551.465:526.61./62 (263)

LONGITUDINAL AND TRANSVERSAL SLOPE OF THE FLORIDA CURRENT

by

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Abstract

The relationship, shown by SANDSTRÖM [10], between the transversal slope of sea surface and the mean speed of the current, is combined with TORRICELLI's theorem. Thus it is shown that the same mean speed of the current between Cat Cay and Miami really gives both the normally accepted transversal height difference of 59 cm and a longitudinal height difference of 7.6 cm between Habana—Key West section and Cat Cay—Miami section, which is close to the corresponding value of 7.0 cm, estimated by MONTGOMERY [7]. On the basis of an empirical scatter diagram between the two height differences, computed from the daily mean values, and by means of the method of least squares in a parabolic scale, it is shown that the former height difference of 59 cm corresponds empirically to the value of 9.8 cm for the latter, which result is rather close to the former value.

For the comparison of longitudinal and transversal surface slopes of the Florida Current there have been, for a few periods of time, the following four tide gauges simultaneously in operation:

Habana (Cuba), (23° 09′ N., 82° 20′ W.)
Key West (Fla.), (24° 33′ N., 81° 48′ W.)
Cat Cay (Bahamas, B. W. I.), (25° 33′ N., 79° 18′ E.)
Miami (Fla.), (25° 46′ N., 80° 08′ W.)

For this consideration the necessary hourly heights of the sea level at the above four stations during 1950 and 1951 were received from the Director of the U.S. Coast and Geodetic Survey, which is stated with acknowledgement.

As shown by SANDSTRÖM [10], there must exist, roughly, the following relationship between the transversal slope of sea surface, i, and the mean speed of the current, $\bar{u}$,

$$i = \frac{\int_0^B \frac{\partial H}{\partial y} dy}{B} = \frac{2\Omega \sin \bar{\varphi}}{g} \bar{u} , \qquad (1)$$

where $\int_0^B \frac{\partial H}{\partial y} dy$ is the transversal height difference,

B the breadth of the current,
Ω the angular speed of the earth,
$\bar{\varphi}$ the mean latitude of the section in question, and
g the acceleration of gravity.

The above formula (1) is strictly valid only when considering steady and frictionless currents. Therefore, when applying it to the conditions in the Florida Current, only the order of magnitude can be obtained.

Several authors have used the above formula (1) to arrive at values for the transversal height difference across the Florida Current. From PILLSBURY's [9] current measurements from the BLAKE during March to May, 1890, the corresponding mean transversal height difference between Cat Cay and Miami during that period was calculated to be 58 cm or, when correcting it for the annual cycle of the mean current, 54 cm was obtained (cf. HELA, [4]). DIETRICH [3] has computed that, on an average, the sea level of Cuba is approximately 45 cm higher than the sea level on the coast of the United States. ISELIN [5] arrived at the value of 55 cm for the transversal height difference between Cat Cay and Miami. The statement of WAGNER (personal communication), based upon practical experience, that the average northerly set between Miami and Cat Cay is 2.9 knots or 149 cm s^{-1}, would give the value of 69 cm for the transversal height difference. When the average »drift current», 2.5 knots or 129 cm s^{-1}, computed from the *Current Atlas of the North Atlantic Ocean, H.O. Misc.* 10,699 (1946), is taken into account, the value of 59 cm is obtained.

Regardless of the actual value of the transversal height difference, one may compute from the tidal data the standard deviation of the transversal height differences. When using the daily means of the available simultaneous records from 1950 and 1951, roughly covering a period of one year, the height difference between Habana and Key West has a standard deviation of 7.8 cm and that between Cat Cay and Miami 8.9 cm. When assuming that the mean height differences for the sections between Habana and Key West, and between Cat Cay and Miami, are 45 cm and 59 cm, that is, when the corresponding mean surface currents would be 57 cm s^{-1} and 129 cm s^{-1}, the standard deviations of the corresponding mean surface currents will be 10.5 cm s^{-1} and 19.5 cm s^{-1}. This would mean that the mean surface current varies in 95 per cent of all cases a) between Habana and Key West between 78 and 36 cm s^{-1}, and b) between Cat Cay and Miami between 168 and 90 cm s^{-1}.

The correlation coefficient, r, between the above two transversal height differences is $+0.79$, and its probable error, F, equals to ± 0.017, since 216 quadruples of daily mean values have been used as a basis for these computations. Since the relation $\left|\frac{r}{F}\right|$ is 47, the correlation is very good indeed (Conrad et al. [2], p. 245). The relatively low value obtained for the coefficient itself is due to local wind effects. Finally, the rectilinear regression between the above two mean transversal height differences is

$$\int_{H}^{KW} \frac{\partial H}{\partial y}\, dy \sim 0.68 \int_{CC}^{M} \frac{\partial H}{\partial y}\, dy + \text{const.} \qquad (2)$$

According to Sverdrup et al. ([12], p. 641), »when attempting a calculation of currents by means of numerous *Atlantis* data from the Caribbean Sea, Parr [8] found that the flow is not directed parallel to the contours of the isobaric surfaces, but between the Lesser Antilles and the Yucatan Channel the current flows uphill. Parr suggested that this feature may be due to piling up of water in front of the narrow Yucatan Channel, caused by the stress excerted on the surface by the prevailing easterly winds. This idea was further examined by Sverdrup [11], who concluded that the piling up of the surface water can be fully explained as the effect of winds blowing with an average velocity of about 10 m s^{-1}, which agrees well with the observed values in spring. A further consequence of this piling up is that in the Gulf of Mexico a higher sea level is maintained than along the adjacent coast of the United States facing the Atlantic

Ocean.» Following a statement of BOWIE [1], MONTGOMERY [6] wrote that »it had been found from leveling survey that sea level is higher in the Gulf of Mexico than along the Atlantic coast. This head or difference in potential between the upstream and downstream ends of the Straits of Florida probably accounts for the major parts of the energy of the Florida Current.» The observed difference in level between Cedar Keys (29° 08′ N., 83° 02′ W.) and St. Augustine (29° 53′ N., 81° 17′ W.) is 19 cm (cf. BOWIE, [1]), which difference, according to TORRICELLI's theorem

$$u_2 = \sqrt{2g \int_1^2 \frac{\partial H}{\partial x} dx} \tag{3}$$

where u_2 is the longitudinal velocity at the Miami section and $\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx$ is the longitudinal height difference or head, between Key West (index 1) and Miami (index 2) transsections, would result a mean velocity of 193 cm s^{-1} at the height of Miami. Later MONTGOMERY [7] paid attention to the incorrect assumption of relatively motionless water at the height of the Key West section. PILLSBURY's [9] measurements show that the surface speed at the axis of the Florida Current on a section about 40 miles west of Key West is 115 cm s^{-1} and on the Miami section 178 cm s^{-1}. From the relationship between these velocities, MONTGOMERY concluded that the drop in head between the two sections would be only 9.4 cm or, between Key West and Miami 7.0 cm, since

$$u_2^2 = u_1^2 + 2g \int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx. \tag{4}$$

If it is assumed that the transversal distribution of surface current speed is at both sections sinusoidal, the values 73 and 113 cm s^{-1} would be obtained for the mean surface currents, which would give a value of only 3.8 cm for the head. However, these mean surface current values certainly cannot be correct. If the above mean surface current values of 57 and 129 cm s^{-1} are used, 6.8 cm is obtained for the value of head, which is rather close to the result of MONTGOMERY [7].

Since it is assumed that the energy of the Florida Current can be derived directly from the difference in sea level between the Gulf of Mexico and the adjacent Atlantic coast, the equations (1) and (4) can be combined into

$$i_2 = \frac{\int\limits_0^{B_2} \frac{\partial H}{\partial y} dy}{B_2} = \frac{2\,\Omega \sin \bar{\varphi}}{g} u_2 =$$

$$\frac{2\Omega \sin \bar{\varphi}}{g} \sqrt{u_1^2 + 2g \int\limits_{x_1}^{x_2} \frac{\partial H}{\partial x} dx}\ ,$$

or

$$i_2^{\,2} = \frac{8\,\Omega^2 \sin^2 \bar{\varphi}}{g} \int\limits_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \frac{4\,\Omega^2 \sin^2 \bar{\varphi}}{g^2} u_1^{\,2}\ . \qquad (5)$$

Since the second member can be omitted, the equation can be written as follows

$$i_2^{\,2} \sim \frac{8\,\Omega^2 \sin^2 \bar{\varphi}}{g} \int\limits_{x_1}^{x_2} \frac{\partial H}{\partial x} dx$$

or

$$i_2^2 \sim k_1 \int\limits_{x_1}^{x_2} \frac{\partial H}{\partial x} dx\ ,$$

where $k_1 = 8.40 \times 10^{-12}\ \mathrm{cm}^{-1}$.
Therefore in this special case with B_2 equal to 40 nautical miles

$$\left(\int\limits_0^{B_2} \frac{\partial H}{\partial y} dy \right)^2 \sim k_2 \int\limits_{x_1}^{x_2} \frac{\partial H}{\partial x} dx\ ,$$

where $k_2 = 460.8$ cm.
Assuming again that the transversal mean height difference at the Cat Cay — Miami section equals to 59 cm, the corresponding mean value of the longitudinal height difference between Habana — Key West section and Cat Cay — Miami section becomes 7.6 cm. The transversal mean height difference 57 cm would give the value 7.0 cm for the longitudinal height difference.

The order of magnitude of the variation of the longitudinal height difference between Habana — Key West section and Cat Cay — Miami section gives another possibility of estimating the true mean value of the longitudinal height difference. First of all, it is evident that a good portion of the changes in the mean height of both sections must occur simultaneously. This is shown through the high degree of correlation, +0.86, between the mean heights of the above two transversal sections, its probable error being ±0.01 and the ratio $\left|\frac{r}{F}\right|$ being equal to 72. The rectilinear regression between the mean heights of the above two transversal sections is

$$\tfrac{1}{2}(H_H + H_{KW}) \sim 0.89[\tfrac{1}{2}(H_{CC} + H_M)] + \text{const.} \tag{8}$$

and the standard deviations of the distribution of the same two mean heights are 8.8 cm and 9.1 cm respectively.

In spite of this high degree of correlation, the longitudinal height difference, expressed by

$$\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \text{const.},$$

varies, its standard deviation being 4.6 cm. However, since the distribution of values of the longitudinal height difference apparently is quite skew, the standard deviation does not indicate the true limits of distribution. In this connection it is worthwhile mentioning that no correlation was found between the mean height of sea level in the area, defined by the above four tide gauges, and the longitudinal height difference.

The empirical scatter diagram shows that there seems to exist a relationship between the daily mean values a) of the transversal height difference at the Cat Cay — Miami section and b) of the longitudinal height difference between Habana — Key West and Cat Cay — Miami sections. According to equation (7), this relationship between the abscissa

$\xi = \left[\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \text{const.}\right]$ and the ordinate $\eta = \int_0^{B_2} \frac{\partial H}{\partial y} dy$ must be parabolic,

and the second rectilinear regression $\xi = \xi(\eta)$ in a parabolic scale gives the empirical relationship

$$\left(\int_0^{B_2} \frac{\partial H}{\partial y} dy\right)^2 = k_3 + k_4 \left(\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \text{const.}\right), \tag{7'}$$

where $k_3 = -9145.8\ \text{cm}^2$ and
$k_4 = 360.4$ cm.

In the case of $\int_0^{B_2} \frac{\partial H}{\partial y} dy = 0$, $\left(\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \text{const.}\right)$ equals to 25.4 cm, and if, on the other hand $\int_0^{B_2} \frac{\partial H}{\partial y} dy = 59$ cm, $\left(\int_{x_1}^{x_2} \frac{\partial H}{\partial x} dx + \text{const.}\right)$ equals to 35.2 cm. Thus the value 9.8 cm is obtained for the mean longitudinal height difference between the Habana — Key West and the Cat Cay — Miami sections, which result is sufficiently close to the other corresponding value, equal to 7.6 cm. One is tempted to see in this difference the effect of the friction, however, the still more plausible explanation to this relatively slight discrepancy is to be found simply in the rather vague accuracy of the rectilinear regression.

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