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On the forecasting of frost with Ångström's formula

By

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The nightly temperature change of the air is usually divided into a static and a dynamic part. The static change is determined by effective outgoing radiation, conduction of heat in the ground and exchange of heat between the surface of the earth and the air. Dynamic change, again, depends upon the advection by transport of cold or warm air along the surface of the earth. The first effect always gives a fall of temperature during the night, the second effect may give either a falling or a rising temperature. The exact differentiation of these parts from each other gives rise to great difficulties as, by changing the temperature and humidity conditions of the atmosphere, the advection occasions changes in the intensity of the effective outgoing radiation and thus also exerts an influence on the magnitude of static cooling.

The static cooling of the air at a fixed place can be forecasted fairly accurately with the aid of meteorological measurements made at the said place. Dynamic cooling, again, can be forecasted with satisfactory accuracy with the help of weather charts only.

To determine the static cooling of the air, a number of methods have been derived, the greater part of which are based on temperature and humidity observations made near the earth's surface. Of these methods, the one derived by ÅNGSTRÖM (1) is probably the best, at least in our conditions.

ÅNGSTRÖM (1) takes the formula of Defant (2) for nightly cooling as his starting-point

$$dT = a\,(T - \Theta)\,dt, \tag{1}$$

where dT is the change of temperature at time dt and T the temperature of the air, Θ a function dependent on the absolute humidity of the atmosphere, and a a constant. By integrating the equation and using some approximate expressions for Θ and for the pressure of saturated water vapour, he arrives at the formula for the diurnal minimum temperature (t_m)

$$t_m = C_1\,t_1 - C_0\,t_0 - k \tag{2}$$

where t_1 is the reading of the wet-bulb thermometer and t_0 the reading of the dry-bulb thermometer of psychrometer. The constants C_1, C_0 and k appearing in the equation, depend, according to ÅNGSTRÖM, upon the fact at which times of the day the psychrometer measurements have been made and, furthermore, on the season, cloudiness, and local factors. ÅNGSTRÖM has obtained values for C_1 that vary between 0.83—1.12, and for C_0 values vacillating between 0.12—0.15.

Equation (2) gives reliable values only if the dynamic cooling is weak, in other words, if the air mass in which the psychrometer observations have been made is prevalent until the following morning. Even if this condition is valid, the values of minimum temperature computed according to the equation, however, generally deviate to a certain extent from the corresponding recorded values. In this investigation it is our purpose to find out what these divergencies are due to.

In studying this, recourse was taken to the observations made in May—August 1946—47 at the aerodrome of Malmi ($\varphi = 60°15'$ N; $\lambda = 25°03'$ E).

The aerodrome is situated on the north coast of the Gulf of Finland, about 10 km from the coastal line. There are some hills rising to the height of 50 m to the east and to the south of the place, otherwise the surroundings are fairly flat. The meteorological station is situated on an even base growing short grass on the eastern side of the aerodrome.

To determine the coefficients of equation (2) the humidity observations given by Assman's aspiration psychrometer and the temperature values measured by an ordinary alcoholic minimum thermometer were used.

The measurings took place in the height of 2 meters. The minimum thermometer had been placed in an English screen, the assman outside the screen in its immediate neighbourhood.

As was mentioned above, the application of equation (2) implies that the air mass in which the psychrometer observations have been made remains prevalent until the following morning. In the investigation, therefore, only observations of such days were made use of, during which (taken from 8 hours p.m. until the following morning 8 hours a.m.) the air mass did not change. During the said period altogether 158 such days occurred. Classified according to the prevalent air masses during the days, the days were divided as follows:

Air mass	Number of days
A	26
m P	50
c P	79
T	3

To determine the constants of equation (2), we, according to ÅNGSTRÖM, first set $C_0 = 1$ and then computed C_1 using the method of the least squares. The values obtained with C_1 varied between 0.20—0.35 and were thus remarkably greater than those obtained by ÅNGSTRÖM. The value of this constant was set roundly $C_1 = {}^1/_4$.

Substituting the above values for the constants C_0 and C_1, we obtain

$$(3) \qquad t_m = t_1 - \tfrac{1}{4}\, t_0 - k.$$

As $\frac{1}{4}\, t_0$ is comparatively small, it is to be expected that the accuracy will not notably suffer, if, instead of this equation, another equation

$$(4) \qquad t_m = t_1 - k',$$

is used where k' is a new constant.

To determine the constants k and k', psychrometer measurements made at 8, 14, 17 and 20 hours were used. The constants were computed separately for clear and almost clear days on the one hand, and for half-clouded and clouded days on the other hand. The day was taken as belonging to the former group if the cloudiness, according to observations at 20 *h*, was 0—4, and to the latter group, if the cloudiness was 5—10. The values of k and k' are to be seen in Table 1.

Table 1. Values of constants k and k'.

cloudiness	time	May			June			July			August		
		k	k'	n	k	k'	n	k	k'	n	k	k'	n
0—4	0800	3°.7	6°.9	22	0°.6	5°.2	18	—0°.3	4°.3	22	2°.3	5°.9	29
	1400	4.7	8.4	22	0.6	6.2	17	0.3	5.9	22	2.6	7.8	30
	1700	4.7	8.5	22	0.8	5.9	18	0.3	5.5	21	2.6	7.7	30
	2000	4.5	7.7	22	0.8	6.1	18	0.0	4.9	22	2.6	6.7	30
5—10	0800	1.3	3.2	14	—1.0	2.5	17	—1.1	3.4	22	—0.4	3.4	14
	1400	1.6	4.2	13	—0.1	3.8	18	—0.6	4.6	22	0.5	5.2	14
	1700	2.2	4.6	14	—0.2	3.7	17	—0.5	4.5	22	0.0	4.3	14
	2000	1.8	3.6	14	—0.6	3.0	17	—0.4	3.8	21	—0.4	3.2	14

As is seen in the table, the value of k, to a large extent, depends on the length of the night. In June—July, when the night is the shortest, it is considerably smaller than in May and August. Cloudiness has a noticeable effect on k, too. When cloudiness is 5—10, k is 1—2° smaller than when cloudiness is 0—4. The time of the psychrometer measurements, on the other hand, has no great effect on k. The values computed from the measurements at 14, 17 and 20 hours are almost the same, whereas the values computed from the measurements at 8 h are about 0°.5 smaller than the above values. We may take it that constant k is almost independent on the daily warming and cooling.

Constant k' is naturally somewhat greater than k. It depends on the season rather less, on the cloudiness and on the time of psychrometer measurements, again, more than k.

In order to find out how much the accuracy deteriorates if, instead of equation (3), the simpler equation (4) is used, the standard errors of the minimum temperature (t_m), which were determined according to both equations, were computed. Here the following formula was used:

$$\delta\,(t_m) = \sqrt{\frac{(t_m - \mathrm{t}_m)^2}{n}} \tag{5}$$

where t is the observed minimum temperature and *n* the number of observations. When all the observation material was made use of, the standard error of minimum temperature — computed with the aid of equation (3) — became 1°.93, and the standard error of minimum temperature computed with equation (4) 2°.00. We see that the error increases 0°.07 only if, instead of equation (3), equation (4) is used.

The following table gives according to MEINANDER (3) the values of *k*' computed from the 15 *h* observations which were made at the aerodrome of Turku in 1938—41. For the sake of comparison, also the corresponding values computed from the 14 *h* observation at the aerodrome of Malmi are given there.

Month	V	VI	VII	VIII
Turku	6°.2	3°.9	4°.0	4°.6
Malmi	6.8	5.0	5.2	6.9

We see that the observations at Turku give about 1° smaller values for *k*' on the average than the observations at Malmi. MEINANDER resulted in the value 2°.9 as the standard error of the computed observations, which value is about 1° greater than the one I obtained. This is probably mainly due to the fact that MEINANDER, in his investigation, also considered such days as witnessed a change of the air mass.

ÅNGSTRÖM's equation involves that the vertical distribution of temperature and humidity is approximately normal, because only in that case the psychrometer measurements made near the earth's surface give a true picture of the temperature and humidity conditions prevalent in the atmosphere. The distribution of the meteorological elements in question, however, often considerably differs from the normal and thus occasions an error in the computed values of the minimum temperature.

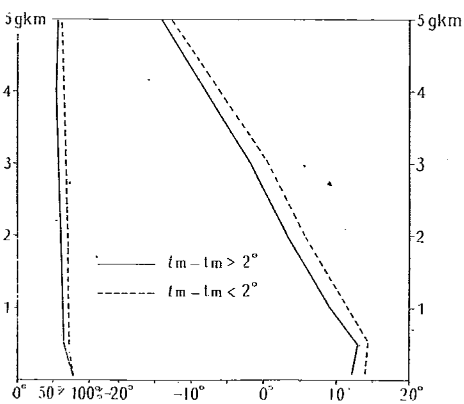
As a certain vertical distribution of temperature and humidity is characteristic of each of the air mass classes, it is to be expected that, in different air masses, different values would be obtained for constant *k*. Owing to the too scanty observation material available for the accurate investigation of this matter, we used the following method: in each air mass group was formed the mean value of the difference between the minimum temperature (t_m) computed with the aid of equation (3) and the min. temperature observed (t_m).

The results were the following:

Air mass	*A*	*mP*	*cP*
$t_m - \mathrm{t}_m$	0°.1	0°.3	— 0°.2

When the values given in Table 1 are used for *k*, we in *A*- and *mP*-masses get slightly too high values for minimum temperature, in *cP* mass again too low values; that is to say, in *A*- and *mP*-masses somewhat greater values should be used for *k*, in *cP*-mass again smaller values. The values of the difference $t_m - \mathrm{t}_m$ are, however, so small that they have no practical significance. This result is, in fact, quite natural, because in the vertical distribution of temperature and humidity in different air masses no great divergencies are traceable in summer. As, in addition, the equilibrium of the air masses in summer is comparatively labile and the vertical mixing therefore pronounced, the temperature and humidity observations made near the surface give a fairly good picture of the conditions prevailing in the whole air mass. In winter, on the other hand, when the air masses are more stable and the stability differences of the air masses greater, *k*, in all probability, depends to a much greater extent on the air mass.

As the grouping according to air mass borders of the differences $t_m - \mathrm{t}_m$ of minimum temperatures led to no result, the following method was further adopted. With the aid of Ilmala's soundings the mean values of temperature and relative humidity at different altitudes were computed during all those days, on which the difference between the minimum temperature, computed from the 8 h observations and the minimum temperature observed, was > 2°, the same was computed as regards those days, during which this difference was < 2°. The figure to the left gives the temperature and humidity values obtained as the function of altitude.



As is seen in the picture, the vertical distribution of temperature is almost the same in both cases. The average temperature gradient from the surface to the height of 5 gkm is 5°.37/km, when the difference t — t_m of the minimum temperatures is $> 2°$, and $5°.45$/km, when this difference is $< 2°$. In the distribution of relative humidity, on the other hand, a small difference is to be observed. On the surface the humidity is in both cases about the same, but, when $t_m - \mathrm{t_m}$ is $> 2°$, the humidity decreases somewhat faster than is the case when $t_m - \mathrm{t_m}$ is $< 2°$. The difference in the distribution of humidity is, however, so small that it only for a small part can explain the fact that the differences of the computed and the observed minimum temperature is not the same in the groups in question.

We thus come to the conclusion that *the consideration of the temperature and humidity conditions prevalent in the upper air layers cannot be expected to improve the accuracy of frost forecasting to any noteworthy degree.*

Ångström's equation implies that dynamic temperature change of the air is so slight that it may be disregarded. Though we in this investigation have used observations of such days only, during which the air mass did not change, it is possible, however, that the temperature of the air mass, due to advection, has changed to a certain degree from the time the psychrometer observations were made until the time the minimum temperature occurred. In addition, some changes in temperature may have taken place, this due to vertical currents and radiation. In order to find out the influence of these temperature changes, the variations of mean temperature of the 3 km-thick, lowest air layer during 24 hours (Δt) were calculated with the aid of soundings carried out at Ilmala, and the differences $t_m - \mathrm{t_m}$ of minimum temperatures — computed according to 8 *h* observations — were grouped according to the values of Δt that were thus obtained. The result was as follows:

Δt	—2°.4	—0°.2	2°.2
$t_m - \mathrm{t_m}$	—0°.2	0°.1	0°.1

As is seen in the table, the values of $t_m - \mathrm{t_m}$ are very small. The temperature changes taking place in a certain air mass seem, accordingly, to have no effect on the minimum temperature. *The minimum temperature is thus, in a way, a rather conservative property of the air mass.*

The nightly temperature in the air layer near the surface quite essentially depends on wind velocity. The weaker the wind, the weaker the transport of heat between the lower and upper air layers, and the stronger the static cooling of the air. The effect of wind on minimum temperature can be considered by writing the equations (3) and (4) into the following form:

$$t_m = t_1 - \tfrac{1}{4} t_0 - k_0 + \Delta k \tag{5}$$

$$t_m = t_1 - k'_0 + \Delta k \tag{6}$$

where Δk is a function of wind force F. Its value is = 0 when F = 0 Beaufort.

Δk was determined from the 8 h and 20 h observations by using values measured at 20 h for wind force. The result was:

F (Beauf.)	0	1	2	3	4	5
Δk	0	0°.9	1°.5	1°.1	2°.0	2°.6
Δk adjusted	0	0°.8	1°.3	1°.7	2°.1	2°.5

The values of constants k_0 and k'_0 were computed by using the adjusted values of Δk given above. The results are shown in Table 2.

Table 2. Values of constants k_0 and k'_0.

cloudiness	time	May			June			July			August		
		k_0	k'_0	n	k_0	k'_0	n	k_0	k'_0	n	k_0	k'_0	n
0—4	0800	4°.6	7°.3	22	1°.8	6°.4	18	1.0	5°.7	22	2°.8	6°.7	29
	1400	5.6	9.3	22	1.8	7.4	17	1.8	7.3	22	3.5	8.9	30
	1700	5.6	9.4	22	2.1	7.1	18	1.7	6.9	21	3.4	8.5	30
	2000	5.4	8.6	22	2.0	7.3	18	1.4	6.3	22	3.4	7.6	30
5—10	0800	2.1	3.9	14	0.4	3.9	17	0.1	4.4	22	0.1	4.1	14
	1400	2.4	4.9	13	1.3	5.2	18	0.4	5.6	22	1.2	5.9	14
	1700	2.9	5.4	14	1.2	5.2	17	0.5	5.5	22	0.7	5.0	14
	2000	2.6	4.4	14	0.8	4.4	17	0.3	4.9	21	0.4	3.9	14

As is seen in the table, constants k_0 and k'_0 depend on cloudiness to a considerable degree. This dependence was examined more accurately still by grouping the values of k_0 and k'_0 according to cloudiness into 4 groups and computing a mean value for k_c and k'_0 in each group. The results are seen in the following table:

	cloudiness	c—1	2—4	5—7	8—10
k_0	....	3°.2	2°.7	2°.2	0°.2
k'_0	....	7°.8	7°.1	6°.0	3°.8

The table shows that k_0 and k'_0 depend on the cloudiness considerably more when cloudiness is great than when it is small. This is due to the fact that at almost cloudy and fully cloudy weather the clouds are usually thicker and lie lower than is the case at almost clear and half-clouded weather. In the former case, therefore, the clouds have a proportionally greater effect on effective outgoing radiation than in the latter case.

In order to find out how much the consideration of wind improves the accuracy, the standard errors of the minimum temperatures computed with the aid of equations (3) and (4) as well as with equations (5) and (6) are displayed in the following table:

Equation	Standard error
(3) (complete equation without wind correction)	2°.00
(4) (simplified » » » »)	1°.93
(5) (complete equation with wind correction)	1°.85
(6) (simplified » » » »)	1°.80

The consideration of wind thus decreases the standard error 0°.13-0°.15.

Table 3. Standard errors of computed minimum temperatures.

time	May		June		July		August		May—August	
	σ	n	σ	n	σ	n	σ	n	σ	n
0800	1°.4	36	1°.8	35	1°.7	44	1°.8	43	1°.66	158
1400	1.4	35	2.3	35	1.6	44	1.9	44	1.85	158
1700	1.6	36	2.4	35	1.6	43	1.9	44	1.89	158
2000	1.5	36	2.0	35	1.8	43	1.9	44	1.82	158
mean	1.51	143	2.13	140	1.70	174	1.85	175	1.80	632

Table 3 still gives the standard error of minimum temperatures computed from equation (5), separately per each month and time of observation.

The standard error to some degree varies in different months being greatest (2°.13) in June and smallest (1°.51) in May. The standard error seems to be almost independent on the time of observation. We do get, it is true, a somewhat smaller values for the standard error according to 08 *h* June observations than at other times of observations but this is, in all probability, mostly due to casual factors. This result is well in harmony with the result obtained earlier, namely that the minimum temperature is not to any noteworthy degree affected by the warming and cooling phenomena taking place in the air mass.

The standard error of minimum temperatures which are computed according to ÅNGSTRÖM is the greatest at 15 h, which, according to him, is due to the fact that convection is most pronounced just then. It is likely that this factor has a certain effect. The fact that ÅNGSTRÖM, with 21 *h* observations, arrived at a much greatery accuracy than with 15 *h* observations is, however, for the most part due to his considering such days, too, when the air mass changed, because the later the psychrometer observations are made, the smaller is the probability that the air mass would change before the moment of the occurrence of the minimum temperature and, accordingly, the greater will be the accuracy of the computed temperature minima.

As the aerodrome of Malmi is situated at the distance of 10 km only from the coast of the Gulf of Finland, it is to be expected that the land and sea-breeze phenomenon occasions an error to the computed minimum temperatures. In order to elucidate this matter, the difference t_m — $\mathrm{t_m}$ of the computed and observed minimum temperatures were, according to the direction of the wind, grouped in three groups. The first group consisted of the days (24 hours), during which the land- and sea-breeze phenomenon occurred distinct, the second of those, during which the wind blew continuously from the continent (direction of wind NW-NE), and the third of those, during which the wind blew from the sea (direction of wind SW-SE). The following table shows the mean values of t_m — $\mathrm{t_m}$ in each group:

Direction of wind	08	14	17	20
Seabreeze by day, landbreeze at night	—0°.4	0°.0	—0°.1	0°.0
Landwind all through 24 hours . .	—0.8	—1.2	—1.0	—1.2
Seawind » » 24 » . .	0.5	0.5	0.5	0.1

The table shows that the computed and observed minimum temperatures are on land- and sea-breeze days quite compatible with each other. When land-wind prevails, the computed values, again, are about 1° too low; at prevailing sea-wind again about 0°.5 too high. This result is quite surprising and not easily explained. The changes in minimum temperatures are in any case so pronounced that the result must be considered as real.

Constant *k* in Ångström's formula is to a certain extent dependent on the thermal conductivity of the ground. As this quantity to a certain degree changes along with the percentage of moisture of the soil, it is to be expected that the difference $t_m - \mathrm{t_m}$ of the computed and observed temperature values would depend on rain conditions. For the investigation of the dependence, for each day the amount of precipitation measured during that day and the two days preceding it was calculated and the values of $t_m - \mathrm{t_m}$ were grouped according to these amounts of precipitation. No distinct difference in the values of $t_m - \mathrm{t_m}$ could, however, be observed in different groups.

Of the factors affecting the difference between the computed and observed minimum temperatures untreated so far, the actual observation errors, and the micro-variations of temperature and humidity may be mentioned. These micro-variations bring about the fact that psychrometer observations give somewhat casual values. The psychrometer measurements made at the aerodrome of Malmi seem quite reliable. In the observations made with a minimum thermometer, on the other hand, a few errors occur. When comparing them with the minima read from the thermograph charts, we could find differences of even two degrees. The influence of the last-mentioned factors is, however, comparatively small.

As a conclusion of the foregoing we can state the following.

1. Temperature and humidity measurements made near the surface give in the warm season a sufficiently accurate picture of the temperature and humidity conditions prevailing in the atmosphere. The use of aerological measurements does not, therefore, essentially improve the accuracy of frost forecasting.

2. The minimum temperature is practically independent on the temperature variations taking place in the air mass.

3. The value of constant *k* in ÅNGSTRÖM's equation is to a considerable degree dependent on cloudiness and wind velocity. The accuracy of frost forecasting is therefore mainly due to the fact with how great an accuracy the cloudiness and wind conditions of the following night can be forecasted.

4. If the air mass, in which the psychrometer observations have been made, remains prevalent until the following morning, the accuracy of minimum temperature computed according to ÅNGSTRÖM's formula is not to any significant degree dependent on the time of psychrometer measurements.

LITERATURE

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