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Amendments to the Computation of the Radiation Error of the Finnish (Väisälä) Radiosonde

Provisional contribution

by

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The radiation error of the Finnish radiosonde is, according to V. VÄISÄLÄ [1], given by the formula

$$(1) \qquad \Delta T = S(h) \sqrt{\frac{280}{v}} \left(\frac{100}{p}\right)^{0.852}$$

wherein

ΔT = radiation error, given as difference between day and night soundings,

$S(h)$ = radiation error function dependent only on the height angle of the Sun,

v = ascending rate of the radiosonde, gm/min.,

p = pressure at the radiosonde, mb.

In the formula (1) the radiation intensity of the Sun at the radiosonde in the stratosphere is regarded as constant. $S(h)$ is regarded as constant and equals 1.41° C for $0 < h \leqq 11°$. For $h > 11°$ $S(h)$ is computed by means of the formula

$$(2) \qquad S(h) = 5° \sin \frac{11}{7} h - 0°.12$$

The pressure exponent

$$\beta = 0.852$$

was determined at two height angles, 35° and 52°. The ascending rate v was determined from level to level. Thus the possible systematic change in v with height is taken into consideration. Generally in practical work a constant mean value of v is used.

To amend the formula (1), VÄISÄLÄ prepared an isopleth plot of the **Sun's Radiation intensity** which reaches the radiosonde at different levels in the atmosphere with different height angles of the Sun [2]. This plot shows that the formula (1) is imperfect in two ways: Firstly, because the night soundings used for (1), are mostly made just after sunset with small negative height angles, there is, in reality, a small radiation error in these soundings which were supposed to be free of it when deriving the formula (1). Secondly, when the radiosonde ascends, the Sun's radiation intensity at the instrument, assumed to be constant in (1), changes as a matter of fact, considerably with height, especially at small Sun's height angles. At higher height angles (35° and more), however, the change described is very small.

To correct the errors described we first introduce the relative Sun's radiation intensity (I) as factor in (1). This intensity which is a function of the Sun's height angle (h) and the pressure at the radiosonde (p), I(h, p), can be read from the isopleth of VÄISÄLÄ [2]. The function S(h) in (1) then represents the radiation error caused by the radiation intensity equal to the solar constant.

Secondly the simple theory of pilot balloons states that the ascending rate of the balloon is inversely proportional to $\sqrt[6]{\varrho}$, where ϱ = density. In practice, this statement as a rule holds approximately for sounding balloons too. Supposing the statement to be true we can write

$$(3) \qquad v = v_1 \left(\frac{\varrho_1}{\varrho}\right)^{\frac{1}{6}} = v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{6}} \left(\frac{T}{T_1}\right)^{\frac{1}{6}},$$

where v_1 denotes the ascending rate at density ϱ_1. Here the variability of the temperature T, in the stratosphere, during one sounding can be totally neglected. Thus, when $p_1 = 100$ mb, we obtain:

$$(4) \qquad v = v_1 \left(\frac{100}{p}\right)^{\frac{1}{6}}$$

Thirdly, in (1) the transition from the normal rate of 280 gm/min. to

300 gm/min. is effected by again substituting S(*h*) for the quantity $S(h)\sqrt{\frac{280}{300}}$.

Taking these three points into consideration in (1) we get

$$\Delta T = S(h)\,.\,I(h,\ p)\sqrt{\frac{300}{v_1}}\,.\,\left(\frac{100}{p}\right)^{\beta-\frac{1}{12}} \tag{5}$$

The ascending rate. The value of ΔT at given h, p and v_1 is, according to (5), constant. This, however, can not be exactly the case. If we consider two instances wherein h, p and v_1 are the same but temperature has two different values T_1 and T_0 the densities ϱ_1 and ϱ_0 also will differ according to

$$\varrho_0 : \varrho_1 = T_1 : T_0 \tag{6}$$

It is evident that the ventilation of the thermometer, ceteris paribus, is better and the radiation error ΔT smaller at greater density i.e. in colder air. This fact can be taken into account in (5) on substituting the momentum $v_1\,\varrho_1$, for the velocity v_1 according to Hergesell [3]. For maintaining homogenity, we at the same time substitute 300 with the momentum 300 ϱ_0 Then, in stead of $\sqrt{\frac{300}{v_1}}$ we get the quantity

$$\sqrt{\frac{300\ \varrho_0}{v_1\ \varrho_1}} = \sqrt{\frac{300\ T_1}{v_1\ T_0}}\ ,$$

where T_0 and T_1 are temperatures at the same pressure e.g. at 100 mb. Now, we introduce a reduced ascending rate

$$v_{01} = v_1\,\frac{\varrho_1}{\varrho_0} = v_1\,\frac{T_0}{T_1} \tag{7}$$

In stead of (5) we then obtain

$$\Delta T = S(h)\ I(h,\ p)\sqrt{\frac{300}{v_{01}}}\left(\frac{100}{p}\right)^{\beta-\frac{1}{12}} \tag{8}$$

According to (7) the velocity v_{01} is a measure of the momentum $v_1\,\varrho_1$, because ϱ_0 is constant. We can show that v_{01} is independent of temperature. Since

(9) $$v_1 = \left(\frac{d\Phi}{dt}\right)_1 = RT_1 \left(\frac{d\ lnp}{dt}\right)_1$$

where the index $_1$ refers to values at pressure $p_1 = 100$ mb, we get from (7)

(10) $$v_{01} = \frac{v_1}{T_1} T_0 = R T_0 \left(\frac{d\ lnp}{dt}\right)_1 = \left(\frac{d\Phi_0}{dt}\right)_1$$

In this equation T_0 stands for a constant temperature which is entirely at our disposal. We have chosen the value

(11) $$T_0 = -56.5°C = 216.7 \text{ K}.$$

v_{01} can be determined by means of pressure observations only, according to (10).

When plotting $\sqrt[6]{p}$ against ascending time we get on differentiating

$$\frac{d\sqrt[6]{p}}{dt} = \frac{1}{6} p^{\frac{1}{6}} \frac{d\ lnp}{dt}$$

On the other hand, it follows from (4) that

$$v = v_1 \left(\frac{100}{p}\right)^{\frac{1}{6}} = \frac{d\Phi}{dt} = RT\frac{d\ lnp}{dt} \sim RT_1\frac{d\ lnp}{dt}$$

Thus taking (10) into account

$$\frac{d\sqrt[6]{p}}{dt} = \frac{1}{6} p^{\frac{1}{6}} \cdot \frac{v_1}{RT_1}\left(\frac{100}{p}\right)^{\frac{1}{6}} = \frac{1}{6}\sqrt[6]{100}\ \frac{v_{01}}{RT_0}$$

or

(12) $$\boxed{v_{01} = \frac{6\ RT_0}{\sqrt[6]{100}} \frac{d\sqrt[6]{p}}{dt}}$$

Because, in practice, the pressure plot $\sqrt[6]{p}$ against time is nearly a straight line the rate v_{01} is given as the sloping coefficient of this line multiplied by a constant.

The new VÄISÄLÄ aerogram paper has on the back side a coordinate system for plotting pressure against time, from which v_{01} can then be read immediately.

The pressure exponent in (1) was determined by *Väisälä* to be $\beta = 0.852$. The exponent in (8) is $\beta - \frac{1}{12} = 0.769$. It should be borne in mind that in determining β the small radiation error in the night soundings has slightly influnced the value of β. After an approximative correction of the night soundings we get the value $\beta - \frac{1}{12} = 0.88$. Because the difference between this exponent 0.88 and the former 0.852 is very small the former exponent is retained in use. Thus the formula

$$\Delta T = S(h) \cdot I(h, p) \cdot \sqrt{\frac{300}{v_{01}}} \left(\frac{100}{p}\right)^{0.852} \tag{13}$$

has been used for calculating radiation error instead of (8).

Determining S(h). The most important changes in S(*h*) are due to variations in the Sun's radiation intensity effective especially at small Sun's height angles. In connection with the total eclipse of the Sun in 1945 soundings were made at short intervals at the Ilmala observatory during four successive nights. During each night two soundings were carried out in darkness without the Sun's radiation and two at *h* between 0° and 5°. Temperature differences ΔT of these »day»-soundings at different pressures against the corresponding night soundings were calculated and each of them was divided by the factor of S(*h*) in (13) for obtaining the value of S(*h*). Mean values of these at different height angles *h* are plotted as dots in Fig. 1.

This plot, Fig. 1, is used for correcting the small radiation error in night soundings which [1] presupposed to be free of radiation error in the determination of S(*h*). The test showed that S(*h*) changed appreciably at small height angles only, for higher height angles as well as the new value of β, being approximately the same as before. Therefore, S(*h*) was amended only as regards smaller *h*-values so as to have the curve join neatly with the former curve at greater *h*-values. This is still a temporary amendment because after the whole sounding material of the eclipse time in 1945 has been worked out the course of S(*h*) still can be slightly changed.

The radiation error nomogram of VÄISÄLÄ [1, p. 46] can, however, be used with due alterations. Formula (13) shows that ΔT, if $v_{01} = 300$, is a known function of *h* and *p*. Thus, we can draw iso-lines of *h* in a

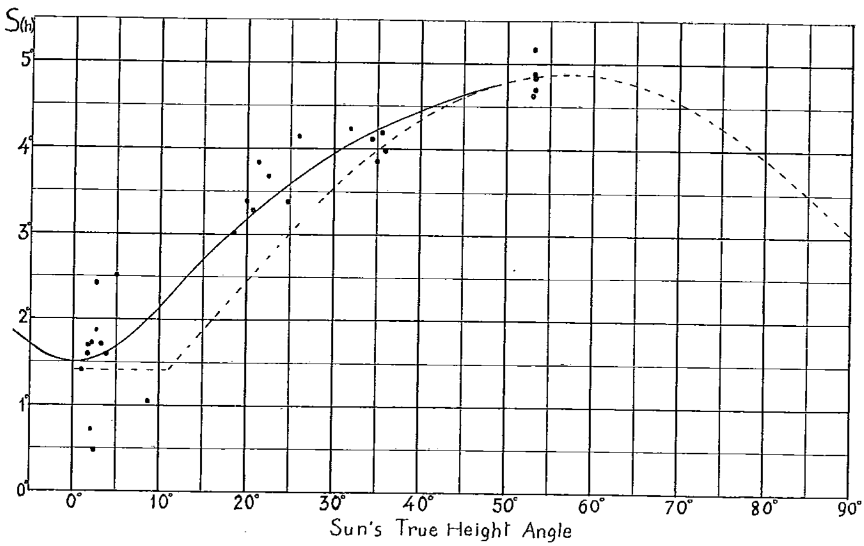

Fig. 1.

coordinate system with log p and log ΔT as coordinates. Such a diagram is printed on the back side of each calibration sheet of the Väisälä radiosondes (Fig. 2).

For each sounding, to be corrected, the Sun's height angle is plotted against pressure on this diagram. The plotted points must still be moved to the right or to the left by multiplying logarithmically with $\sqrt{\frac{300}{v_{01}}}$. For this purpose a scale is provided at the bottom of the diagram. A curve is drawn through the corrected points and the radiation error above 200 mb is read off this curve. Below 200 mb the radiation error correction is determined as previously on the aerogram sheet supposing it diminishes linearily and vanished at 500 mb.

Because $S(h)$ reaches a maximum value at $h = 57°$ the iso-lines are not drawn for greater h-values. These iso-lines can be regarded as perpendicular straight lines and their positions are marked as a scale at the bottom of the diagram.

Acknowledgement. In this connection I have the pleasure of cordially thanking Prof. Dr Vilho Väisälä for the interest he has shown in my work as well as for writing this contribution in English.

Helsinki, October 1950.

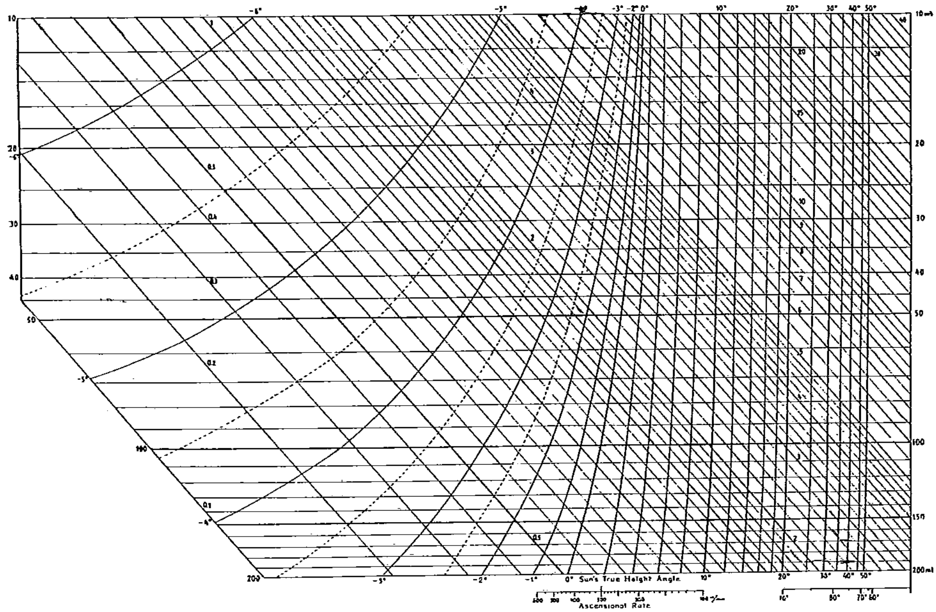

Fig. 2.

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