551.508.71

The Lag Coefficient of Hygroscopic Hygrometers

Supplementary report

by

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Abstract

See the report of the Symposium in Helsinki, May 1953, this issue of *Geophysica*, p. 146.

In a previous paper on this subject [1] I developed a theory of the lag coefficient of hygroscopic hygrometers, especially of the hair hygrometer. There the following two formulae[1]) for the lag, (23) and (27) were derived:

$$(1) \qquad u - u_s = \gamma \frac{dx}{d\tau} \text{ with } \gamma = k \frac{p}{E} \text{ and } k = \frac{3\,cd}{4\,\varepsilon f} \frac{\Delta\, l_o}{l},$$

$$(2) \qquad u - u_s = \beta \frac{d\,u_s}{d\tau} \text{ with } \beta = \frac{\gamma}{u'_s}.$$

Here

u = relative humidity of the air ($u \leqq 1$), $U = 100\, u$,

u_s = » » » » » which is in equilibrium with the water content of the hair,

$u_s = u_s(x)$ is the Gay-Lussac function,

$x = \dfrac{\Delta\, l}{\Delta l_o}$ = the relative lengthening of the hair, primarily recorded,

[1]) These formulae contained two insignificant errors, which are corrected here.

τ = time,
l = length of the dry hair,
Δl = lengthening of the hair from 0 % to U % humidity,
Δl_0 = maximal lengthening of the hair (at 100 %),
d = diameter of the dry hair,
p, T, ϱ = as usual,
E = saturation vapor pressure at the air temperature (T),
c = a constant, $\varepsilon = 0.622$,
f = $f(\varrho, v)$ = the ventilation function, meaning the amount of air per unit surface area of the hair which during unit time comes into contact and into water exchange with the hair.

It is this ventilation function, $f(\varrho, v)$, which our further development concerns. Evidently, this function is at least nearly proportional to the air density ϱ. Therefore we may write

$$f(\varrho, v) = \varrho \cdot \varphi(v). \tag{3}$$

The function φ of the ventilating speed v probably has the property of becoming nearly constant even at very low speeds. Then from (1) we get

$$\gamma = \frac{3\,cd}{4\,\varepsilon\varphi}\,\frac{\Delta l_0}{l}\,\frac{p}{\varrho E}.$$

As $p = R\varrho T$, we have further

$$\gamma = \varkappa \cdot \frac{T}{E} \quad \text{with} \quad \varkappa = \frac{3\,cdR}{4\,\varepsilon\varphi}\,\frac{\Delta l_0}{l}. \tag{4}$$

Here the quantity $\varkappa$ can be regarded as a constant even for moderate ventilating speeds. Both $\varkappa$ and the lag are directly proportional to the thickness (d) of the hair, which is very plausible. The relative maximal lengthening, $\frac{\Delta l_0}{l}$, is nearly the same for different hairs. Therefore thinner hairs are quicker in responding to changes in air humidity.

In my previous paper [1] I supposed the ventilating function $f(\varrho, v)$ to be constant at surface pressure, which cannot hold with very widely varying temperatures because of the variation of density with temperature.

The usual lag coefficient β (2) now becomes

$$\beta = \frac{\varkappa}{u'_s}\,\frac{T}{E}. \tag{5}$$

We see from (4) that γ, for one and the same hair, depends upon the temperature only, and from (5) that β depends upon the relative humidity too, being inversely proportional to the derivative u'_s of the Gay-Lussac function.

To prove the dependence of β upon temperature we use the same measurements of NYBERG as in [1], Table 4. The product $\frac{E}{T}\beta$ should be

Table 1. The lag coefficient at 80—90 % humidity (NYBERG).

T° C	20	18	7	4	0	—5	—7	—11	—16	—18	—22	—25	—32
T° K	293	291	280	277	273	268	266	262	257	255	251	248	241
E mb	23.3	20.6	10.0	8.1	6.1	4.2	3.61	2.63	1.74	1.48	1.05	0.80	0.41
β sec.	25	30	130	70	140	230	130	120	390	320	750	800	1 260
$\frac{E}{T}\beta$	1.98	2.13	4.64	2.06	3.15	3.62	1.77	1.22	2.65	1.85	3.14	2.58	2.16

constant because the measurements are made at about the same relative humidity of 80—90 %; hence u'_s is nearly constant and approximately = 2 (Table 2). The agreement is better than in the case of the constancy of βE in [1] Table 4. The mean value is

(6) $$\frac{E}{T}\beta = 2.53 \pm 0.25.$$

We now get from (5):

(7) $$\varkappa = \frac{E}{T}\beta\, u'_s = 2.53 \times 2 = 5.06 \pm 0.5 \sim 5.$$

Thus we obtain from (4) and (5) the following a p p r o x i m a t e values for the two lag coefficients

(8) $$\gamma = 5\,\frac{T}{E}\ \text{sec. and}\ \beta = \frac{5}{u'_s}\,\frac{T}{E}\ \text{sec.}$$

The derivative of the Gay-Lussac function u'_s is given in Table 2. In the same table also the quotient $u'_s : u_s$ is shown. As we can see, this quotient is nearly constant for the usual humidities 30—100 %. This means that the Gay-Lussac function is nearly a logarithmic one. If we take the value

Table 2. Gay-Lussac function $u_s = u_s\ (x)$.

$100\, u_s$ %	0	10	20	30	40	50	60	70	80	90	100
$100\, x$ %	0	20.9	38.8	52.8	63.7	72.2	79.2	85.2	90.5	95.4	100

u'_s	0.479	0.559	0.715	0.917	1.18	1.43	1.67	1.89	2.04	2.18
$u'_s : u_s$	9.6	3.7	2.86	2.62	2.62	2.60	2.57	2.52	2.40	2.30

U_s (10)	—	14.0	21.8	31.0	40.7	50.3	60.0	69.7	79.6	90.0	100.9
Error δU_s	—	+4.0	+1.8	+1.0	+0.7	+0.3	0	—0.3	—0.4	0.0	+0.9

$$(9) \qquad u'_s : u_s = 2.5 \text{ or } u'_s = 2.5\ u_s$$

and postulate that the logarithmic and Gay-Lussac functions must coincide at 60 %, we get

$$(10) \qquad \log U_s = 1.086\, x + 0.918.$$

In Table 2 we have also given values of U_s according to (10), as well as the differences of these values against the Gay-Lussac scale. The agreement is fairly good between 30 and 100 %. It is evident that the value (10) for U_s can be used practically always when considering the lag. Thus we get from (5) and (9)

$$(11) \qquad \beta = \frac{\varkappa}{2.5\ u_s} \frac{T}{E} \sim \frac{2}{u_s} \frac{T}{E} \text{ sec.}$$

With this value the lag equation (2) can be written (for U) approximately:

$$(12) \qquad U = U_s + \frac{200}{U_s} \frac{T}{E} \frac{d U_s}{d\tau}.$$

This relation depends upon the assumption (9). The coefficient 2 in (11) and 200 in (12) may obtain other, somewhat different values after further observations.

In practical soundings the lag of the hair is a very unfortunate fact. At low temperatures the response of the hair to humidity variations is very small. To get some rational corrections the value of the derivative $\frac{d U_s}{d\tau}$

must be measured accurately. It has very small values at low temperatures. As it is not usually U_s, but the relative hair length increase x (or X as a percentage) that is recorded directly, the derivative $\frac{dX}{d\tau}$ can be directly measured from the record. Therefore form (1) of the lag is better suited for correcting the recorded humidities than (2) and (12). This fact becomes yet more convincing as the lag coefficient γ (4) is independent of the humidity but β (5) not. Therefore we write (with (4)):

$$(13) \qquad U = U_s + \gamma \frac{dX}{d\tau} = U_s + \varkappa \frac{T}{E} \frac{dX}{d\tau}$$

or approximately (7):

$$(14) \qquad U = U_s + 5 \frac{T}{E} \frac{dX}{d\tau}.$$

Let us apply this formula to a sounding made on board the S/S NAVIGATOR 30. 8. 1939 at $\varphi = 29°.0$ N, $\lambda = 19°.4$ W.

To get the derivative $dX : d\tau$ (in % /sec.) we measure on the record by means of the tangent method the corresponding value in mm per min. Let this be η'. Taking into account the hyperbolic scale of our radiosonde and the calibration curve of the hair hygrometer we get

$$\frac{dX}{d\tau} = 0{,}0573 \left(\frac{z + 90}{100}\right)^2 \eta'$$

where z is the hyperbolic reading of the record. z varies between 104 and 118.

Table 3 contains data for the sounding. I have chosen this sounding because the record of the falling radiosonde almost reaches sea level. It is very interesting to analyse the falling part of the record and compare it with the rising part. We have extended the analysis from sea level up to nearly 500 mb. A trade inversion of 4° C was observed between 853 and 827 mb during the ascent and of 5° C between 883 and 843 mb during the descent of the radiosonde. When ascending the RS reached a sc cloud at 871 mb. It is to be noted that the mean ascending rate of the balloon was 350 m/min. but the descending rate about twice as great, 690 m/min. The bursting height of the balloon was at 29 mb.

Table 3. Sounding S/S NAVIGATOR 30. 8. 1939.

No.	τ min.	P mb	U_s %	η'	$dX : d\tau$	γ sec.	ΔU%	U%
0	0	1020	—	—	—	—	—	80
1	1.0	978	88	—0.40	+0.088	60	+ 5	93
3	2.2	931	86	+0.46	— .102	75	— 8	78
5	3.1	897	91	—0.60	+ .132	83	+11	102
6	3.7	871	98	—0.12	+ .026	90	+ 2	100
8	4.5	841	78	+1.80	— .405	80	—32	46
9	5.0	827	61	+1.10	— .255	75	—19	42
10	5.5	808	53	+0.65	— .152	75	—11	42
11	6.5	771	43	+0.33	— .079	82	— 6	37
12	7.5	740	40	+0.23	— .056	85	— 5	35
15	11.5	625	37	+0.06	— .015	150	— 2	35
17	15.0	536	34	+0.03	— .007	270	— 2	32
18	99.4	518	31	+0.01	— .003	315	— 1	30
20	101.9	652	32	—0.09	+ .022	146	+ 3	35
23	104.7	805	34	0	0	75	0	34
25	105.5	865	34	—0.74	+ .184	80	+15	49
26	105.9	883	41	—1.45	+ .350	94	+33	74
27	106.3	921	49	—0.81	+ .192	80	+15	64
28	106.7	951	60	—1.70	+ .395	73	+29	89
29	107.0	973	71	—1.22	+ .277	70	+19	90
30	107.4	1002	76	—0.19	+ .043	64	+ 3	79

As we can see, the corrections due to the lag can be very great, up to 33 % in our case. Fig. 1 shows the result more clearly. The originally recorded curves of ascent and descent diverge very greatly from each other. After correction the similarity of the curves is striking, especially in view of the meaningless original downward curve as well as the difference of 20—30 mb in the inversion levels. The tangent of the recorded curve has become dominant.

It is a well known fact that an extreme point of a record with lag is not the extreme value of the element but is a correct point. The extreme value of the element is to be found a small time interval before the recorded extreme. The point (944 mb, 91 %) is a recorded maximum but the corrected curve shows the maximum already at the foregoing point (978 mb, 93 %). In the same manner the point (915 mb, 80 %) is a recorded minimum but the corrected curve shows the minimum at the point (931 mb, 78 %).

There is still one significant feature to be observed. The corrected

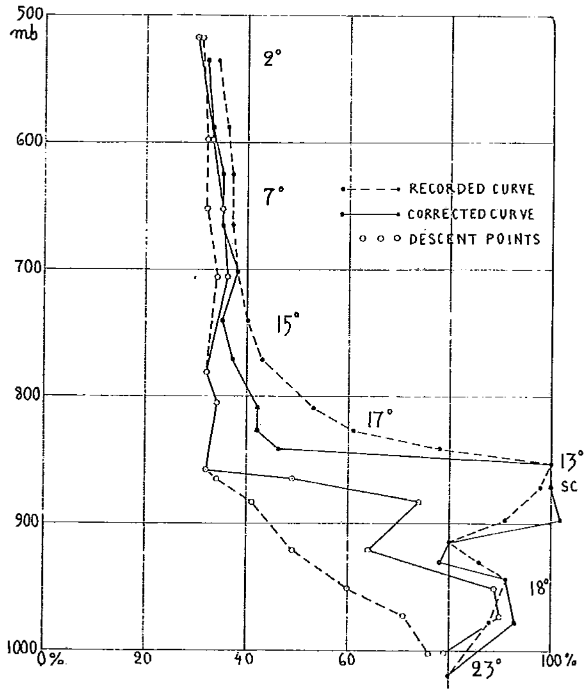

Fig. 1. Humidity sounding made on board the S/S NAVIGATOR, 30. 8. 1939 ($\varphi = 29°.0$ N, $\lambda = 19°.4$ W).

curves coincide from about 740 mb upwards. Just above the inversion up to 740 mb the corrected ascent curve shows a diminishing humidity greater than the descent curve. In other words, after a very great change of humidity the hair does not at once attain the conditions of our theory. In the same manner, just after passing the inversion the corrected descent curve does not reach 100 % humidity (supposing that a cloud also exists where the RS comes down), but lower down the curve coincides with the ascent curve. This phenomenon is perhaps due to the inner parts of the hair which only reach equilibrium with the new humidity after a certain time, whereupon the hair again acts in accordance with the theory.

REFERENCES.

1. VÄISÄLÄ, V., 1952: Theory of the lag coefficient of hygroscopic hygrometers. *Univ. of Helsinki, Institute of Meteorology, Mitteilungen — Papers* No 71, 10 pp.