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A COMPARISON BETWEEN THE GEOSTROPHIC AND GRADIENT WIND IN A CASE OF WESTERLY JET

by

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Abstract:

Three cross sections normal to the flow in a cold trough are presented. The zonal wind speed has been computed first by using the purely geostrophic assumption and then by attempting to consider some of the acceleration components. Unfortunately actual wind measurements were not available in this case. In spite of this, it still seems evident that the cyclostrophic component deserves consideration when computing wind speeds in the vicinity of a jet stream with the aid of pressure data.

At the end of the paper the relationship between the location of the jet center and the isentropic surfaces has been examined.

1. *Introduction*

Even today the best available estimation of the wind speeds in the upper troposphere is based on geostrophic computations. For one thing, optical balloon observations are very inaccurate for strong winds and lack of radar equipment is regrettably common in large areas of the aerological network. Moreover, the network for upper-air observations is not close enough.

In recent years many papers have been published proving that the assumption of geostrophic balance gives a fair approximation to the true wind only in the lower parts of the troposphere, though the presence of isallobaric and frictional terms makes the result very questionable even in these conditions. Particularly as regards the upper troposphere the scarcity of upper-wind observations constitutes the greatest disadvantage in a comparative work of this kind. Moreover, efforts to explain theoretically the effect of different acceleration terms have sometimes produced contradictory results. Reference may be made to three papers of BRUNT-DOUGLAS [1], ERTEL [3] and BYERS [2] dealing with the direction of the isallobaric component.

According to LOEWE and RADOK [6], the actual wind velocities in the vicinity of a jet center are much lower than those computed from isobaric contours. Often the wind speeds as calculated directly from the pressure field may reach the maximum rate of 100 m sec^{-1}. But such speeds have very seldom been directly measured. It is obvious that in cross sections

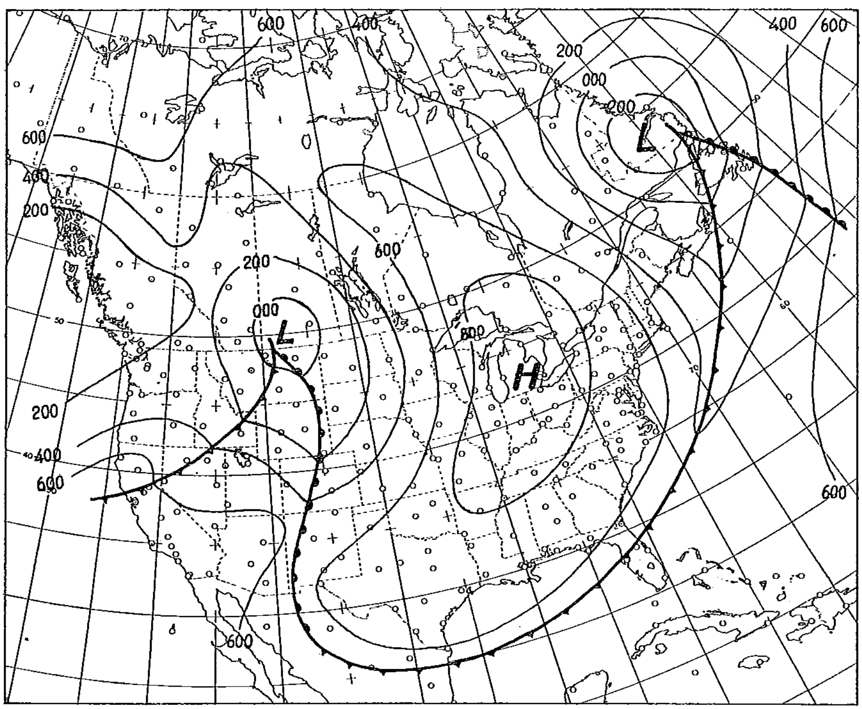

Fig. 1. 1000-mb chart 22.1.51 0300 GCT.

of this kind, we are dealing with a considerable ageostrophic wind component, and the problem is which of the acceleration components has caused this nongeostrophic effect.

2. *Synoptic situation*

The following figures illustrate the synoptic situation chosen as the object of investigation. Fig. 1 indicates the 1000-mb chart of 22.1.51 0300 GCT, showing an outbreak of cold polar air far to the south over

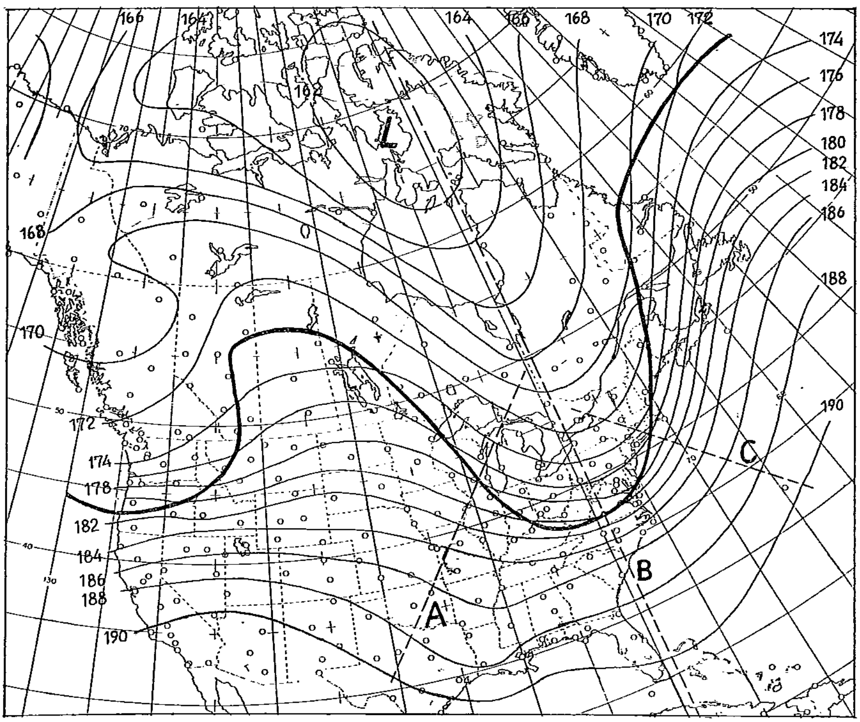

Fig. 2. 500-mb chart 22.1.51 0300 GCT. Contours are drawn for every 200 ft. The polar front is shown as a heavy, solid line; dashed lines indicate the locations of the cross sections A, B and C.

North America. Fig. 2 gives the corresponding 500-mb chart. The location of the polar front is clearly seen in Fig. 3, where the isotherms are drawn at intervals of 2 °C. Figures 4, 5 and 6 represent three cross sections through the cold trough as indicated by dashed lines in Fig. 2. The cross

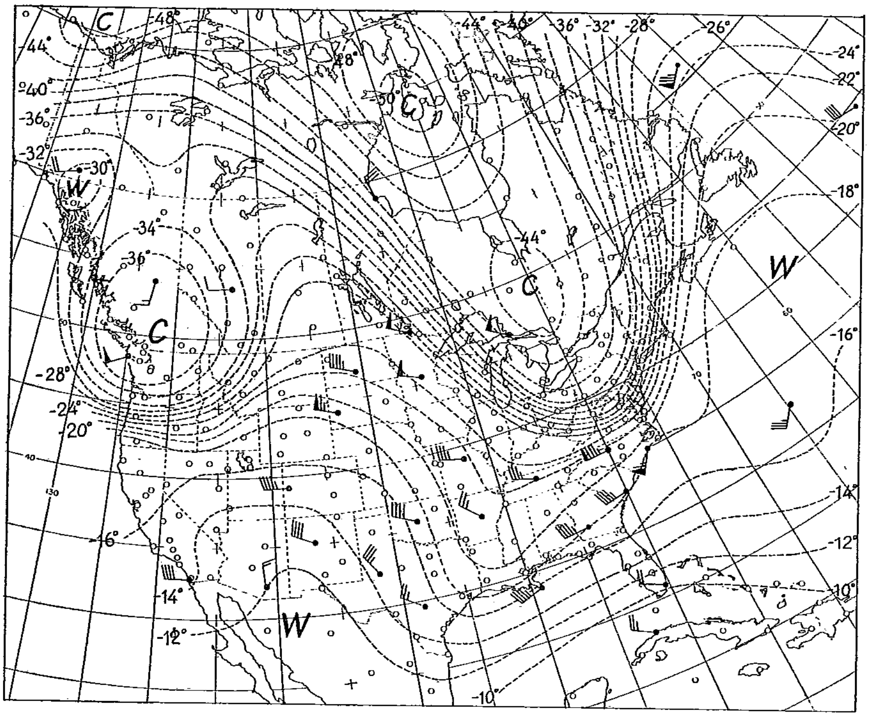

Fig. 3. Isotherms at 500-mb level, 22.1.51 0300 GCT, labelled for every 2 °C.

sections indicate the geostrophic velocities computed tangential to the frontal surface and one can see that these amount to considerable maximum values. The calculations are performed using the levels of 1000, 850, 700, 500, 400, 300, 200, and 100 mb.

3. *Method of reduction*

A frictionless and horizontal motion has been assumed and the local acceleration neglected. As to the advective acceleration, only the component normal to the velocity has been taken into account. Thus, according to PETTERSSEN [10], we may write:

$$v_g - v = \frac{v^2}{fr_i}\left(1 - \frac{c\cos\beta}{v}\right), \tag{1}$$

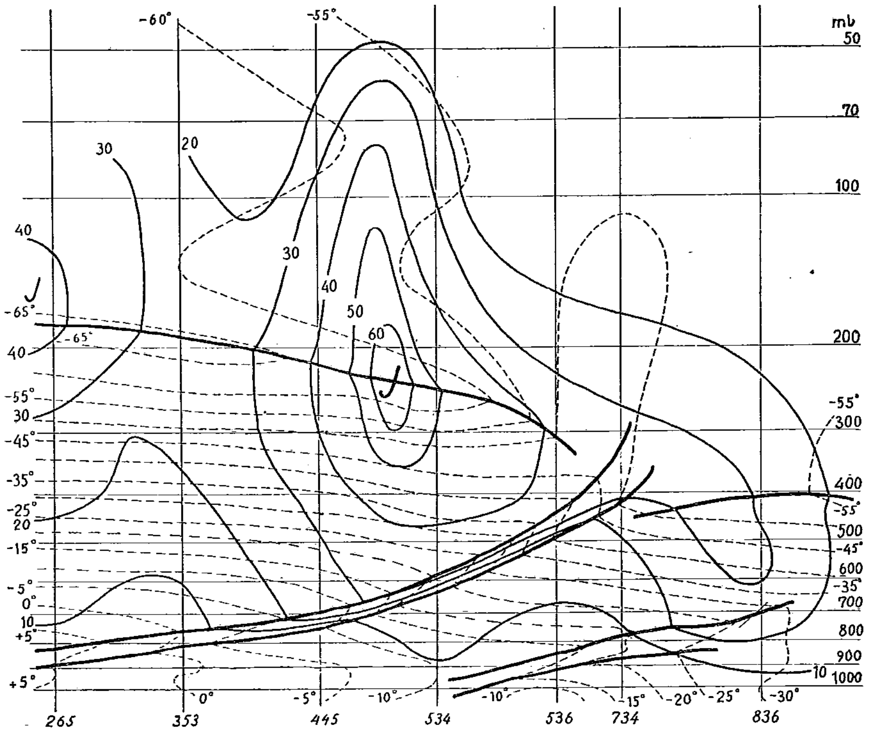

Fig. 4. Cross section A. Frontal boundaries and tropopauses are indicated by heavy, sol lines and isotherms by dashed lines in degrees centigrade. Thin, solid lines indicate th geostrophic wind velocity in m sec^{-1} normal to the cross section.

where the following symbols have been used:

v_g = geostrophic wind speed
v = gradient wind speed
f = Coriolis parameter
c = velocity of the isobar system, assumed to preserve its form.
β = the angle between the path of the pressure field and the tangent to the isobars; the isobars are assumed to be parallel and the angle β to be uniform in space and time.
r_i = radius of curvature of the isobars.

The righthand side of eq. (1) may be considered as a purely cyclostrophic component, but if we still wish to take into account the »difluence» of the isobars, the ageostrophic component thus computed *to some extent* includes the effect of the tangential acceleration.

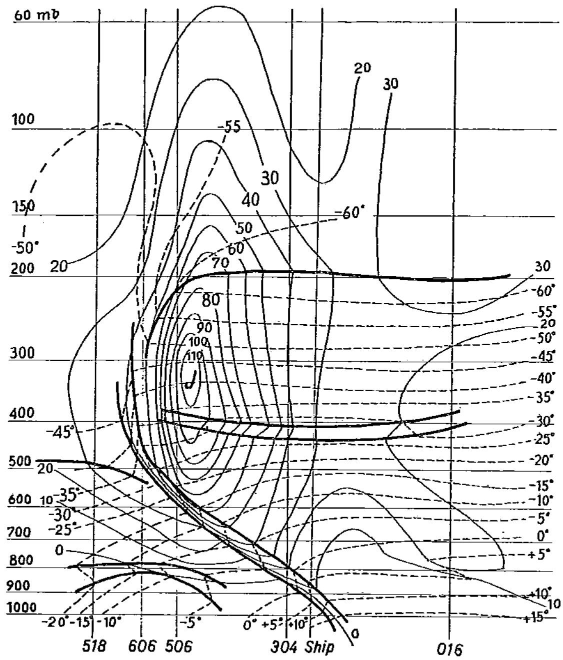

Fig. 5. Cross section C. Notations as in Fig. 4.

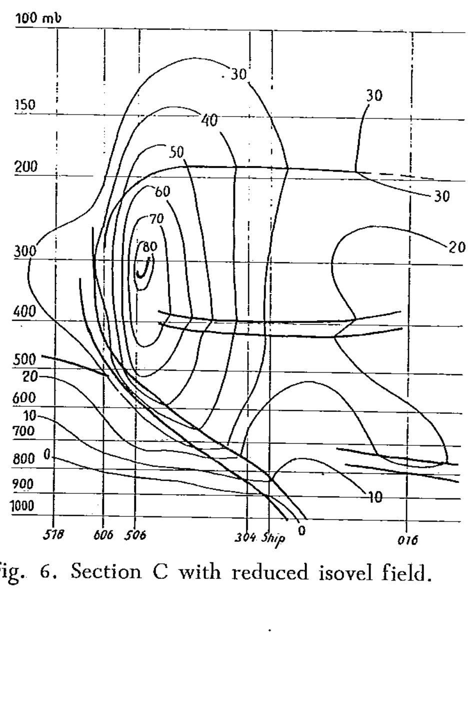

Fig. 6. Section C with reduced isovel field.

The component of c parallel to the isobars is denoted by c_i and the normal component to the right of the direction of the isobars is denoted by c_n. According to HOGBEN [4] the ageostrophic component may be written as

$$v_g - v = \frac{v^2}{f}\left[\left(1 - \frac{c_i}{v}\right)\frac{1}{r_i} - \frac{c_m}{v} \cdot d_i\right], \tag{2}$$

where d_i denotes the difluence of the isobars, $d_i = \dfrac{d\beta}{dn}$.

The writer has carried out the reduction according to formula (1), utilizing the above-mentioned pressure surfaces. Formula (2) has been used in certain special cases.

It is to be noticed that comparative work of this kind comprises many factors, e.g. when measuring the curvature from a pressure field, a certain subjectivity cannot be avoided. According to NEIBURGER *et.al.* [7], however, estimations made by several meteorologists independently of each other established that the percentage deviation from the mean ageostrophic component was smaller for strong winds. The investigation referred to was carried out using winds at the 700-mb level; accordingly the wind speeds in question naturally remained considerably below the wind speeds dealt with in this paper.

The velocity of the trough was in this situation 17 m sec^{-1} directly east. Figures 7 and 8 indicate the reduced isovel field in cross sections B and C. An inspection of the figures shows that the greatest absolute deviation, 44 m sec^{-1}, appears at the 400-mb level in section B. Hence the gradient wind here amounts only to 59 percent of the value obtained by geostrophic approximation. In section C the curvatures are generally smaller, and consequently the deviations are less considerable. The greatest difference here is 32 m sec^{-1} at the 300-mb level. Evidently the reduction leads to a decrease of maximum speeds, in section B from 110 m sec^{-1} to 75 m sec^{-1}, and in section C from 115 m sec^{-1} to 82 m sec^{-1}. At all pressure levels treated here, the absolute deviation was generally greatest in the area of maximum speed. The considerable decrease of latitudinal shear is also to be noted. Considering the vertical distribution of the deviations it is shown that the difference is greatest in the vicinity of the jet, whereas at the 700-mb level, for example, the deviations are so small that they are no longer significant.

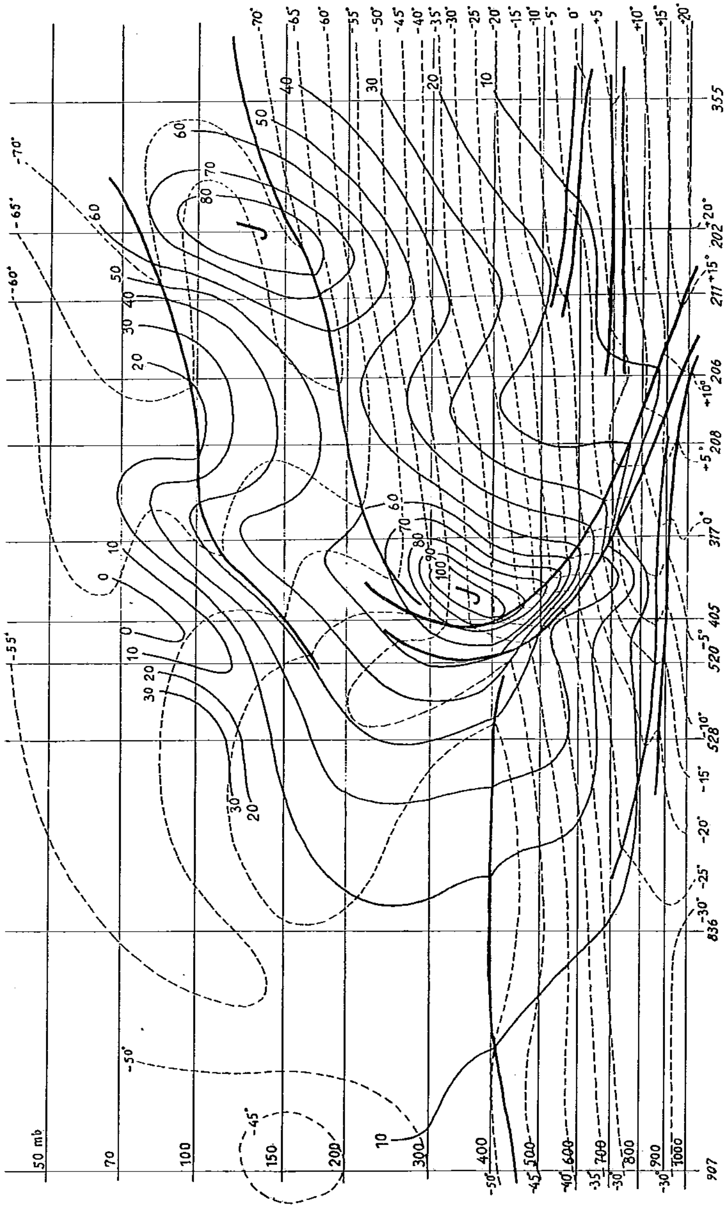

Fig. 7. Cross section B. Notations as in Fig. 4.

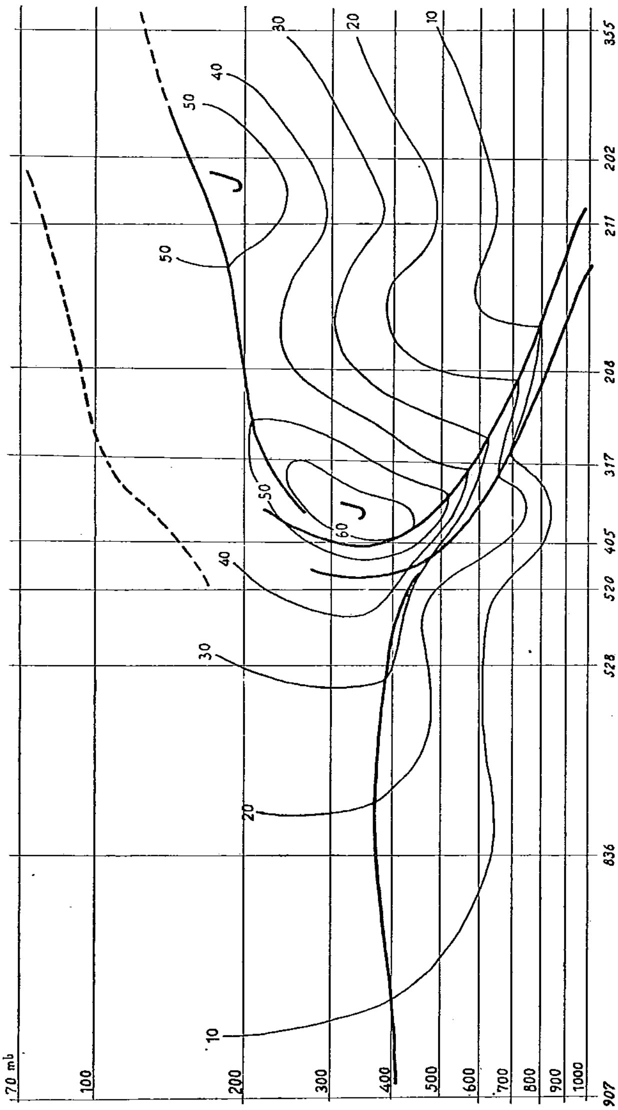

Fig. 8. Section B with reduced isovel field.

When using the geostrophic speeds a considerable tangential accelerat- ion ought to occur between the cross sections *A* and *B*. The wind maxi- mum appearing in section *A* is 64 m sec^{-1}, and considering that both the curvature and the isobaric difluence at this point are relatively small, this speed may fit as an appropriate approximate value. The geostrophic

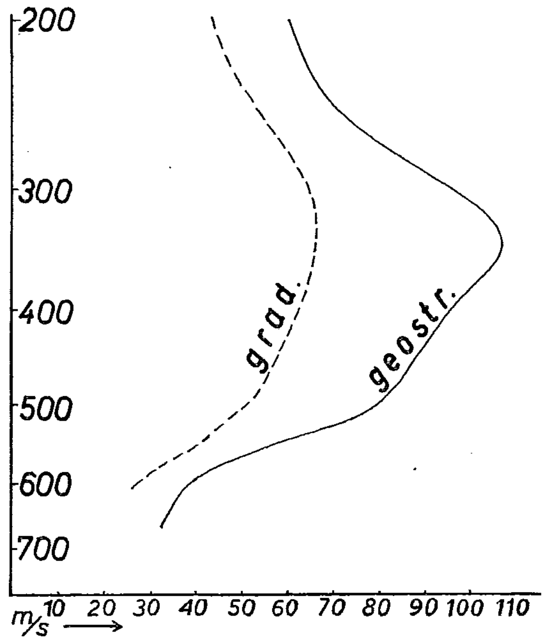

Fig. 9. Vertical distribution of geostrophic and gradient wind in the vicinity of the jet along the cross section B.

maximum speed in section *B* is, however, 110 m sec^{-1}, which means that the wind speed has increased by 45 m sec^{-1} in a distance of 1000 km. The reduction carried out along the trough line, however, reduces the wind speed in a way that makes it justifiable to assume that the effect of the tangential acceleration term is much less than that caused by the curvature.

The application of equation (2) indicates that at any rate in the case of our investigation the ageostrophic component caused by difluence was much smaller than the pure cyclostrophic component. The corrections obtained with the aid of this equation and corresponding to the different pressure levels were at most 10 per cent of the maximum speeds derived from the Petterssen's formula (1).

4. *Location of the geostrophic wind maximum*

The author has examined the few available papers presenting several simultaneous cross sections through a trough. Comparison of these showed

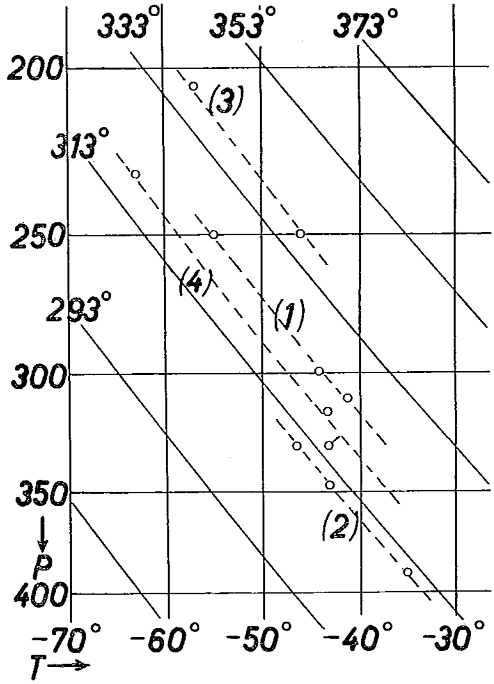

Fig. 10. Correlation between the jet centers and the potential temperature in some troughs (the cases listed in the text). Sloping lines indicate dry-adiabatics and small circles the location of the jet centers in the aerodiagram. Dashed lines show clearly how in each of the troughs the jet centers independent of the portion of the trough have almost exactly the same potential temperature.

that in all cross sections through different portions of a trough the jet centers connected with the trough appeared at the same isentropic surface.

Fig. 10 indicates an aerodiagram to which the jet cores have been transferred from the cross sections presented in the papers listed below:

1) PALMÉN-NEWTON . ([9], Fig. 3)
2) YI-PING HSIEH ([5], Fig. 10)
3) PALMÉN ([8], Fig. 9)
4) Situation discussed above in this paper.

Fig. 11 represents an isobar analysis of the isentropic surface $\Theta = 318°K$, which according to Fig. 10 seems to be the potential temperature in the jet cores discussed in this paper.

It is to be noticed that the jet coincides fairly precisely with the »low

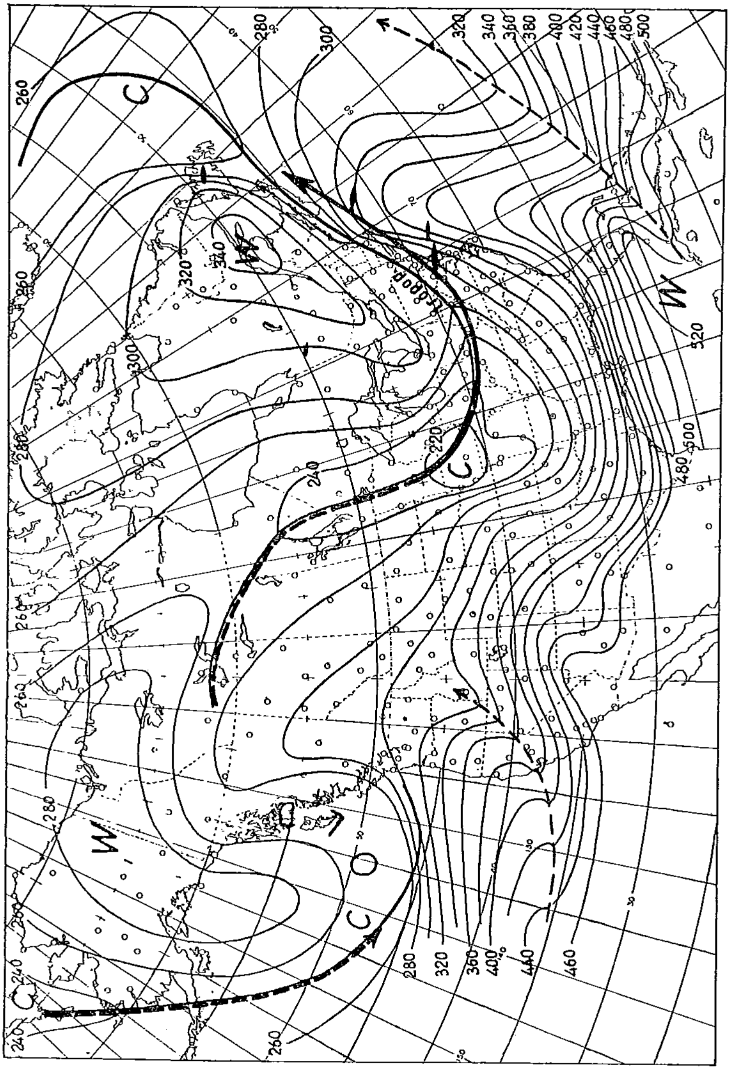

Fig. 11. Isobars at the isentropic surface $\Theta = 318°K$. The jet stream is shown as heavy, solid line when distinct, and as heavy, dashed line when not distinct; thinner solid lines indicate the tropopauses. Thinner, dashed lines indicate the location of depressions evidently connected with the subtropical wind maximum.

pressure belt» in this isentropic pressure field. Evidently a corresponding connection must prevail between the subtropical wind maximum and depressions situated further south in Fig. 11. In order to understand this figure one has to consider the equation of geostrophic wind for an isentropic surface:

$$(3) \qquad v_g = \frac{1}{f}\frac{\partial \psi}{\partial n} = \frac{1}{f}\left[\frac{1}{\varrho}\left(\frac{\partial p}{\partial n}\right)_\Theta + \left(\frac{\partial \Phi}{\partial n}\right)_\Theta\right],$$

where ψ denotes the MONTGOMERY acceleration potential for an isentropic surface, ϱ the density and Φ the geopotential. The subindex Θ means that the gradients are taken along the isentropic surface.

If the jet is to have a definite core along the vertical and if the motion is geostrophic, then, according to the thermal wind equation, in the center of the core the following relationship must hold:

$$(4) \qquad \left(\frac{\partial T}{\partial n}\right)_p = 0 \text{ or } \left(\frac{\partial p}{\partial n}\right)_\Theta = 0$$

An isobar in the core is also a tangent to the isentrope. According to Fig. 11, the tangential isentrope is in this case below the isobar except in the center.

However, if we assume that the jet stream has a definite core along the vertical, and if the motion is geostrophic, then the slope of the isobaric surfaces must increase with height below the jet stream core. Statically this requires warm air to the south and cold air to the north of the jet axis below the isobaric surface passing through the core, and *vice versa* above this surface. Thus, if the axis of the wind maxima at different pressure surfaces is exactly vertical, in a thin layer in the immediate vicinity of the jet center no remarkably cold column can exist.

When, however, the axis of the wind maxima slopes along a steeply southward-rising tropopause, the core is located in the centre of colder air and thus the isentrope passing the jet center also runs below the isobar passing the center elsewhere but in the core. Both the cases discussed above are illustrated in Fig. 12.

One may assume that the situation represented in Fig. 11 resembles case *b* in Fig. 12, in which the shaded area indicates the relatively cold column in the vicinity of the jet center.

It is a known fact that at the pressure surfaces just above the jet stream there is a band of cold air to the south of the jet. This fact is also visible in Fig. 12b at the surface P_{j-1}. According to Fig. 11 a similar cold band also exists at the pressure surfaces just below the jet stream but here coinciding with the jet or the region a little to the north of it. This fact is also in agreement with the statical presumptions according to Fig. 12b at the

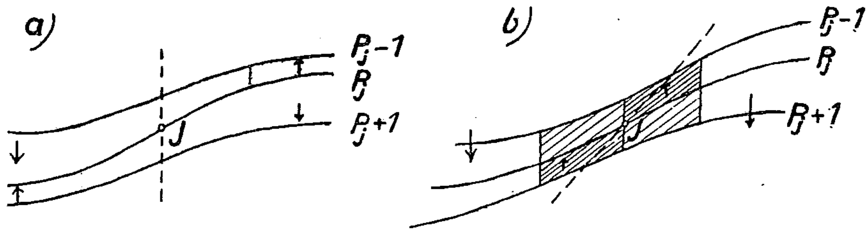

Fig. 12. Schematic picture showing the axis of wind maximum (dashed lines) at the isobaric surfaces Pj+1, Pj (i.e. the surface at which the jet center J exists) and Pj-1. In the case a) the axis is vertical, in b) it is parallel with a steeply southward-rising tropopause. Shaded areas depict the location of cold layers. A probable vertical circulation has been illustrated by small arrows.

surface *Pj + 1*. The areas of very warm air both to the north and the south of the jet in Fig. 11 are presumably produced by a strong descent of air in these regions. When the probable vertical motion capable of contributing to the development of a situation such as that presented in Fig. 11 is examined, this circulation pattern is by no means in disagreement with the model of »direct» circulation above and »indirect» circulation below the jet stream.

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