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On the Stress of the Wind on the Water Surface

By

ILMO HELA

The first tide-gauge on the coast of Finland was engaged in Hanko on the 20th of July in 1887. Of its records, the hourly readings of the years 1897—1903 were published first in the series Finnländische Hydrographisch-Biologische Untersuchungen (JOHANSSON, 1909). In the preface of that volume written by the director of the Meteorological Central Institute at that time, G. MELANDER, we read:

— Eine Vorgeschichte und Beschreibung des Limnigraphen nebst einer Kritik der Leistungen desselben ist jetzt von dem Assistent D:r Osc. V. Johansson ausgeführt und als Einleitung zu den Tabellen hinzugefügt.

These lines written 40 years ago refer to the critical considerations on the statistics of the height of sea-level written by our jubilee celebrator in his youth. It seems, therefore, appropriate to discuss, within the limits of Geophysica 3, shortly also the variations in the height of sea-level.

1. *The Frequency (Duration Percentages) of the Rates of Water-Level Variations.*

In an earlier paper (STENIJ, HELA, 1947) a graphical representation is given, by means of which the frequency distribution of the water heights at any point on the Finnish coast can be calculated. Even at first sight of the frequency table it is noticed that the variations are much smaller in the centre part of the Baltic than in the inner parts of the Gulf of Bothnia and the Gulf of Finland.

Accordingly, also the rate of water-level variation must, in general, be smaller in the centre part of the Baltic than in the inner parts of the Gulf of Bothnia and the Gulf of Finland. Fig. 1 gives the frequency (duration percentages) of the rates of sea-level variations at Kemi, at Degerby, and at Helsinki. The frequency of the variation rates in percentages is given in cm in 4 hours. (It is evident that a corresponding calculation in

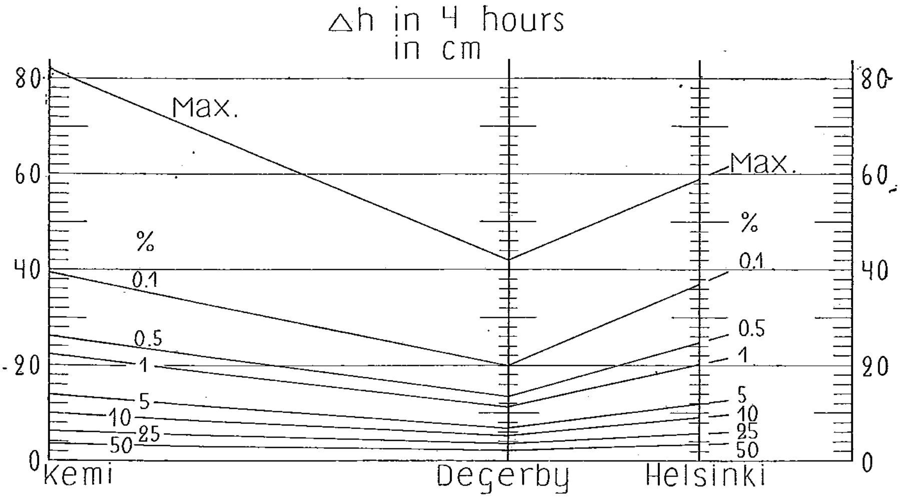


Fig. 1. The frequency (duration percentages) of the rates of water-level variations on the Finnish coast.

1 hour would have given, at least in extreme cases, relatively, slightly greater values for the rates of water-level variations.) The frequency (duration percentages) recorded on each station, Kemi, Degerby, and Helsinki, is based on 21 912 cases of variations in the height of sea-level observed during 4 hours.

For some practical purposes, the frequency (duration percentages) of the rates of water-level variations thus obtained is of importance. Linear interpolation no doubt gives the right result of the frequency of the rates of variations also at other points on the coast with adequate accuracy, since also the regional distribution of the frequency of water heights is of the same kind. The frequency table obtained shows that rapid, considerably large variations in the height of sea-level occur very rarely. In this paper we consider one extreme case.

2. Tidal Graph of Kemi from the 1st until the 2nd November 1934.

The water-level rose at Kemi on the 1st of November 1934 between 16^h and 20^h, E.E.T., 82 cm, in other words 20.5 cm per hour. Fig. 2 shows the tidal graph of this and the following day. In order to under-

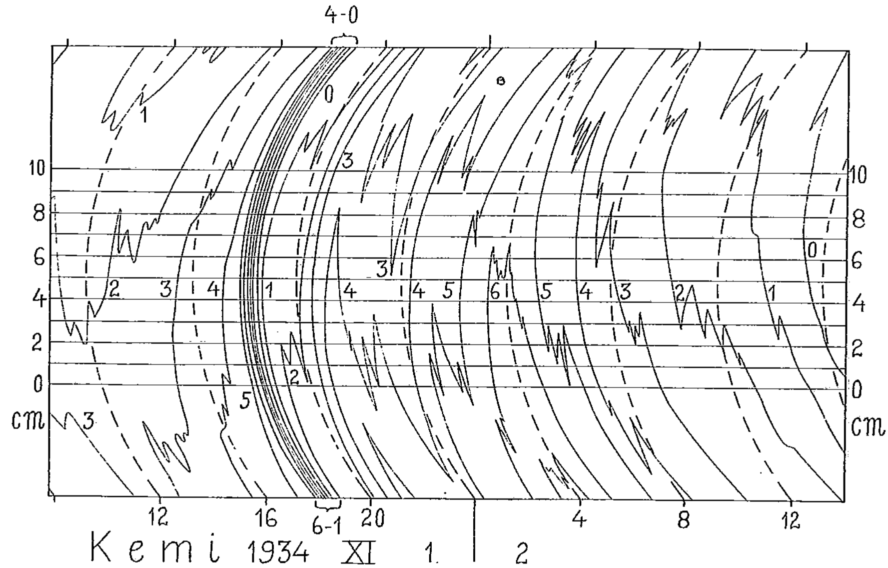


Fig. 2. Reduced reproduction of tidal graph of Kemi between 1—2 Nov. 1934.

stand the figure it is best to study Fig. 3, designed by H. Renqvist. The figure shows a tide-gauge of Renqvist-Witting type, which is engaged, for example, just at Kemi. In the well of the tide-gauge a float is swimming and bronze cable B is fastened on this float. Cable B goes over wheel 1 on to wheel C, which is in horizontal position and moves round a vertical axis. Cable B twists round the thread groove in the top part of wheel C. The length of the thread is exactly one meter. On the other hand, cable E goes out of the other groove on the frame of wheel C over wheel 2 and, by way of counterbalance 9, to a fixed point. Counterbalance 9 tightens the cables so that, as the water-level goes up, wheel C turns with the sun, and, as the water-level goes down, it turns counterclockwise. A registering sheet of the breadth of 28.8 cm goes out of paper roller H and over roller G on to collecting roller J. Clock K turns roller G so that the paper moves 12 mm per hour. Ten pencils of different

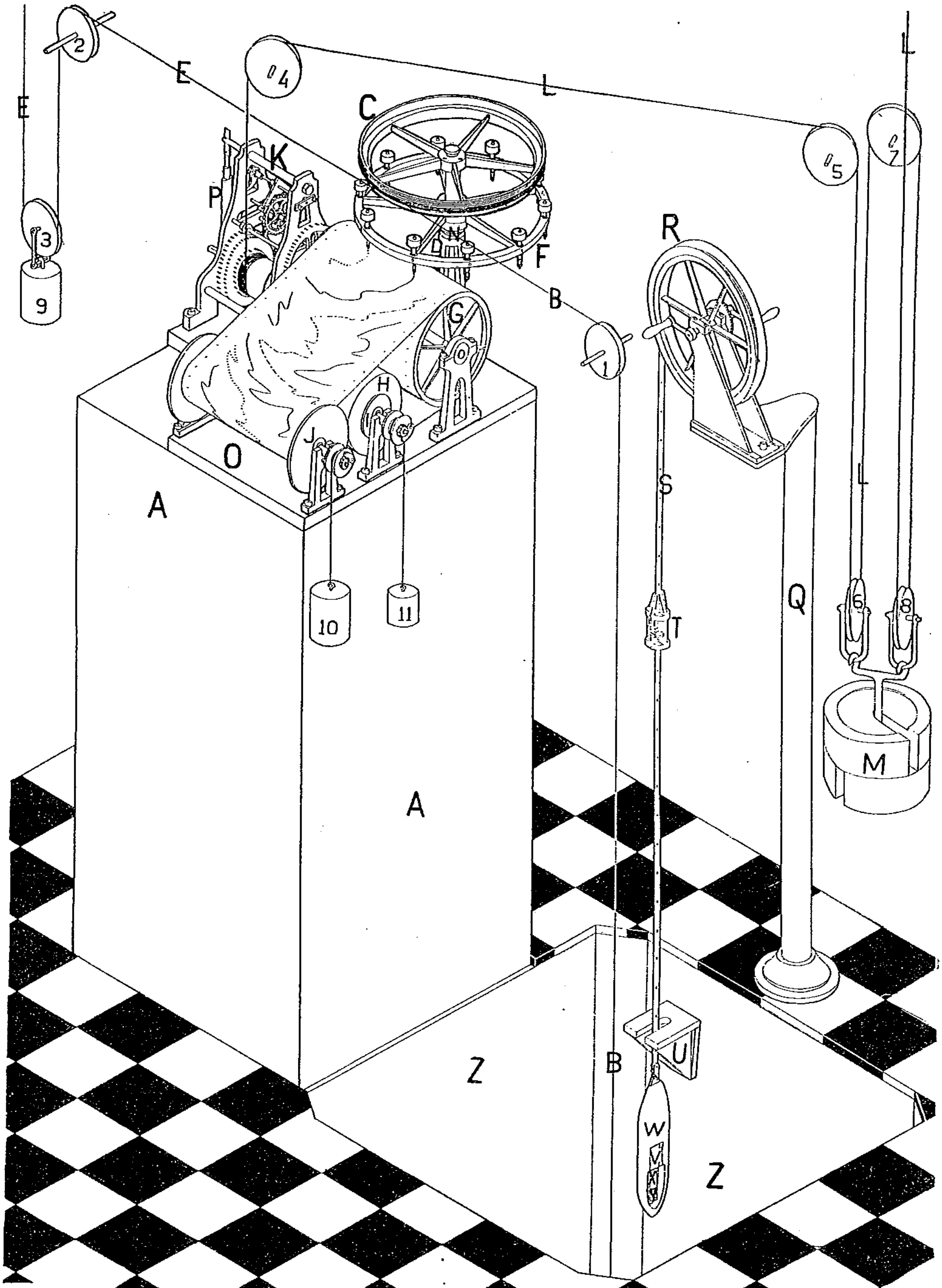


Fig. 3. The automatic recording tide-gauge, constructed by H. Renqvist and Rolf Witting.

colours have been fastened on the lower edge of wheel C so that they are equidistant from each other on a circle, the circumference of which is 1 meter long.

The tide-gauge thus draws the variations of height of water-level in full size. A curve drawn by a pencil of a definite colour corresponds to a known decimeter of water height. Every moment the pencils draw at least two curves on the paper as is seen also from Fig. 2. Given in middle water the curve of the rectangular coordination represented by Fig. 4 is obtained. As regards this curve it is to the point to pay attention to the following details especially:

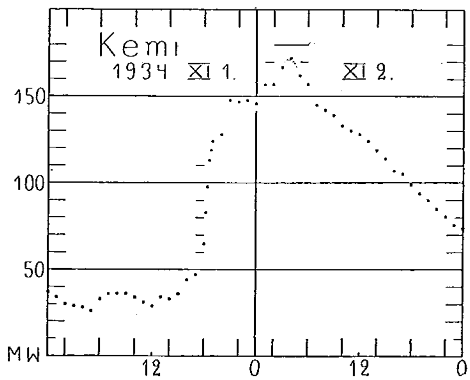


Fig. 4. The variations of the water height at Kemi 1—2 November 1934.

— The water-level rose at Kemi 59 cm on the 1st of November 1934 18^h—19^h (E.E.T.). (A look at Fig. 2 shows also that the function of the tide-gauge was surely not restrained and so the rapid rise is without doubt quite real.)

— A close inspection of Fig. 2 shows that the rise of water-level was, when most rapid, 48 cm per 36 min. or $^4/_3$ cm per minute.

— The height of water-level was, 2 November 1934 4^h (E.E.T.), MW + 172 cm, which is the most extreme value known for the present.

3. *Weather Conditions and the Height of Sea-Level.*

A rapid rise of the height of water-level is associated with the distribution of wind and air pressure distribution in the following manner.

Fig. 5 shows the weather map 1 November, 1934, 9^h. A deepening cyclone moving in NNW direction with the velocity of 60 km per hour,

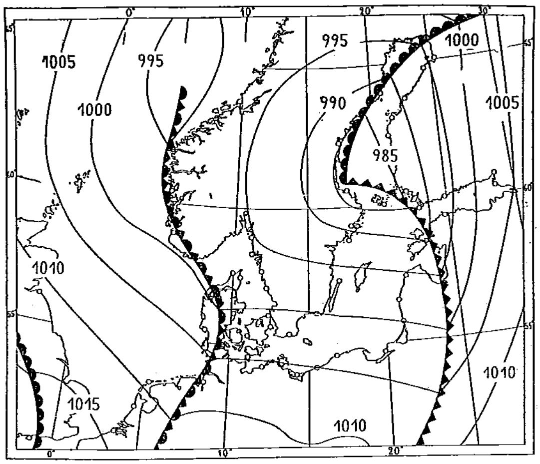
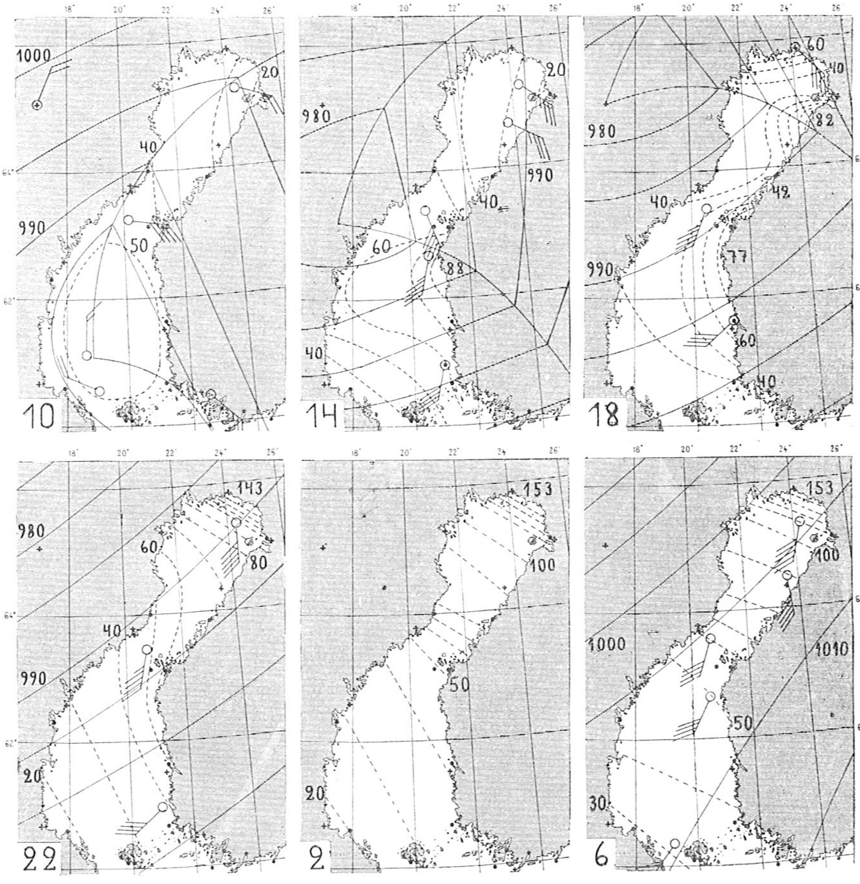


Fig. 5. Weather map, 1 November, 1934, 9^h, E.E.T.

approximately, exists in the south-western part of the Gulf of Bothnia. Figures 6—11 elucidate also the movement of the cyclone and its occlusion process. 2 November, 1934, 9^h, we find the center of our occluded cyclone near to the Lofot Islands.

The regional variation of air pressure, winds, and, on the other hand, of the heights of water-level comes out clearly in Figs. 6—11. — Fig. 6. 1 November 10^h, when the center of the cyclone is in the open south part of the Gulf of Bothnia, the water height is at all coast stations of the south part of the Gulf about the same. It seems obvious that the water masses have to some extent accumulated on the open part of the sea. In the north part of the Gulf of Bothnia, the easterly wind blowing at the back of the warm front gives rise to a lowering of the water-level on the Finnish coast. — Fig. 7. 14^h, the cold front is already near to the 63. latitude. A wave of high water has been formed in the area of the cold front. — Fig. 8. 18^h, the height of the high water wave, associated with the cold front, now near to Raahe, is about 82 cm. In the front part of the cold front, at Toppila, the water height is as low as 27 cm, whereas at Kemi the height of water-level 18^h is 61 cm already, west



Figs. 6—11. Weather maps with distribution of the height of sea-level 1 November, 1934, 10^h, 14^h, 18^h, 22^h and 2 November, 2^h and 6^h, E.E.T.

of Kemi in all likelihood somewhat more. — Fig. 9. 22^h the cold front has travelled entirely to the north of the region of our map. A strong S or SW wind is blowing all over the area of the Gulf of Bothnia. — Figs. 10—11. The comparison of the wind observations and the water height curves indicates that the situations of 2 November 1934 2^h and 6^h may without doubt be considered nearly stationary and so we may use them as basis for the following considerations.

The maps indicate that the rapid rise of water height at Kemi between 18^h and 19^h has taken place just before the turning of the wind from

SE (ESE) to S or SSW which occurred in connection with the cold front. Until to the reach of the stationary situation, the water height has further risen on the northern coast of the Gulf of Bothnia.

The problem of the stress of the wind still deserves great attention. It is our endeavor to derive an empiric formula for the relationship between wind velocity, the slope of the sea-level, and the depth of the sea area, especially for the case when the depth of the sea is considerably smaller than the so-called frictional depth, as the case fairly often is in the sea regions of Finland when a strong wind is blowing.

4. The Influence of the Election of the Depth Value.

PALMÉN and LAURILA (1938), successfully investigating a similar storm, wrote

— Theoretisch liegt es nahe, dass die Beziehung zwischen Windgeschwindigkeit und Windstau eine quadratische ist, und zwar von der Form

(1) $$i = \frac{a}{d} W^2,$$

where i is the slope of the sea-level (nondimensional),
W is wind velocity (cm sec^{-1}),
d is the depth of the sea (cm), and
a a constant (cm^{-1} sec^2).

In this formula, a priori at least, a is a constant that can be computed by means of the slope of the sea-level, ascertained empirically in a stationary case.

In order to reach finally the values of the tangential stress of the wind and the frictional coefficient we have to know, on the one hand, that according to EKMAN (1906)

(2) $$i = \frac{\lambda \tau_a}{g \varrho d},$$

where λ is a nondimensional constant, of magnitude 1 so that, if the depth of the sea d is equally great or greater than the so-called frictional depth D, the value of the constant is about 1, otherwise it is greater;
τ_a is the tangential stress exerted by the wind against the sea-

level (g cm^{-1} sec^{-2}),
g acceleration of gravity, and
ϱ density of water.

In our case we can set $g\varrho = 10^3$ g cm^{-2} sec^{-2}.

EKMAN's formula assumes that the depth of the sea is the same over the whole area under consideration. It is just here that, the greatest difficulty lies, our sea area being by no means constant in its depth. However, the entire value of the results depends upon the right choice of the depth value. The difficulties arising here are indicated, for example, by the following:

1) The average depth of the south part (Selkämeri, Bottenhavet) of the Gulf of Bothnia is 63 m, according to WITTING (1908).

2) PALMÉN and LAURILA (1938) discuss this difficulty in a study of great value:

— Offenbar kann dieser Wert nicht ohne weiteres benutzt werden. Wahrscheinlich hat man mit einem kleineren Wert als mit der in gewöhnlicher Weise bestimmten mittleren Tiefe zu rechnen. Wir benutzen den Ausdruck

$$d' = n \frac{1}{\sum \frac{1}{d_n}}, \tag{3}$$

wo d_n die als konstant angenommene Tiefe in einem gewissen kleinen Teilgebiet bezeichnet. Es ist natürlich unmöglich einen exakten Wert für d' in unserer Form zu erhalten; eine Schätzung ergibt, dass für das hier behandelte Meeresgebiet eine Reduktion der mittleren Tiefe mit wenigstens 20 % nötig ist, um die Einwirkung der variablen Tiefe zu eliminieren. Unter solchen Verhältnissen könnte man also mit einer Tiefe von etwa 50 m in den obigen Gleichungen rechnen.

3) The mean depth of the longitudinal section in the direction of the slope of area A (Fig. 12), which we have chosen to represent the open sea area of the south part of Gulf of Bothnia, is 72.5 m.

4) The mean depth of the longitudinal section (in the direction of the greatest slope), the length of which section has here been received from the triangle Björn-Draghällan-Mäntyluoto, is 48 m.

5) The depth value computed from the last-mentioned longitudinal section by using formula (3) would be 21.8 m.

These varying depth values show how difficult, although seemingly unimportant, this task is. To avoid this difficulty we have, in our considerations, tried to chooze the sea areas under inspection so that their depths do not vary very much, and so we have found it possible to use the mean depth of longitudinal section in the direction of the greatest slope as the depth value.

An additional difficulty is worth mentioning and e.g. HELLSTRÖM (1940) has paid attention to it when he writes that it may be concluded that the values of the wind force previously computed for the Baltic are somewhat high owing to the stratification of the water. This complication occurs no more in the beginning of November 1934, but it has probably still had a small influence by the beginning of October 1936 (cf. PALMÉN and LAURILA, 1938). As, in their case, the sea area under consideration, especially in the direction of the greatest slope, varies on the depth range 0—60 m, we have — in a comparison calculation in the end of this article — used value $^1/_2\,(48 + 21.8) = 34.9$ m.

5. Consideration of the Relationship between the Wind and the Heights of Water-Level.

The object of our consideration is 1934 November 2. 2^h and 6^h, on the one hand, and, on the other, A) the open sea area of the south part of the Gulf of Bothnia (Selkämeri, Bottenhavet), B) the narrow sea area between the two wider parts of the Gulf of Bothnia (Merenkurkku, Kvarken), C) the open sea area of the north part of the Gulf of Bothnia (Perämeri, Bottenviken), and D) the shallow northern part of the Perämeri (Bottenviken). (Fig. 12 shows the areas.) Table 1 contains the primary material of our consideration. The static effect of pressure has already been eliminated from the slope values.

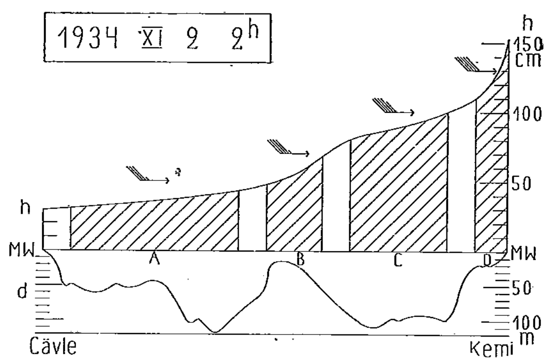


Fig. 12. Upper part: the slope of the sea-level. Lower part: the corresponding depths of the sea areas.

Table 1. The Primary Material of our Consideration.

	i	d (cm)	W (cm sec^{-1})	$a = \frac{i\,d}{W^2}$ (cm^{-1} sec^2)	D (cm)	$D - d$ (cm)
2^h A)	5.5×10^{-7}	7,250	1,250	2.55×10^{-9}	10,150	2,900
2^h B)	29.4×10^{-7}	2,650	1,820	2.35×10^{-9}	14,650	12,000
2^h C)	12.2×10^{-7}	9,900	2,150	2.61×10^{-9}	17,200	7,300
2^h D)	79.0×10^{-7}	1,500	2,350	2.15×10^{-9}	18,700	17,200
6^h A)	6.8×10^{-7}	7,250	1,250	3.16×10^{-9}	10,150	2,900
6^h B)	25.0×10^{-7}	2,650	1,680	2.35×10^{-9}	13,550	10,900
6^h C)	12.3×10^{-7}	9,900	1,990	3.08×10^{-9}	15,900	6,000
6^h D)	85.7×10^{-7}	1,500	2,350	2.33×10^{-9}	18,700	17,200

The computation of the constant a happens by starting from the equation $i = a' W^2$ assuming that a' has the form $a' = \frac{a}{d} + b$.

Here we come to the formula

$$(4) \qquad i = \frac{2.68 \times 10^{-9}}{d} W^2.$$

In »the storm of the year 1872» (COLDING, 1881), treated by EKMAN (1906), the value of the constant in the formula was 4.8×10^{-9}. In the case treated by PALMÉN-LAURILA (1938), again, 3.15×10^{-9}. These differences can depend a) upon the too small depth of the sea areas considered in comparison with the frictional depth and upon the different methods of paying attention to this fact, b) upon the unsatisfactory values of the »mean depth», c) upon the incorrect wind observations, d) upon the different homogeneity conditions of the water and e) upon the non-stationary conditions.

The values of the frictional depth have been put down in the table. They have been obtained by using the formula

$$(5) \qquad D = \frac{7.6\,W}{\sqrt{\sin \varphi}}.$$

The table gives values for the quantity $D - d$, too, which shows how much smaller the depth of the sea has been than the frictional depth.

The relationship between »constant» a and quantity $D - d$ - assuming that the depth of the sea area under consideration d is much smaller than the frictional depth D - may be expressed in the following form

$$a = 1.9 \times 10^{-9} + \frac{4.98 \times 10^{-6}}{D - d},$$

which result can be substituted in formula (1).

However, we arrive at a simpler result by finding the »constant» in formula (1) as a function $a = a(W)$. This relationship, too, is apparently inverse,

$$a = \frac{10^{-6}}{W} + 1.87 \times 10^{-9},$$

the substitution of which gives finally the result

$$i = \frac{1.87 \times 10^{-9} W^2}{d} + \frac{10^{-6} W}{d}. \tag{6}$$

PALMÉN (1936) finds for the sea area under his consideration, a formula for the case with weak winds:

$$i = 0.21\, W + 0.032\, W^2,$$

where the slope of the water-level has been given in units cm per 100 km and the wind velocity in units m per sec. — If we in our formula (6) use the same units and substitute for the depth, not the real mean depth, but, on trial, the depth value 50 m, we obtain

$$i = 0.20\, W + 0.037\, W^2.$$

In our case, the formula we reached as result of the consideration of strong winds resembles the result, at which PALMÉN arrived by considering weak winds, with the difference, however, that, in our case, the second order term begins to dominate earlier. Our considerations, however, have been concentrated in the first place on areas in which the depth is very much smaller than the frictional depth.

Formula (6) can also be written in the form (Fig. 13)

$$\text{(6 a)} \qquad i = \frac{1.87 \times 10^{-9}\, W^2}{d} \left(1 + \frac{535}{W}\right).$$

In Fig. 13 the slope of sea-level is the abscissa and the depth of the sea area is the ordinate. Each of the hyperboli represents a different wind velocity (in Beaufort degrees).

Our original eight slope values have been marked on the figure (with small circles) and such is the case with the corresponding values of PAL-

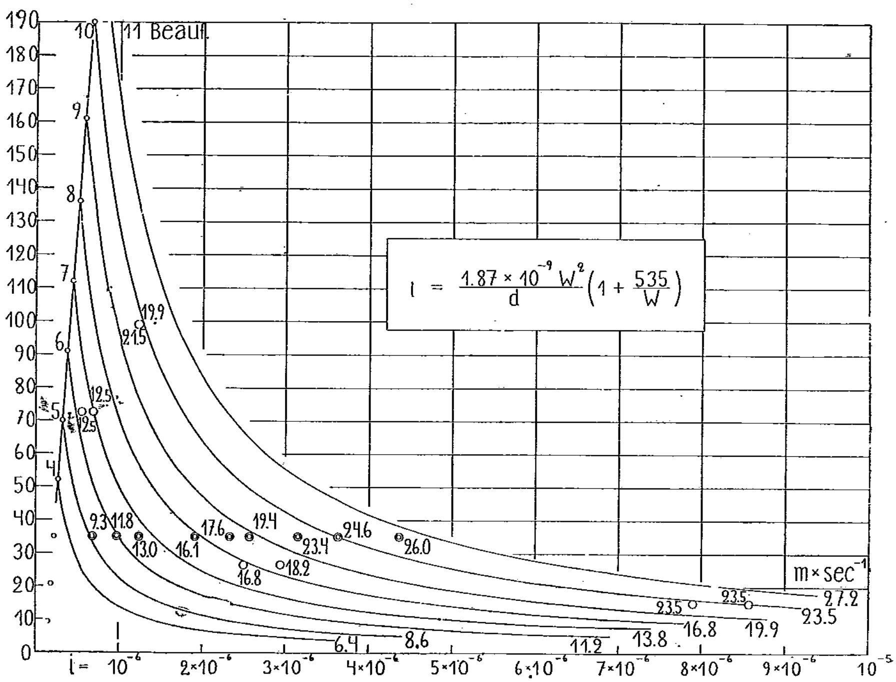


Fig. 13. The relationship between the slope in sea-level, the wind velocity, and the depth of the sea area.

MÉN-LAURILA, too; the latter values have been, however, recalculated by using the approximate depth value mentioned above.

We obtain, furthermore,

$$i = \frac{1.87 \times 10^{-9} W^2}{d} + 1.31 \times 10^{-7} \frac{D}{d} \sqrt{\sin \varphi},$$

from which we can further derive, in the case that the depth of the sea area equals to the frictional depth and especially on the latitude of the Gulf of Bothnia, that

$$(7) \qquad i = \frac{1.87 \times 10^{-9} W^2}{d} + 1.25 \times 10^{-7}.$$

By means of this formula the slopes of sea-level at different strong wind velocities are obtained in those cases when the depth of the sea equals to the frictional depth. The curves in Fig. 13 are on the left bounded by the points, in which, according to formula (7), the depth and the frictional depth meet. The empiric formula (6) is valid in the first place only below the graphical representation of the function $i = i(W, d)$ in formula (7), in other words, on condition that the depth of the sea area is smaller than the frictional depth.

By means of formula (7) the magnitude of the tangential stress of the wind is obtained. According to formula (2)

$$\lambda \tau_a = g\varrho id = 10^3 id.$$

In this case constant $\lambda = 1$, and thus $\tau_a = 10^3 id$. According to TAYLOR (1916)

$$(8) \qquad \tau_a = \varkappa \varrho' W^2 = 1.3 \times 10^{-3} \varkappa W^2,$$

where ϱ' is the density of the air and

$\varkappa$ is the frictional coefficient (nondimensional). We obtain thus further

$$\varkappa = 7.69 \times 10^5 \frac{i\,d}{W^2}.$$

Table 2. The Values of the Frictional Coefficient and Wind Stress.

i (nondim.)	W (Beauf.)	W (cm sec^{-1})	$D = d$ (cm)	$\varkappa$ (nondim.)	τ_a (gcm^{-1} sec^{-2})
6.68×10^{-7}	10	2,350	19,000	1.76×10^{-3}	12.63
5.83×10^{-7}	9	1,990	16,100	1.82×10^{-3}	9.37
5.10×10^{-7}	8	1,680	13,600	1.89×10^{-3}	6.93
4.39×10^{-7}	7	1,380	11,200	1.99×10^{-3}	4.92
3.79×10^{-7}	6	1,120	9,100	2.12×10^{-3}	3.44

Table 2 shows that, in case of wind velocities 10—25 m per sec, the frictional coefficient is about 1.92×10^{-3}. So we obtain

(9) $$\tau_a = 0.0019\, \varrho' W^2 \text{ (g cm}^{-1} \text{ sec}^{-2}).$$

The corresponding result of PALMÉN and LAURILA was

(10) $$\tau_a = 0.0024\, \varrho' W^2.$$

We may assume that this difference is again mostly due to the different ways of the computation of the depth. If we change the depth 50 m into another approximate depth 34.9 m that we suggested above, the stress of wind becomes, instead of formula (10),

(10 a) $$\tau_a = 0.0017\, \varrho' W^2.$$

It might be of interest, finally, to compare how the obtained values of wind stress agree with those of SVERDRUP (1942) and, on the other hand, with those of PALMÉN and LAURILA (1938). SVERDRUP states namely the »Stress of the wind corresponding to stated wind velocities at a height of 15 m and assuming the surface to be rough».

Table 3. Comparison of Results.

Beauf.	10	9	8	7	6
PALMÉN-LAURILA	17.20	12.18	8.80	5.92	3.91
— » —, red.	12.20	8.76	6.24	4.20	2.77
SVERDRUP	15.40	11.00	8.00	5.40	3.50
HELA	12.63	9.37	6.93	4.92	3.44

The values now obtained are considerably smaller than those of PALMÉN-LAURILA. This might be mostly due to the different methods of the computation of the depths and partly also to the different homogeneity conditions of the water. For the sake of comparison, also reduced values which have been obtained by using the smaller depth value, mentioned before as the basis of PALMÉN-LAURILA's considerations, have been included in the table. Also the values given by SVERDRUP are, particularly at greatest wind velocities, greater than those now obtained. The conformity is, however, mediocre, when considering the inaccuracy of the evaluated wind velocities, and also the fact that the results have been reached by altogether different methods.

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