \alpha_i = x_i + \vartheta_i \\ \beta_i = y_i + \varepsilon_i \\ \Sigma\vartheta_i = \Sigma\varepsilon_i = 0 \end{cases} \tag{2}$$

The correlation coefficient between the observed values is:

$$R_{\alpha\beta} = \frac{\sigma_x\sigma_y R_{xy} + \sigma_y\sigma_\vartheta R_{y\vartheta} + \sigma_x\sigma_\varepsilon R_{x\varepsilon} + \sigma_\vartheta\sigma_\varepsilon R_{\vartheta\varepsilon}}{\sigma_\alpha\sigma_\beta} \tag{3}$$

where σ = standard error. Since, by hypothesis, ϑ and ε are entirely independent of each other and of X and Y, all the terms in the numerator except the first vanish and thus:

$$R_{xy} = \frac{\sigma_\alpha\sigma_\beta}{\sigma_x\sigma_y} R_{\alpha\beta} \tag{4}$$

where:

$$\begin{cases} \sigma_\alpha^2 = \sigma_x^2 + \sigma_\vartheta^2 \\ \sigma_\beta^2 = \sigma_y^2 + \sigma_\varepsilon^2 \end{cases} \tag{5}$$

This means that the true value of the correlation coefficient, R_{xy}, is higher than the observed value $R_{\alpha\beta}$, which contains the random errors ϑ and ε.

Let us further suppose that $\sigma_\alpha = \sigma_\beta$ and $\sigma_\vartheta = \sigma_\varepsilon$. This condition is fulfilled if our data are from similar sources, *e.g.* recorded at the same height, along the same direction and with the same recorder and identical sensors. We now readily see that:

$$\frac{R_{\alpha\beta}}{R_{xy}} = 1 - \left(\frac{\sigma_\vartheta}{\sigma_\alpha}\right)^2 \tag{6}$$

Example:

Some estimates were made of the scale of thermal turbulence in an earlier paper by the author [2]. These estimates were made during stable and unstable conditions, and the length l_2 of Eq. 1 was computed as a result. It appeared, however, that the extrapolation of the values of R_y, for zero separation of the sensors during stable conditions did not give the theoretical value of 1.0 but about 0.70.

The standard deviation of the temperature fluctuations during stable conditions was 0.119°C. This corresponds to σ_α in Eq. 6, while $R_{\alpha\beta} =$ 0.70 and $R_{xy} = 1.00$. Thus, assuming that the low value $R_{\alpha\beta} = 0.70$ is due to random errors, we get from Eq. 6 $\sigma_\vartheta = 0.065$°C.

There were three different sources for random errors in the recording and evaluation system. Firstly, the repeatability of a recording system, if not perfect, is liable to give rise to random errors. In this particular case, a single-channel recorder with a mechanical switch was used. The standard deviation of the recorded points was found to be about 0.2 mm or 0.05°C during the calibration of the system. Secondly, the points measured on the recording paper were read to the nearest 0.5 mm, which corresponds to 0.125°C. The third source of random errors was the smoothing of the final temperature values to the nearest 0.1°C. Both the second and third error are smoothing errors. The deviations of the smoothed values from the correct ones belong to a uniform distribution with a standard deviation:

$$\sigma_u = \frac{d}{\sqrt{12}} \tag{7}$$

where d is a division to be smoothed or 0.125°C in the second and 0.1°C in the third error above. We thus get $\sigma_1 = 0.05$°C, $\sigma_2 = 0.036$°C and $\sigma_3 = 0.029°C$. Assuming further that σ_1, σ_2 and σ_3 are independent of each other, we get the combined standard deviation due to random errors:

$$\sigma_\vartheta = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = 0.068°C \quad (8)$$

This is close to the value $\sigma_\vartheta = 0.065°C$ which was estimated from Eq. 6. It is thus probable that the apparent reduction of the correlation coefficient to 0.70 for zero separation is mostly due to random errors of observation, although some of the assumptions may not be strictly correct.

During unstable conditions, on the other hand, the standard deviation of the temperature fluctuations described in [2] was 0.290°C. Assuming $\sigma_\vartheta = 0.065°C$, $\sigma_\alpha = 0.290°C$, we get from Eq. 6 $R_{\alpha\beta}/R_{xy} = 0.95$.

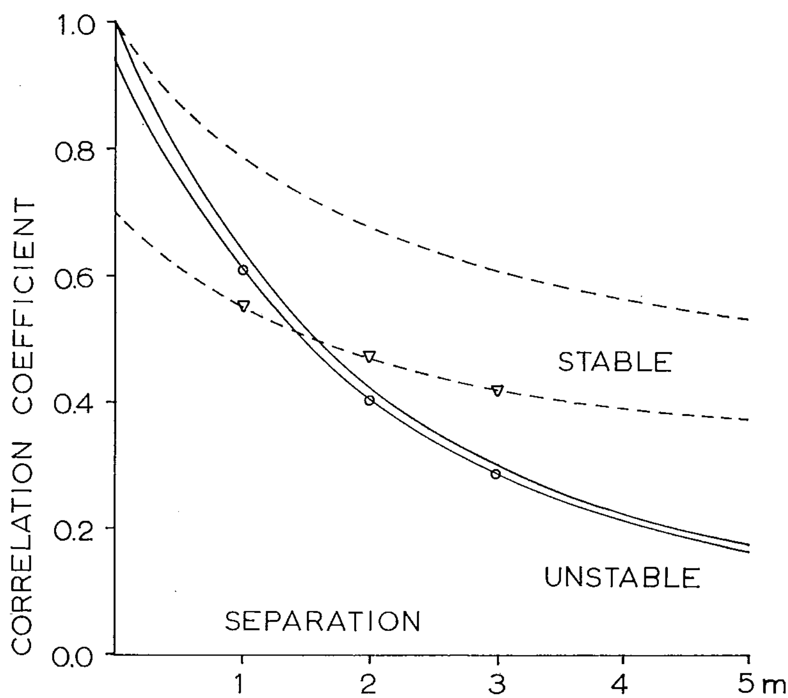

Fig. 1. Correction for random errors in the curves of correlation coefficient versus sensor separation in a study of the scale of thermal turbulence. The original curves for unstable (full curves) and stable (dotted curves) situations are those with circles and triangles (Fig. 5 in [2]), whereas the corrected curves are those starting at $R = 1.0$.

The observed and corrected values of the correlation coefficient are plotted in Fig. 1.

A review of some other papers [3, 4] on the scale of turbulence revealed sharp bends in the curves of the correlation coefficient versus separation which are probably due to errors similar to those described above. Such errors are most liable to occur in observations of temperature and wind (particularly v' and w' components) fluctuations during stable conditions. Referring to Eq. 6 we recommend resolutions of 0.01°C and 1 cm/s, instead of 0.1°C and 0.1 m/s, which are still the best resolutions of many commercial devices for recording fluctuation data.

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